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An Algorithm for Estimating Minimum Strength of Thin-Walled Structures to Resist Elastic Buckling under Pressure

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Алгоритм оценки минимальной прочности тонкостенных конструкций под давлением по критерию упругого схлопывания

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При проектировании оболочечных конструкций важно учитывать их упругое схлопывание, поскольку оно может привести к разрушению конструкций. В частности, для обеспечения надежности проектирования следует учитывать изменение уровня нагрузки потери устойчивости вследствие уменьшения толщины стенок конструкции. Для решения подобных задач используется конечноэлементный метод длины дуги. Однако с помощью этого метода не всегда можно оценить траекторию изменения нагрузки схлопывания. Метод длины дуги был использован при упругом схлопывании пологого фрагмента сферической оболочки. Предложен новый алгоритм в явной конечноэлементной постановке для оценки минимальной прочности тонкостенных конструкций, согласно которому начальная деформация задается путем прижатия горизонтальной жесткой плиты к верхней точке схлопывающейся конструкции и ее перемещения по вертикали. Предполагается, что метод обеспечит хорошее инженерное решение для оценки минимальной нагрузки, при которой происходит частичное упругое схлопывание оболочечных конструкций общего вида под давлением.

Ключевые слова: тонкостенная конструкция, корпус корабля, напряжение упругого схлопывания, метод длины дуги, метод конечных элементов, пологая сферическая оболочка.

Introduction. Elastic buckling, such as snap-through, that occurs in thin-walled structures subject to pressure is an important issue in the design of thin-walled structures. Radha and Rajagopalan [1] studied and identified failures in shell structures that occur because of elastic buckling. Huang [2] and Thurston [3] pointed out that dispersions in buckling strength caused by reducing the wall thickness of a structure is especially important in terms of design safety.

In general, the arc-length method, which is a finite element method (FEM) of analysis, is applied to solve this problem. We confirmed the effectiveness of the arc-length method in solving the elastic snap-through buckling problem in a partial shallow spherical shell. We focused on the dispersion of the buckling strength and found that the arc-length method is effective in calculating the lowest value of buckling mode in most cases. However, we also confirmed there were some cases in which the calculation cannot be solved. Cerini and Falzon [4] studied and confirmed the reliability of the method. Therefore, we formulated a new algorithm to estimate the minimum buckling load in a shallow partial spherical shell and report it below. This method is expected to be used to estimate the minimum load on the partial elastic snap-through buckling load of general thin-shell structures under pressure.

Figure 1 shows the specific buckling problem studied in this paper. The main aim of this study is to evaluate this problem, which exerts pressure on the upper side of a shallow spherical shell. This subject has already been reported by Huang [2] and Thurston [3]. As Fig. 2 shows, experimental buckling loads are significantly smaller than the numerical solutions given in classical buckling theory. It also shows the dispersion of the experimental results. In other words, this means the experimental results are unstable and the strength of the structure is smaller compared with the theoretical solution, as the wall thickness decreases.

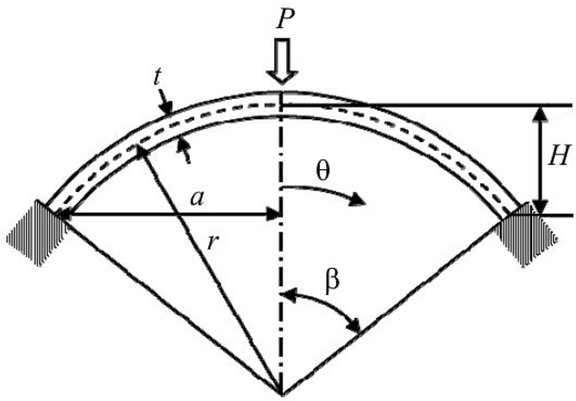

Fig. 1. Geometry of clamped shallow spherical shells.

Judging from our experience, it may lead to a misinterpretation that these differences are considered to be a special case because we cannot say the lower buckling mode never triggers a dynamic collapse deformation at large due to the disruption of the balance of whole structures by the partial elastic snap-through.

In this study, we show that in most cases, the arc-length method is effective in estimating the lower limit of buckling loads as stated earlier. We have derived a solution for this problem by applying the arc-length method (implicit LS-DYNA method) that is easier for designers to use and obtained the same results as those proposed by Thurston [3] regarding converged cases. However, we could not obtain a solution in the case of less thickness and deeper profile.

Figure 3 shows elastic buckling with snap-through. Point A shows the upper limit strength for buckling, point E shows the minimum buckling strength, which is the focus of this study, and point F shows imperfection effects, which have been studied by many researchers [4–7].

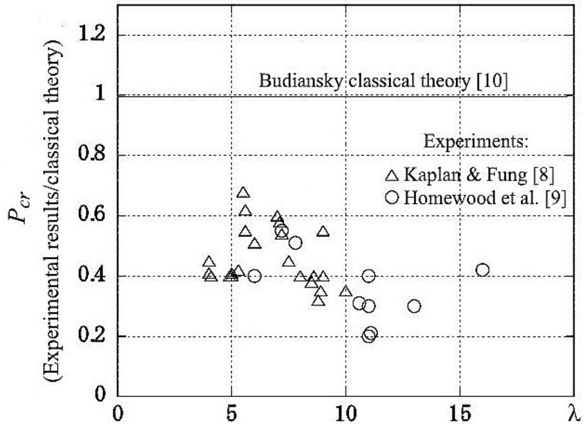

Fig. 2. Calculated solutions for classical theory and experimental results for clamped shallow spherical shells.

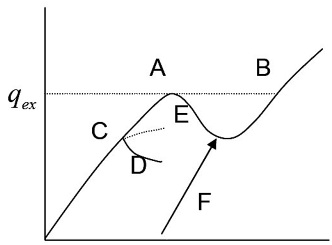

Fig. 3. Pressure–deflection curve of clamped shallow spherical shells.

We have been recognized from our experience that the buckling calculation with changing shape as initial imperfection must be careful.

In this paper, we propose a new algorithm using initial deformation to calculate the minimum buckling strength without changing the initial shape of structures. This method is applied to a case in which pressure is exerted on upper side of a partial spherical shell, as shown in Fig. 1. The aim of this study is also to reduce the number of procedures required for calculating correct bifurcation buckling as marked by points *C* or *D* in Fig. 3. This algorithm assumes a rigid wall in the center of a partial spherical shell that slightly deforms the center in the downward direction as a result of the contact to give an effective initial deformation. Furthermore, this method causes deformations such as snap-through, which are generated by changing the amount of indentation on a rigid wall by the FEM method. That is, we propose an algorithm to calculate the path of *F* in Fig. 3.

1. **Buckling with Snap-Through in a Clamped Partial Spherical Shell**. Huang [2] and Thurston [3] studied and compared theoretical solutions of classical buckling theory and buckling experimental results. They define a, r, t, H, β (angle), and P (distributed load) in Fig. 1.

Value of λ (horizontal axis in Fig. 2) is a standardized shape parameter of Eq. (1) and each variable correspond to the ones in Fig. 1. Value of ν is Poisson's ratio and 0.3 is the value under consideration. The increase of λ corresponds to a reduction in the wall thickness of the partial spherical shell. The actual load value decreases remarkably in comparison with the theoretical value given by the axisymmetric theory and also varies greatly.

Consider Eqs. (1)–(3):

$$\lambda = 2[3(1-\nu^2)]^{0.25}\left(\frac{H}{t}\right)^{0.5}, \tag{1}$$

$$P_{cr} = \frac{q_{ex}}{q_0}, \tag{2}$$

$$q_0 = \frac{2E}{\sqrt{3(1-\nu^2)}}\left(\frac{t}{r}\right)^2, \tag{3}$$

where P_{cr} on the vertical axis is the standardized load parameter defined in Eq. (2), q_{ex} is an experimental value, q_0 is a numerical solution for the classical buckling theory [7], and E is the Young modulus (210 GPa). The other variables also correspond to the letters in Fig. 1.

The reduction in wall thickness corresponds to the increase of λ on the horizontal axis, as shown in Fig. 2. As λ increases, the minimum buckling load of the actual structure becomes notably smaller. This tendency is generally common to the thin-wall structures. As shown in Fig. 3, elastic buckling is triggered when the relative thickness of the structure is decreased, even if structures are made in the same way. The load rarely reaches A as shown in Fig. 3, but snap-through buckling occurs when the load nears C in the figure before reaching A. The path depends on the influence of the initial imperfection, which generates various results. In Fig. 3, F is an example of the path.

2. **Calculation of Minimum Buckling Strength by the Arc-Length Method**. The calculation result is shown below by using λ in Eq. (1), which is affected by shape and thickness parameters in Fig. 1.

Consider Eqs. (4) and (5):

$$x = \frac{p}{q_0}, \tag{4}$$

$$y = \left(\frac{\bar{u}}{t}\right). \tag{5}$$

The objects of the examination are cases in which λ varies from 3.8 to 9.2. We show the results of a study in which it is assumed that r is 200 mm, β is 0.31 rad, p in Eq. (4) is pressure (MPa), $X = x/1.6523$ is a standardized load parameter corresponding to Eq. (2), $\bar{u}$ in Eq. (5) is the downward displacement (mm) of the arc center, and y is a standardized displacement parameter. Thurston calculated the problem under these conditions and reported the results that are shown in Fig. 4. He checked the minimum buckling load value against Eqs. (1)–(3), coordinated the relation to Fig. 2, and showed the results when λ is 6.7 and 8.9 in Fig. 4. However, the scale of the horizontal axis in the study of Thurston is squared λ.

Next, we show the result of applying the arc-length method. Figure 5 shows the convergence results when $\lambda = 3.8$ and 4.6, and the result that did not converge when $\lambda = 6.5$. As far as we determined, the calculation converges when $\lambda \leq 4.6$ and does not converge when $\lambda > 5.3$. The double underlined numbers in the figure are minimum buckling load values. Figure 4 shows these results as a whole. The square symbols in Fig. 4 are the results of a calculation using the arc-length method. Solutions calculated by Thurston in Fig. 4 and the convergence solution obtained by the arc-length method are in good agreement. This is the reason we decide that it is effective to apply the arc-length method to calculate the minimum snap-through buckling load in Fig. 5 when the calculation converges.

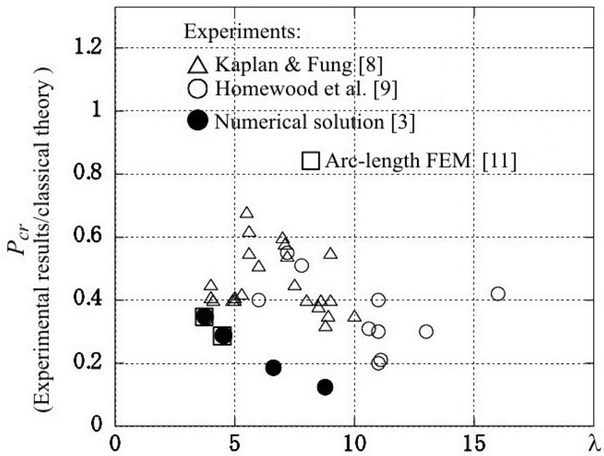

Fig. 4. Calculated lowest buckling pressure for an axisymmetric mode by Thurston and the arc-length solution with the experimental results.

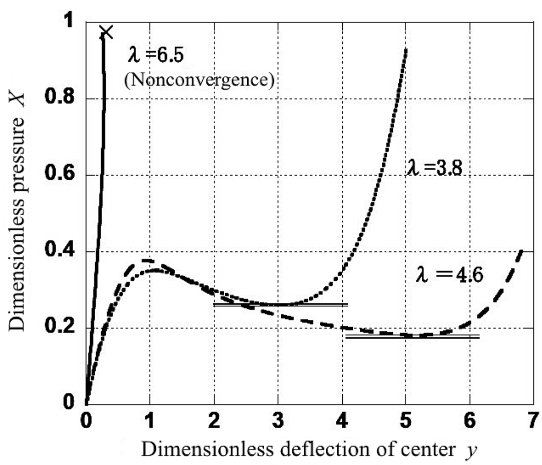

Fig. 5. Calculated critical snap-through arc-length solution results for clamped shallow spherical shells.

Therefore, we investigated whether there is an effective method for a case in which $\lambda > 5.3$. As we described, an analysis of snap-through buckling by applying an explicit method has been studied and we can find many reports regarding these studies. However, these studies focus on the deformation that occurs when load A

jumps or after the load A jumps, and they do not show a method to calculate the minimum snap-through buckling load that is shown as C in Fig. 3.

We have elaborated a new method to calculate the effective minimum snap-through buckling value in the case that the arc-length method does not converge. We present this method in the next section.

3. **Proposed New Algorithm and Evaluation**. As described previously, a calculation performed by using the arc-length method does not converge when $\lambda \geq 5.3$. This means it is impossible to calculate the minimum value that is shown in Fig. 3 as an evaluation value of the buckling load. Here, we propose a novel method, instead of using the arc-length method, which overcomes this challenge and calculates the lower value as described in next section.

3.1. ***Algorithm***.

(i) Calculate the 1st eigenmode under the pressure loading to obtain a point of maximum displacement and its unit vector. (This procedure is not included in this study.)

(ii) A rigid wall (Fig. 6) is placed in contact with the maximum displacement point, which was obtained in (i). Match the movement vector with the unit vector and move the rigid wall slightly. The movement velocity is given incrementally in one second, that is, it should be a quasi-static loading. This calculation is conducted using the explicit method. A specific value is decided as the movement distance, which is from almost 1 to 6 times the wall thickness.

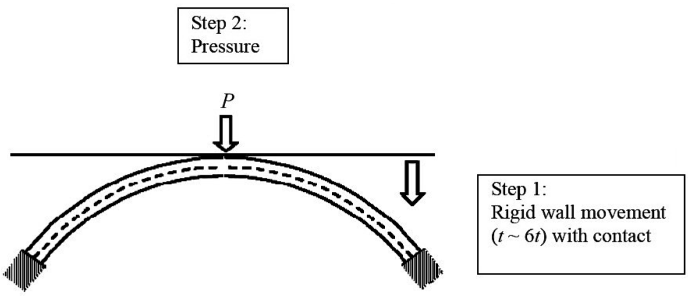

Fig. 6. Scheme of the proposed method.

(iii) A pressure load is given incrementally in one second, that is, it should be a quasi-static loading.

(iv) Plot the results of (iii) against the maximum displacement obtained in (i). The load value when snap-through buckling occurs is decided as the evaluation load.

(v) Change the movement distance and then iterate from (ii) to (iv). This calculation omits the procedure to obtain the 1st eigenmode because it is clear that the mode by which the center of Fig. 1 moves the furthest downward is the 1st eigenmode.

3.2. ***Evaluation of Cases Applicable for the Arc-Length Method***. Here, we show that the minimum snap-through buckling strength agrees with the solution in Fig. 4 as Thurston calculated [3] and the results calculated by the arc-length method. Figure 7 shows the results of a calculation in which $\lambda = 3.8$, which agrees with the results calculated by Thurston and the results calculated by the arc-length method. Value of t in Fig. 7 indicates the pushed distance described in Section 3.1.

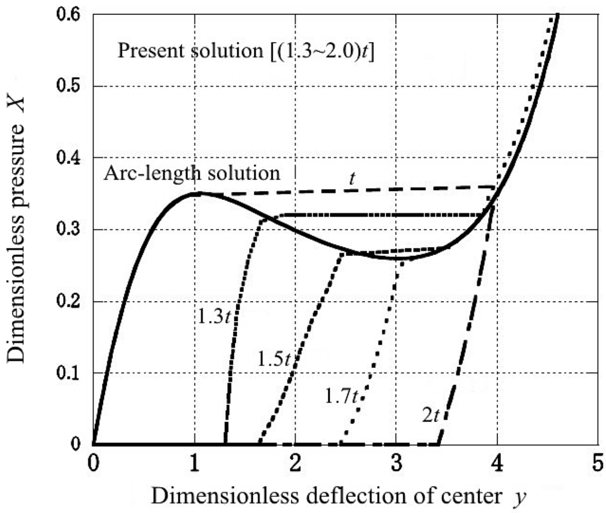

Fig. 7. Calculated critical snap-through arc-length solution results for $\lambda = 3.8$ by present solutions (1.3*t*, 1.5*t*, 1.7*t*, and 2*t*), explicit solution (*t*), and arc-length solution for clamped shallow spherical shells.

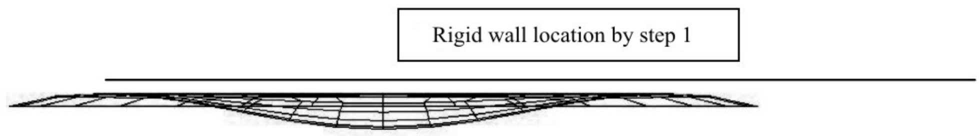

Fig. 8. Buckling shape of the top of a spherical shell by the procedure outlined in step 1 of the proposed method for the solution (2*t*).

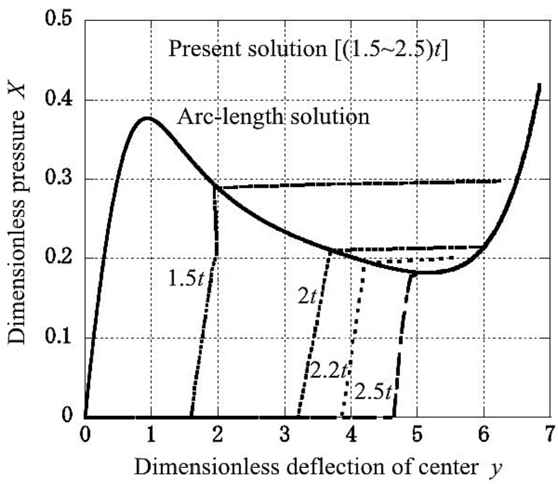

Fig. 9. Calculated critical snap-through arc-length solution results for $\lambda = 4.6$ by the proposed procedure [(1.5–2.5)*t*] and the arc-length solution for clamped shallow spherical shells.

The point of which $X = 0$ in Fig. 7 is non-displacement of the center point when the rigid wall force into the structure with the given distance. However the transverse value of the figure does not correspond to pushed distance because elastic buckling has already occurred at this time. Figure 8 shows a deformed state. The maximum displacement of the center in Fig. 8 corresponds to y in Fig. 7.

Figure 9 shows the results for $\lambda = 4.6$. They are the same results as when $\lambda = 3.8$. As described above, we can show the same results as those calculated by the arc-length method when $\lambda = 3.8$ and 4.6, and therefore, not only the arc-length method but also the proposed method can evaluate the minimum snap-through buckling strength.

3.3. ***Evaluation for Cases Not Applicable for Arc-Length Method***. Next we will show that this calculation method can be applied to the case that cannot be solved by using the arc-length method. Figure 10 shows the calculation results for $\lambda = 9.2$. This shows that this calculation method can be applied in a case where the wall thickness is very low. As described above, the shell thickness is considerably low when $\lambda \geq 4.6$. Figure 11 combines the results from both Figs. 4 and 10.

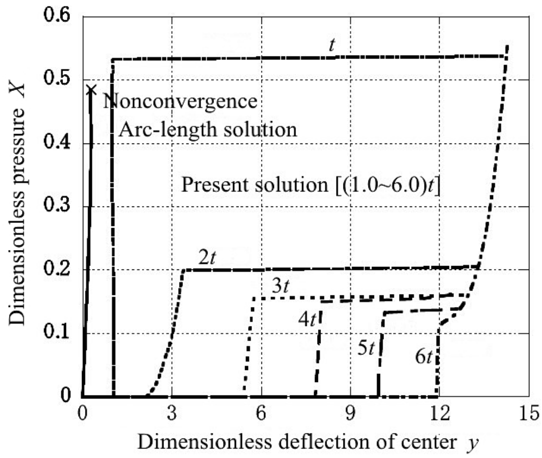

Fig. 10. Calculated critical snap-through arc-length solution results for $\lambda = 6.5$ by the proposed procedure [$(1.0–6.0)t$] and arc-length solutions (nonconvergence) for clamped shallow spherical shells.

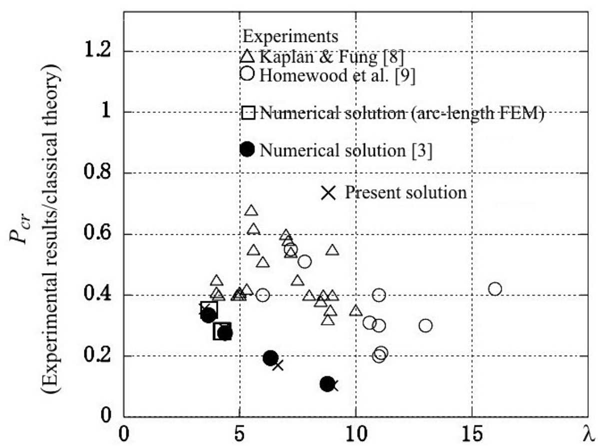

Fig. 11. Calculated lowest buckling of solutions, and arc-length proposed method and experimental results.

In such cases, the current method of estimating the minimum strength of elastic snap-through buckling is inadequate and the proposed method is effective as an analytical method.

Conclusions. We proposed a method to estimate the minimum buckling load of the partial snap-through that occurs in thin-wall structures such as marine structures that are subject to external pressure. We investigated the method by solving the minimum buckling strength of elastic snap-through that occurs in clamped thin partial spherical shell structures under external pressure. In addition, we presented a study of current theoretical analysis, the experimental results and the relation between FEM analysis by the arc-length method and theoretical analysis and related experiments to show the effectiveness and limitations of the arc-length method. This study shows that the proposed method can be applied even when the arc-length method does not converge and allows one to reduce the wall thickness to make light products. We expect the proposed method to be applied as a practical method.

Резюме

При проектуванні оболонкових конструкцій важливо враховувати їх пружне схлопування, оскільки воно може призвести до руйнування конструкцій. Зокрема, для забезпечення надійності проектування необхідно враховувати зміни рівня навантаження втрати стійкості внаслідок зменшення товщини стінок конструкції. Для розв'язання подібних задач використовується скінченноелементний метод довжини дуги. Однак за допомогою цього методу не завжди можливо оцінити траєкторію зміни навантаження схлопування. Метод довжини дуги використовували при пружному схлопуванні пологого фрагменту сферичної оболонки. Запропоновано новий алгоритм у явній скінченноелементній постановці для оцінки мінімальної міцності тонкостінних конструкцій, згідно з яким початкова деформація задається шляхом притиснення горизонтальної жорсткої плити до верхньої точки конструкції, що схлопується, та її переміщення по вертикалі. Припускається, що метод забезпечить добрий інженерний розв'язок для оцінки мінімального навантаження, за якого відбувається часткове пружне схлопування оболонкових конструкцій загального вигляду під тиском.

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