M.I.A.Othman[1], R. M. Farouk[2], H.A. El Hamied[3]

THE EFFECT OF MAGNETIC FIELD AND THERMAL RELAXATION ON 2-D PROBLEM OF GENERALIZED THERMOELASTIC DIFFUSION

[1, 2, 3] Zagazig University, P.O. Box 44519, Zagazig, Egypt.
E-mail: m_i_othman@yahoo.com, rmfarouk1@yahoo.com, hemat7000@yahoo.com

Abstract. The generalized magneto-thermoelasticity is developed. The formulation is done under two theories: the generalized thermoelasticity coupled theory and Lord – Shulman theory with one relaxation time. The normal mode analysis is used to obtain the expressions for temperature, displacement components, the thermal stresses distributions and concentration of diffusion. The variations of the considered variables are represented graphically. A comparison is made with the results predicted by two theories in presence and absence of the magnetic field.

Key words: Generalized thermoelasticity, thermoelastic diffusion, Lord – Shulman, electromagnetic field.

§1. Introduction.

The propagation of waves in thermoelastic materials has many applications in various fields of science and technology, namely, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessel, aerospace, chemical pipe and metallurgy. The importance of thermal stresses in causing structural damages and changes in functioning of structure is well recognized whenever thermal stress environments are involved. Therefore, the ability to predict electrodynamics stress induced by sudden thermal loading in composite structures is essential for the proper and safe design and the knowledge of its response during the last three decades, non classical theories of thermoelastic so-called generalized thermoelasticity.

Biot [2] developed the coupled theory of thermoelastic to deal with a defect of the uncoupled theory that mechanical causes have no effect on the temperature. However, this theory shares a defect of the uncoupled theory in that it predicts infinite speeds of propagation for heat waves. Lord and Shulman (L – S) [7] introduced the theory of generalized thermoelasticity with one relaxation time for the special case of an isotropic body. This theory was extended by Sherief [20] and Dhaliwal and Sherief [4] to include the anisotropic case. In this theory, a modified law of heat conduction including both the heat flux and its time derivative replaces the conventional Fourier's law. The heat equation associated with this theory is hyperbolic and hence eliminates the paradox of infinite speeds of propagation inherent in both the uncoupled and coupled theories of thermoelasticity. For this theory, Ignaczak [5] studied uniqueness of solution; Sherief [21] proved uniqueness and stability. Othman [13] used L – S theory under the dependence of the modulus of elasticity on the reference temperature in two dimensional generalized thermoelasticity. Othman [14] studied the effect of rotation on plane waves in generalized thermoelasticity with two relaxation times.

The theory of magneto-thermoelasticity is concerned with the influence of the magnetic field on the elastic and thermoelastic deformations of a solid body. This theory has aroused much interest in recent years, because of its application in various branches of science and technology. Electro-magneto-thermoelasticity investigates the interaction between temperature, strain, stress and electromagnetic field in a solid elastic body. The origin of magneto-elasticity as a separate discipline can be traced back to the work of Rikitake [19], who studied the excitation magneto-hydrodynamic waves by the seismic waves which pass through the field core of the earth permeated by geomagnetic field. This in spired Cagniard [3] to suggest the possible influence of the earth's magnetic field on the propagation of the seismic waves and open a new field of research on the magneto-elastic waves in solids. A systematic study in this area started with the important work of Knopoff [6] who developed the basic equation governing the magneto-mechanical interactions in electrically conducting materials and used them to study the influence of the terrestrial magnetic field on the propagation of seismic waves in the interior of the earth. An excellent summary of the work done in magneto-elasticity and magneto-thermoelasticity in the fifties and sixties of the present centuries was gives by Paria [18]. Among the authors whom considered the generalized magneto-thermoelastic equations are Nayfeh and Nasser [8] whom studied the propagation of plane waves in a solid under the influence of an electromagnetic field, Agarwal [1] considered thermoelastic and magneto-thermo-elastic plane wave propagation an infinity elastic medium with two relaxation times, Othman [15] studied electro-magneto-thermoelastic plane waves with thermal relaxation in a medium of prefect conductivity, Sherief and Helmy [23] studied a two-dimensional problem in electro-magneto-thermoelasticity for a half-space, whose surface in subjected to a non-uniform thermal shock and is stress free in the presence of a transverse magnetic field, Othman et al. [16] investigated the effect of diffusion on two-dimensional problem of generalized thermoelasticity with Green – Naghdi theory.

The thermo-diffusion in elastic solids is due to coupling of the fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with environment. Nowacki [9 – 12] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Diffusion can be defined as the random walk, of an ensemble of particles, from regions of high concentration to regions of lower concentration. There is now a great deal of interest in the study of this phenomenon, due to its many applications in geophysics and industrial applications. In integrated circuit fabrication, diffusion is used to introduce "do pants" in controlled amount into semiconductor substrate.

In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors, form the source drain regions in MOS transistors and dope ploy-silicon gates in MOS transistors. In most of these applications, the concentration is calculated using what is known as Fick's law.

This is a simple law that does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the effect of the temperature on this interaction. In this theory, the coupled thermoelastic model is used. This implies infinite speeds of propagation of thermoelastic waves. Sherief et al. [22] developed the theory of generalized thermoelastic diffusion that predicts finite speeds of propagation for thermoelastic and diffusive waves. Sherief and Saleh [24] worked on a one dimensional problem of a thermoelastic half-space with a permeating substance in contact with the bounding plane in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Recently, Othman et al. [17] have studied the dependence of the modulus of elasticity on reference temperature in the theory of generalized thermoelastic diffusion with one relaxation time.

In this paper, we shall formulate the normal mode analysis of a two-dimensional problem of electro-magneto-thermoelasticity under Lord – Shulman theory in a perfectly conducting medium.

The exact expressions for temperature distribution, thermal stress and displacement components are obtained, and represented graphically in the presence and absence of the magnetic field for the different theories.

§ 2. Formulation of the problem.

We consider an isotropic homogeneous, linear, thermally and perfectly conducting elastic medium. The whole body is at constant temperature T_0 and it is acted on throughout by a constant magnetic field $H = (0, H_0, 0)$ which is oriented towards the positive direction of y-axis. We are being our consideration with the linearized equation of electromagnetism, valid for slowly moving media [5]. We assume that all quantities are functions of the coordinates x, z and time t and independent of coordinate y. So the displacement vector will have the components $u_x = u(x,z,t)$ and $u_z = w(x,z,t)$. The electric intensity vector is normal to both that the magnetic intensity and displacement vector. Thus, it has the components

$$E_x = E_1, \quad E_y = 0, \quad E_z = E_3 , \tag{2.1}$$

$$\operatorname{curl} h = J + \varepsilon_0 \dot{E} , \tag{2.2}$$

$$\operatorname{curl} E = -\mu_0 \dot{h} , \tag{2.3}$$

$$E = -\mu_0 \left(\dot{u} \wedge H \right), \tag{2.4}$$

$$\operatorname{div} h = 0 . \tag{2.5}$$

The basic equations for electro-magneto-thermoelastic, in a homogeneous, in a homogeneous, isotropic solid without body forces, can be written as:

1) equation of motion

$$\rho \ddot{u}_i = \sigma_{ij,j} + F_i \quad \left(F_i = \mu_0 \left(J \wedge H \right)_i \right); \tag{2.6}$$

2) strain displacement equation relation

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i} \right); \tag{2.7}$$

3) the constitutive law for the theory of generalized thermoelasticity

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}\left[\lambda e_{kk} - \beta_1 \left(T - T_0\right) - \beta_2 C \right], \tag{2.8}$$

where, σ_{ij} the components of stress tensor and λ, μ are lame's constants, ρ is the density and β_1, β_2 are material constants given by $\beta_1 = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion and $\beta_2 = (3\lambda + 2\mu)\alpha_c$, α_c is the coefficient of linear diffusion expansion;

4) heat conduction equation

$$K T_{,ii} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right)\left[\rho C_E + \beta_1 T_0 e_{kk} + a T_0 C \right], \tag{2.9}$$

where K is the thermal conductivity, τ_0 is the thermal relaxation time, C_E is the specific heat at constant strain, a is a measure of thermo-diffusion effect, T is the absolute temperature, T_0 is a reference temperature assumed to obey the inequality $\left| \left(T - T_0\right)/T_0 \right| << 1$;

5) the equation of diffusion has the form

$$d\,\beta_2 e_{kk,ii} + d\,a T_{,ii} + \dot{C} + \tau \ddot{C} - d b C_{,ii} = 0 , \tag{2.10}$$

where d is the diffusion coefficient, b is a measure diffusion effect, τ is the diffusion relaxation time and C is the concentration of diffusive material in the elastic.

We introduce the displacement potentials $\varphi(x,z,t)$ and $\psi(x,z,t)$ through the relations

$$u = \varphi_{,x} + \psi_{,z}, \quad w = \varphi_{,z} - \psi_{,x} \tag{2.11}$$

$$(\nabla^2 \psi = u_{,z} - w_{,x}, \quad e_{kk} = \nabla^2 \varphi). \tag{2.12}$$

From Eqs. (4), (11) and (12) we have

$$E = \mu_0 H_0 (\dot{w}, 0, -\dot{u}), \tag{2.13}$$

From Eqs. (3) and (13) we have

$$h = -H_0 e_{kk} = -H_0 \nabla^2 \varphi, \tag{2.14}$$

From Eqs. (2.2) and (2.13) we have

$$F_1 = -\mu_0 H_0 \left(\frac{\partial h}{\partial x} + \varepsilon_0 \mu_0 H_0 \frac{\partial^2 u}{\partial t^2} \right); \tag{2.15}$$

$$F_2 = 0; \tag{2.16}$$

$$F_3 = -\mu_0 H_0 \left(\frac{\partial h}{\partial z} + \varepsilon_0 \mu_0 H_0 \frac{\partial^2 w}{\partial t^2} \right). \tag{2.17}$$

From Eq. (2.8), it follow that the components of stress tensor have the form

$$\sigma_{xx} = 2\mu e_{xx} + \lambda e_{kk} - \beta_1 (T - T_0) - \beta_2 C; \tag{2.18}$$

$$\sigma_{zz} = 2\mu e_{zz} + \lambda e_{kk} - \beta_1 (T - T_0) - \beta_2 C; \tag{2.19}$$

$$\sigma_{xz} = 2\mu e_{xz}; \tag{2.20}$$

$$\sigma_{yy} = \lambda e_{kk} - \beta_1 (T - T_0) - \beta_2 C; \tag{2.21}$$

$$\sigma_{xy} = \sigma_{yz} = 0. \tag{2.22}$$

The governing equations can be put in a more convenient form by using the following non-dimensional variables:

$$(x', z') = c_1 \eta_0 (x, z); \ (u', w') = c_1 \eta_0 (u, w); \ t' = c_1^2 \eta_0 t; \ C' = \frac{\beta_2 C}{\rho c_1^2}; \ \theta = \frac{\beta_1 (T - T_0)}{\rho c_1^2};$$

$$\sigma'_{ij} = \frac{\sigma_{ij}}{\rho c_1^2}; \ \tau'_0 = c_1^2 \eta_0 \tau_0; \ \tau' = c_1^2 \eta_0 \tau; \ E' = \frac{\varepsilon \eta_0 E}{\sigma_0 H_0 \mu_0^2 c_1}; \ h' = \frac{\varepsilon \eta_0 h}{\sigma_0 H_0 \mu_0};$$

$$c_1^2 = \frac{(\lambda + 2\mu)}{\rho} \quad \left(\eta_0 = \frac{\rho C_E}{K} \right). \tag{2.23}$$

Using Eq. (2.23) into Eqs. (2.6), (2.9) and (2.10) we have

$$\ddot{u} = \beta_0^2 \nabla^2 u + \left(1-\beta_0^2\right)\frac{\partial e}{\partial x} - \frac{\partial \theta}{\partial x} - \frac{\partial C}{\partial x} - A_0 \frac{\partial h}{\partial x} - B_0 \ddot{u}\,; \tag{2.24}$$

$$\ddot{w} = \beta_0^2 \nabla^2 w + \left(1-\beta_0^2\right)\frac{\partial e}{\partial z} - \frac{\partial \theta}{\partial z} - \frac{\partial C}{\partial z} - A_0 \frac{\partial h}{\partial z} - B_0 \ddot{w}\,; \tag{2.25}$$

$$\left[\nabla^2 - \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\right]\theta = \varepsilon \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) e_{kk} + a_1 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) C\,; \tag{2.26}$$

$$\nabla^2 e + \alpha_1 \nabla^2 \theta + \alpha_2 \left(\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2}\right) C - \alpha_3 \nabla^2 C = 0\,, \tag{2.27}$$

where

$$c_0^2 = \frac{(\lambda + 2\mu)}{\mu}; \quad \beta_0^2 = \frac{1}{c_0^2}; \quad A_0 = \frac{\sigma_0 \mu_0^2 H_0^2}{\varepsilon \rho \eta_0 c_1^2}; \quad B_0 = \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho}; \quad \varepsilon = \frac{\beta_1}{\rho c_E};$$

$$a_1 = \frac{a c_1^2}{c_E \beta_2'}; \quad \alpha_1 = \frac{a \rho c_1^2}{\beta_1 \beta_2}; \quad \alpha_2 = \frac{\rho c_1^2}{\beta_2^2 d \eta_0}; \quad \alpha_3 = \frac{b \rho c_1^2}{\beta_2^2}.$$

Using Eqs. (2.23) into Eqs. (2.18) – (2.21) we have

$$\sigma_{xx} = \frac{\partial u}{\partial x} + \left(1 - 2\beta_0^2\right)\frac{\partial w}{\partial z} - \theta - C\,; \tag{2.28}$$

$$\sigma_{zz} = \frac{\partial w}{\partial z} + \left(1 - 2\beta_0^2\right)\frac{\partial u}{\partial x} - \theta - C\,; \tag{2.29}$$

$$\sigma_{xz} = \beta_0^2 \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right); \tag{2.30}$$

$$\sigma_{yy} = \left(1 - 2\beta_0^2\right)\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) - \theta - C\,. \tag{2.31}$$

From Eqs. (2.11), (2.12) into Eqs. (2.24) – (2.27) we obtain

$$\left[\alpha \nabla^2 - \gamma \frac{\partial^2}{\partial t^2}\right]\varphi = \theta + C\,; \tag{2.32}$$

$$\left[\nabla^2 - \frac{1 + b_0^2}{\beta_0^2}\frac{\partial^2}{\partial t^2}\right]\psi = 0\,; \tag{2.33}$$

$$\left[\nabla^2 - \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\right]\theta = \varepsilon \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\nabla^2 \varphi + a_1 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) C\,; \tag{2.34}$$

$$\nabla^4 \varphi + \alpha_1 \nabla^2 \theta + \alpha_2 \left(\frac{\partial}{\partial t} + \tau \frac{\partial^2}{\partial t^2}\right) C - \alpha_3 \nabla^2 C = 0\,, \tag{2.35}$$

where $\alpha = (1 + A_0 H_0)$, $\gamma = (1 + B_0)$.

The initial conditions of the problem are taken to be homogeneous while the boundary conditions are assumed to be

$$\sigma_{zz}(x,z,t)\big|_{z=0} = f_1(x,t); \ \ \sigma_{xz}(x,z,t)\big|_{z=0} = 0; \ \ \frac{\partial C}{\partial z}\big|_{z=0} = 0; \ \ \frac{\partial \theta}{\partial z}\big|_{z=0} = 0, \tag{2.36}$$

where $f_1(x,t)$ is known function of *x* and *t*.

The solution of considered physical variable can be decomposed in terms of normal modes as the following:

$$\left[\varphi, \psi, \theta, \sigma_{ij}, C\right](x,z,t) = \left[\varphi^*, \psi^*, \theta^*, \sigma_{ij}^*, C^*\right](z)\, e^{(\omega t + ikx)}; \tag{2.37}$$

$$\left(\alpha D^2 - s_1\right)\varphi^* = \theta^* + C^*; \tag{2.38}$$

$$\left(D^2 - m^2\right)\psi^* = 0; \tag{2.39}$$

$$\left(D^2 - s_2\right)\theta^* = s_3\left(D^2 - k^2\right)\varphi^* + s_4\, C^*; \tag{2.40}$$

$$\left(D^2 - k^2\right)^2 \varphi^* + \alpha_1\left(D^2 - k^2\right)\theta^* + \left(s_5 - \alpha_3 D^2\right)C^* = 0, \tag{2.41}$$

where s_n $(n = 1,2,3,4,5)$ and m^2 are defined in the appendix.

Eqs. (2.38), (2.40) and (2.41) form a coupled system of φ^*, θ^*, C^*, while Eq. (2.39) un-coupled, where $D = d/dz$.

Eliminating θ^* and C^* between Eqs. (2.38), (2.40) and (2.41) we obtain

$$\left(D^6 - \ell_0 D^4 + \ell_1 D^2 - \ell_2\right)\varphi^*(z) = 0. \tag{2.42}$$

In a similar manner we arrive at,

$$\left(D^6 - \ell_0 D^4 + \ell_1 D^2 - \ell_2\right)\left(\theta^*, C^*\right)(z) = 0, \tag{2.43}$$

Eq. (2.42) can be factorized as

$$\left[\left(D^2 - k_1^2\right)\left(D^2 - k_2^2\right)\left(D^2 - k_3^2\right)\right]\varphi^*(z) = 0; \tag{2.44}$$

$$\left(D^2 - k_i^2\right)\varphi_i^*(z) = 0, \ \ i = 1,2,3, \tag{2.45}$$

where k_i^2 $(i = 1,2,3)$ are the roots of the following characteristic equation

$$\left(k^6 - \ell_0 k^4 + \ell_1 k^2 - \ell_2\right)\varphi^*(z) = 0. \tag{2.46}$$

The solution of Eq. (2.44)

$$\varphi^*(z) = \sum_{i=1}^{3} \varphi_i^*. \tag{2.47}$$

The solution of Eq. (2.45) as $z \to \infty$ is given by

$$\varphi_i^* = G_i e^{-k_i z}. \tag{2.48}$$

From Eq. (2.48) into Eq. (2.47)

$$\varphi^*(z) = \sum_{i=1}^{3} G_i\, e^{-k_i z}. \tag{2.49}$$

Similarly,

$$\theta^*(z) = \sum_{i=1}^{3} G_i'\, e^{-k_i z}; \tag{2.50}$$

$$C^* = \sum_{i=1}^{3} G_i'' e^{-k_i z} . \quad (2.51)$$

The solution of Eq. (2.39) has the form as $z \to \infty$

$$\psi^* = B\, e^{-mz} , \quad (2.52)$$

where G_i, G_i' and G_i'' are some parameters.

Substituting from Eqs. (2.49) – (2.51) into Eqs. (2.38), (2.40) and (2.41) we get the following relation:

$$G_i' = \frac{\left[s_4 \left(\alpha\, k_i^2 - s_1 \right) + s_3 \left(k_i^2 - k^2 \right) \right]}{\left(k_i^2 + s_4 - s_2 \right)} G_i ; \quad (2.53)$$

$$G_i'' = \frac{\left[\left(\alpha\, k_i^2 - s_1 \right) \left(k_i^2 - s_2 \right) - s_3 \left(k_i^2 - k^2 \right) \right]}{\left(k_i^2 + s_4 - s_2 \right)} G_i . \quad (2.54)$$

Substituting from Eqs. (2.53), (2.54) into Eqs. (2.50), (2.51) respectively, we obtain

$$\theta^* = \sum_{i=1}^{3} H_i G_i e^{-k_i z} \quad (2.55)$$

$$\left(C^* = \sum_{i=1}^{3} R_i G_i e^{-k_i z} \right), \quad (2.56)$$

where H_i and R_i are defined in the appendix.

Using Eqs. (2.37), (2.49) and (2.52) into Eq. (2.11) we can obtain the displacement components u^*, w^*

$$u^* = i\, k \sum_{i=1}^{3} G_i e^{-k_i z} - m\, B\, e^{-mz} ; \quad (2.57)$$

$$w^* = -\sum_{i=1}^{3} G_i k_i e^{-k_i z} - i\, k\, B\, e^{-mz} . \quad (2.58)$$

Using Eq. (2.37) into Eqs. (2.28) – (2.31) we obtain

$$\sigma_{xx}^* = i\, k\, u^* + (1 - 2\,\beta_0^2) \frac{\partial w^*}{\partial z} - \theta^* - C^* ; \quad (2.59)$$

$$\sigma_{zz}^* = \frac{\partial w^*}{\partial z} + i\, k\, (1 - 2\,\beta_0^2) u^* - \theta^* - C^* ; \quad (2.60)$$

$$\sigma_{xz}^* = \beta_0^2 \left[\frac{\partial u^*}{\partial z} + i\, k\, w^* \right] ; \quad (2.61)$$

$$\sigma_{yy}^* = (1 - 2\,\beta_0^2) \left[i\, k\, u^* + \frac{\partial w^*}{\partial z} \right] - \theta^* - C^* . \quad (2.62)$$

From Eqs. (2.55) – (2.58) into Eqs. (2.59) – (2.62) we obtain the components of stress tensor

$$\sigma_{xx}^* = \sum_{i=1}^{3} \left[\left(1 - 2\,\beta_0^2 \right) k_i^2 - k^2 - H_i - R_i \right] G_i\, e^{-k\, z_i} - 2\, i\, m\, k\, B\, \beta_0^2\, e^{-mz} ; \quad (2.63)$$

$$\sigma_{zz}^* = \sum_{i=1}^{3} \left[k_i^2 - \left(1 - 2\,\beta_0^2 \right) k^2 - H_i - R_i \right] G_i\, e^{-k\, z_i} + 2\, i\, m\, k\, B\, \beta_0^2\, e^{-mz} ; \quad (2.64)$$

$$\sigma_{yy}^{*}=\sum_{i=1}^{3}\left[\left(1-2\beta_0^2\right)\left(k_i^2-k^2\right)-H_i-R_i\right]G_i\, e^{-k z_i}\,; \tag{2.65}$$

$$\sigma_{xz}^{*}=\beta_0^2\left[-2i\sum_{i=1}^{3}k_i k\, G_i\, e^{-k z_i}+\left(m^2+k^2\right)B\, e^{-mz}\right]. \tag{2.66}$$

In order to determine the parameters $G_i\ (i=1,2,3)$ substituting into the following boundary conditions at $z=0$.

From Eq. (2.36) into Eqs. (2.55), (2.56), (2.64) and (2.66) we have

$$\sum_{i=1}^{3}A_iG_i+A_4B=f_1^{*}\,; \tag{2.67}$$

$$\sum_{i=1}^{3}k_iG_i+E\,B=0\,; \tag{2.68}$$

$$\sum_{i=1}^{3}B_iG_i=0\,; \tag{2.69}$$

$$\sum_{i=1}^{3}E_iG_i=0\,, \tag{2.70}$$

where A_i, B_i and E_i are defined in the appendix.

Solving Eqs. (2.67) – (2.70) we can obtain the parameters $G_i\ (i=1,2,3)$ and B by using the mat lab program.

§3. Numerical results.

The copper material was chosen for purposes of numerical evaluations. The material constants of the problem are thus given by SI units Thomas (1980);

$$T_0=293K\,;\ \sigma=8954\,\mathrm{Kg/m^3}\,;\ \tau_0=0{,}02s\,;\ \tau=0{,}02s\,;\ C_E=383{,}1\ J/KgK\,;$$

$$\alpha_t=1{,}78(10)^{-5}\ K^{-1};\ K=386\ \mathrm{W/(mK)};\ \lambda=7{,}76(10)^{10}\ \mathrm{Kg/(ms^2)};$$

$$\mu=3{,}86(10)^{10}\ \mathrm{Kg/(ms^2)};\ \rho=8{,}950(10)^{3}\ \mathrm{Kg/m^3};\ \alpha_c=1{,}98(10)^{-4}\ \mathrm{m^3/Kg};$$

$$d=0{,}85(10)^{-8}\ \mathrm{Kg\,s/m^3};\ a=1{,}2(10)^{4}\ \mathrm{m^2/(s^2\,K)};\ b=0{,}9(10)^{6}\ \mathrm{m^5/(Kg\,s^2)}.$$

Figs. 1 – 5 depict the influence of magnetic field on the temperature θ, the components of displacement u, w, the component of stress σ_{zz} and the concentration C under (CD) and (L – S) theories, when $\alpha=1$ (i.e. $H_0=0$), $\alpha=1{,}8$ (i.e. $H_0=10^8$).

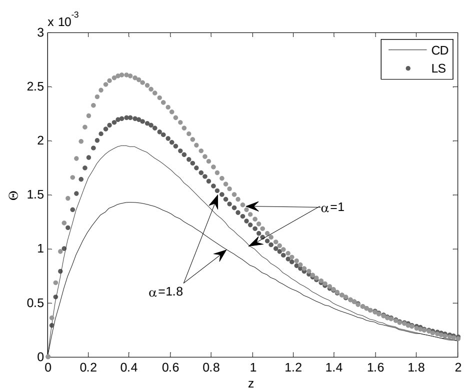

Fig. 1. Variation of the temperature θ for $\tau_0=0$, $\tau_0>0$.

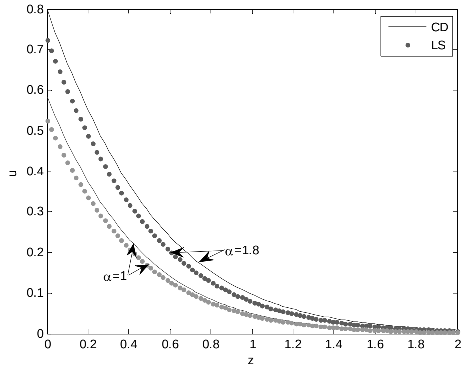

Fig. 2. Variation of the displacement component u *for* $\tau_0 = 0$ *,* $\tau_0 > 0$ *.*

We see from Fig. 1 that the magnetic field has decreasing effect on the temperature and converges to zero with increase the distance *z*.

Figs. 2 and 3 show that the magnetic field has increasing effect on the components of displacement u, w and converges to zero with increase the distance *z*.

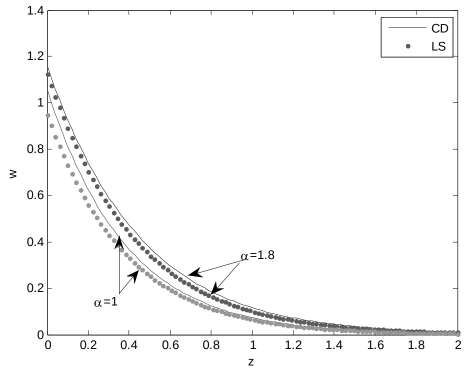

Fig. 3. Variation of the displacement component w *for* $\tau_0 = 0$ *,* $\tau_0 > 0$ *.*

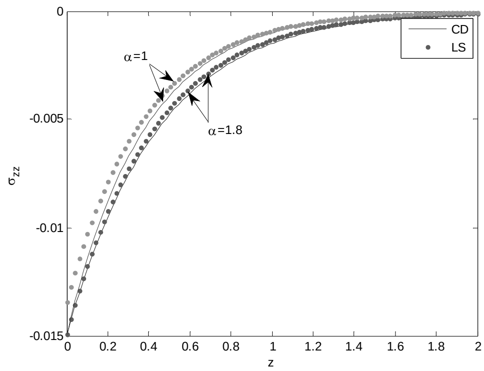

Fig. 4. Variation of the stress component σ_{zz} *for* $\tau_0 = 0$ *,* $\tau_0 > 0$ *.*

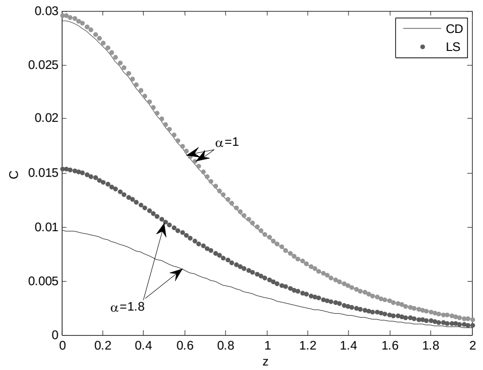

Fig. 5. Variation of the concentration C for $\tau_0 = 0$ *,* $\tau_0 > 0$.

Fig. 4 depicts that the magnetic field has decreasing effect on the stress component σ_{zz} and converges to zero with increase the distance *z*. It is evident from Fig. 5 that the magnetic field has decreasing effect on concentration *C* and converges to zero with increase the distance *z*.

In Figs. 1 – 10, we notice that all the curves converge to zero with increase the distance *z*. Fig. 1, 4 and 5 indicate that the curves under Lord – Shulman theory are greater than those of the coupled theory.

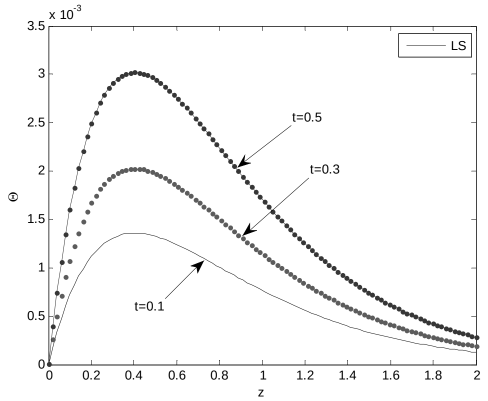

Fig. 6. Variation of the temperature θ *for* $\tau_0 > 0$ *,* $\alpha = 1.8$.

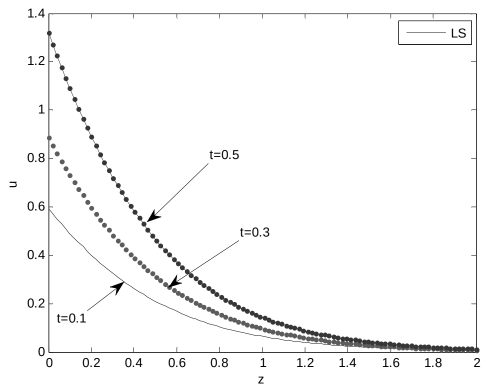

Fig. 7. Variation of the displacement component u *for* $\tau_0 > 0$ *,* $\alpha = 1.8$.

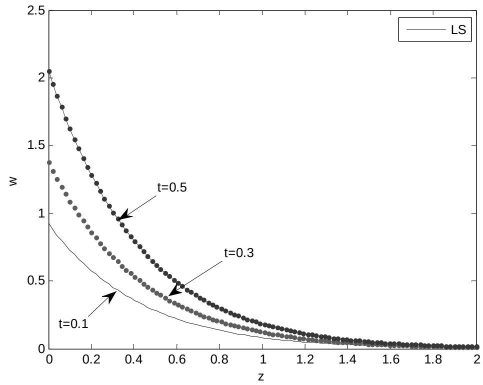

Fig. 8. Variation of the displacement component w *for* $\tau_0 > 0$ *,* $\alpha = 1.8$ *.*

Figs. 2 and 3, show that the curves under the coupled theory are greater than those of Lord – Shulman theory.

The variations of the temperature θ, the components of displacement u, w the component of stress σ_{zz} and the concentration C with the distance z in the presence of magnetic field for different values of time under (L – S) theory are shown in Figs. 6 – 10 at $\alpha = 1{,}8$; $t = 0{,}1$; $t = 0{,}3$ and $t = 0{,}5$. Fig. 6 shows that the temperature θ increases with increase t for $0 < x < 0.4$ but for $0.4 < x$ the temperature decrease with increase t and converges to zero with increasing the distance z.

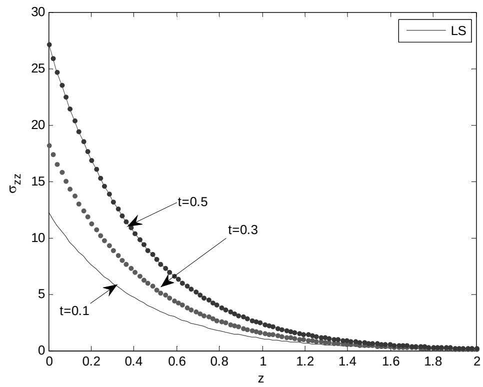

Fig. 9. Variation of the stress component σ_{zz} *for* $\tau_0 > 0$ *,* $\alpha = 1.8$ *.*

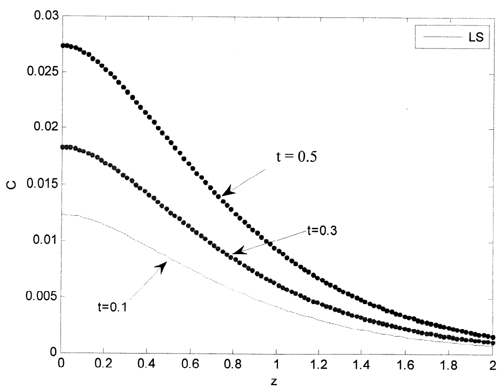

Fig. 10. Variation of the concentration C for $\tau_0 > 0$ *,* $\alpha = 1.8$ *.*

In Figs. 7 – 10 we notice that the components of displacement u, w, the component of stress σ_{zz} and the concentration C are increasing with increase the time t and converges to zero with increase the distance z.

§4. Conclusion.

In this paper, normal mode method is used to study the problem of the effect of magnetic field and thermal relaxation on two-dimensional problem of generalized thermoelastic diffusion under Lord – Shulman theory.

We can obtain the following conclusions according to analysis above.

1. The values of distributions of all physical quantities converge to zero with increasing the distance z.
2. The magnetic field has great influence on the distribution of physical quantities.
3. It is clear from Figs. 6 – 10 that the different values of the time play a significant role in all the physical quantities except temperature.
4. All the physical quantities satisfy the boundary conditions and initial conditions.
5. The curves in the context of (CD) and (L – S) theories decrease exponentially with increase z, this indicates that the thermoelastic diffusion waves are untenanted and non-dispersive. Where purely thermoelastic diffusion waves undergo both attenuation and dispersion.

Appendix.

$$s_1 = \left(\alpha k^2 + \gamma \omega^2\right); \; s_2 = k^2 + \left(\omega + \tau_0 \omega^2\right); \; s_3 = \varepsilon\left(\omega + \tau_0\omega^2\right), \; s_4 = a_1\left(\omega + \tau_0\omega^2\right);$$

$$s_5 = \alpha_2\left(\omega + \tau\,\omega^2\right) + \alpha_3\,k^2\,; \; s = \frac{\left(1 + B_0\right)\varpi^2}{\beta_0^2}; \; m = \sqrt{k^2 + s}\,;$$

$$\ell_0 = \frac{1}{\left(1 + \alpha\alpha_3\right)}\left[2k^2 + s_2 - s_4 - \alpha_3\left(s_1 + s_3 + \alpha\, s_2\right) - \alpha_1\left(s_3 + \alpha\, s_4\right) - \alpha\, s_5\right];$$

$$\ell_1 = \frac{1}{\left(1 + \alpha\alpha_3\right)}\left[2k^2 s_2 + k^4 + s_1 s_5 + s_3\left(\alpha_3 k^2 + s_5\right) + s_2\left(\alpha_3 s_1 + \alpha\, s_5\right) - \right.$$

$$\left. - \alpha_1 s_4\left(\alpha\, k^2 + s_1\right) - 2k^2\left(s_4 + \alpha_1 s_3\right)\right];$$

$$\ell_2 = \frac{1}{\left(1 + \alpha\alpha_3\right)}\left[s_3 s_5 k^2 + s_2 k^4 + s_1 s_2 s_5 - k^4\left(s_4 + \alpha_1 s_3\right) - \alpha_1 s_1 s_4 k^2\right];$$

$$H_i = \frac{\left[s_4\left(\alpha\, k_i^2 - s_1\right) + s_3\left(k_i^2 - k^2\right)\right]}{\left(k_i^2 + s_4 - s_2\right)}, \; R_i = \frac{\left[\left(\alpha\, k_i^2 - s_1\right)\left(k_i^2 - s_2\right) - s_3\left(k_i^2 - k^2\right)\right]}{\left(k_i^2 + s_4 - s_2\right)};$$

$$A_i = \sum_{i=1}^{3}\left[k_i^2 - \left(1 - 2\beta_0^2\right)k^2 - H_i - R_i\right]; \; A_4 = 2\,i\,m\,k\,\beta_0^2\,; \; B_1 = k_1 H_1\,; \; B_2 = k_2 H_2\,;$$

$$B_3 = k_3 H_3\,; \; E_1 = k_1 R_1\,; \; E_2 = k_2 R_2\,; \; E_3 = k_3 R_3 \text{ and } \; E = \frac{i}{2k}\left(k^2 + m^2\right).$$

Р Е З Ю М Е . Розвинуто узагальнену теорію магнітотермопружності. Сформульовано задачу в рамках двох підходів: теорії узагальненої зв’язаної термопружності та теорії Лорда – Шульмана з одним часом релаксації. Використано аналіз нормальної моди для отримання виразів для температури, компонентів зміщень, розподілу температурних напружень та концентрації дифузії. Зміну вказаних вище змінних представлено графічно. Порівняно результати в рамках обох теорій за умови наявності та відсутності магнітного поля.

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