SYMMETRIC AND DIPOLAR ELECTROMAGNETIC WAVES IN COAXIAL STRUCTURE FILLED BY NON-UNIFORM DISSIPATIVE PLASMA WITH AZIMUTH MAGNETIC FIELD

V.P. Olefir, A.E. Sporov

V.N. Karazin Kharkov National University, Institute of High Technologies, Department of Physics and Technology, Kharkov, Ukraine
E-mail: vpolefir@gmail.com

The aim of this report is the investigation of influence of effective collision rate, the value of direct current that flows along the central conductor of the waveguide structure, waveguide geometric parameters and plasma density radial profile on the dispersion, spatial attenuation and radial wave field structure of high-frequency electromagnetic waves. It was shown that mentioned parameters can be used for effective control of the dispersion and attenuation of the studied waves.
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1. INTRODUCTION

At present time it has been carried out the intensive study of electrodynamics properties of coaxial plasma-metal waveguide structures that are widely used in the devices of plasma electronics [1] and also as discharge chambers for gas discharge sustaining [2,3]. The properties of waves that propagate in such waveguides are determined by the azimuth wave field structure [4]. The theoretical study of dispersion and spatial attenuation of electromagnetic eigen waves with complex azimuth wave field structure that propagate along coaxial structure partially filled by dissipative non-uniform plasma with central metal conductor is insufficient.

2. TASK SETTING

The considered waveguide system consists of the metal rod of radius R_1, which is placed at the axis of waveguide system that is enclosed by the cylindrical plasma layer of radius R_2. The vacuum gap $(R_2 < r < R_3)$ separates the plasma layer from outer waveguide metal wall with radius R_3. The direct current J_z flows along the inner metal rod and creates radially non-uniform azimuth magnetic field $H_0(r)$. Plasma was considered in the hydrodynamic approach as cold slightly dissipative medium with constant effective collision rate ν ($\nu/\omega < 1$, where ω is wave frequency). It was supposed that plasma density vary slightly along the plasma column and possesses radial profile $n(r)$ in the bell-shaped form: $n(r) = n(r_{\max})\exp\left(-\delta(r - r_{\max})^2 / r_\delta^2\right)$. The non – uniformity parameter δ describes the plasma density shape and varies from $\delta = 0$ (radially uniform profile) to $\delta = 1$. The parameter $r_{\max}$ is radial coordinate, where plasma density culminates its maximum, and parameter r_δ characterizes the width of bell-shaped profile. The components of permittivity tensor of magnetized collisional plasma $\varepsilon_{1,2,3}$ depend on radial position r and can be found in [5].

The solution of the Maxwell equations in the cylindrical coordinates can be found in the form:

$$E, H = E(r), H(r)\exp\left(i[k_3 z + m\varphi - \omega t]\right), \qquad (1)$$

where k_3 is complex axial wave number, m is azimuth wave number.

In the region of cylindrical plasma layer ordinary differential equations that govern tangential wave components of the considered wave can be written in the following form:

$$\begin{cases} \dfrac{dE_\varphi}{dr} = -\dfrac{E_\varphi}{r} + F_1 H_\varphi - F_2 E_z + F_3 H_z \\ \dfrac{dH_\varphi}{dr} = -F_4 E_\varphi - F_5 H_\varphi + F_6 E_z - F_2 H_z \\ \dfrac{dE_z}{dr} = -F_7 H_\varphi - F_8 E_z - F_1 H_z \\ \dfrac{dH_z}{dr} = -F_9 E_\varphi + F_4 E_z \end{cases}, \qquad (2)$$

here $F_1 = i\dfrac{m}{r}\dfrac{k_3}{k\varepsilon_1}$, $F_2 = \dfrac{m}{r}\dfrac{\varepsilon_2}{\varepsilon_1}$, $F_3 = i\left(k - \dfrac{m^2}{r^2}\dfrac{1}{k\varepsilon_1}\right)$,

$F_4 = i\dfrac{m}{r}\dfrac{k_3}{k}$, $F_5 = \dfrac{1}{r} - k_3\dfrac{\varepsilon_2}{\varepsilon_1}$, $F_6 = \dfrac{i}{k}\left(\dfrac{m^2}{r^2} + k^2\dfrac{\varepsilon_2^2 - \varepsilon_1^2}{\varepsilon_1}\right)$,

$F_7 = \dfrac{i}{k\varepsilon_1}\left(k_3^2 - k^2\varepsilon_1\right)$, $F_8 = k_3\dfrac{\varepsilon_2}{\varepsilon_1}$, $F_9 = \dfrac{i}{k}\left(k_3^2 - k^2\varepsilon_3\right)$,

$k = \omega / c$ is the vacuum wave number. The solutions of this system for arbitrary parameters can be found only with the help of numerical methods.

In the vacuum region the system of Maxwell equations possesses analytical solutions expressed in terms of modified Bessel functions. These solutions contain constants that can be obtained with the help of boundary conditions consisting in continuity of tangential wave field components at plasma – vacuum interface:

$$\begin{cases} C_1 = A_1 H_\varphi^p(R_2) - A_2 E_z^p(R_2) - A_3 H_z^p(R_2) \\ C_2 = -A_4 H_\varphi^p(R_2) + A_5 E_z^p(R_2) + A_6 H_z^p(R_2) \\ C_3 = -A_1 E_\varphi^p(R_2) + A_3 E_z^p(R_2) - A_2 H_z^p(R_2) \\ C_4 = A_4 E_\varphi^p(R_2) - A_6 E_z^p(R_2) + A_5 H_z^p(R_2) \end{cases}, \qquad (3)$$

here $A_1 = i\dfrac{\Delta\kappa_v}{k}K_m(\Delta)$, $A_2 = \Delta K_m^{'}(\Delta)$, $A_3 = i\dfrac{mk_3}{k}K_m(\Delta)$,

$A_4 = i\frac{\Delta\kappa_v}{k}I_m(\Delta)$, $A_5 = \Delta I_m^{'}(\Delta)$, $A_6 = i\frac{mk_3}{k}I_m(\Delta)$, $\Delta = \kappa_v R_2$, $\kappa_v^2 = k_3^2 - k^2$, $E_z^p(R_2)$, $H_z^p(R_2)$, $E_\varphi^p(R_2)$, $H_\varphi^p(R_2)$ are the values of wave field components at plasma – vacuum interface, obtained by the numerical integration of the system (2), prime denotes the derivative with respect to the argument.

The boundary condition for $E_z(r)$ and $E_\varphi(r)$ wave field components at the waveguide metal wall $r = R_3$ gives the dispersion equation in such form:

$$\begin{cases} C_1 I_m(\kappa_v R_3) + C_2 K_m(\kappa_v R_3) = 0 \\ C_3 I_m^{'}(\kappa_v R_3) + C_4 K_m^{'}(\kappa_v R_3) = 0. \end{cases} \quad (4)$$

3. MAIN RESULTS

The main attention in this report was focused on the properties of the symmetric wave with $m = 0$ because this wave is most frequently used in different practical applications [1-4]. In this case the system of Maxwell equations breaks up into two independent subsystems for H and E – waves. Let's study the properties of the E – wave that widely used in practice and possesses three components E_z, E_r and H_φ.

The influence of effective collision rate, direct current value and waveguide geometric parameters on the dispersion and attenuation of the waves considered was studied for the case of radially uniform plasma. For the considered set of problem's parameters the dispersion equation for the symmetric E –wave possesses only one solution.

The influence of the effective electron collision frequency ν on the spatial attenuation coefficient $\alpha = \mathrm{Im}(k_3)R_1$ is shown on the Fig. 1.

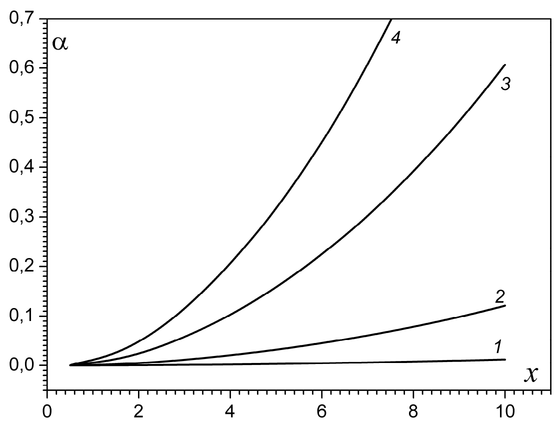

Fig. 1. The dependence of spatial attenuation coefficient $\alpha = \mathrm{Im}(k_3)R_2$ on dimensionless wave number $x = \mathrm{Re}(k_3)R_2$. Numbers 1-4 correspond to ν values: 0.001, 0.01, 0.05, 0.1. Other parameters are: $R_1/R_2 = 0.1$, $R_2\omega/c = 0.5$, $R_3/R_2 = 2.0$, $j = 4.0$

It was obtained that the increase of the effective collision rate value ν leads to the increase of the wave attenuation coefficient α, especially in the region of the short waves ($x > 5$). The spatial attenuation coefficient essentially depends on the value of direct current that creates external non-uniform magnetic field. When current value increases in two times and other problem's parameters remain unchanged the value of α in the case of uniform plasma increases almost in two times.

The value of the normalized direct current ($j = eJ_z/2mc^3$) substantially affects the symmetric mode dispersion (Fig. 2).

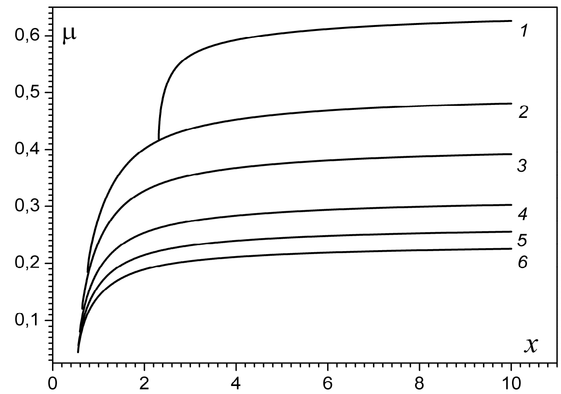

Fig. 2. The dependence of normalized wave frequency $\mu = \omega/\omega_p$ on the wave number x. Numbers 1-6 correspond to j values 0.1, 0.5, 1.0, 2.0, 3.0, 4.0. Other parameters are the same as for the Fig. 1, except $\nu = 0.001$

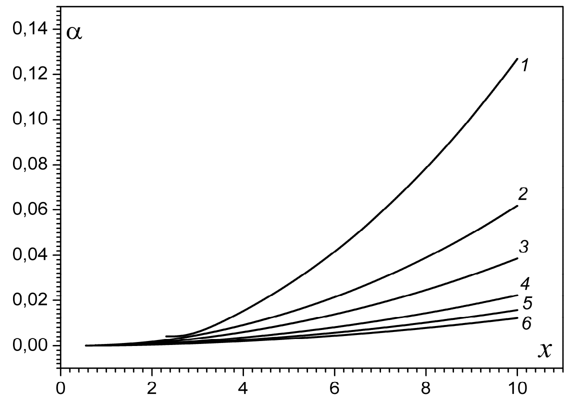

Fig. 3. The dependence of attenuation coefficient α on the wave number x. Numbering of the curves and problem's parameters are the same as for the Fig. 2

When the value of current j increases from 0.1 up to 4.0 the frequency μ maximum value arrives at the region of big axial wave numbers was decreased in 3 times. For every j value some critical axial wave number value exists. The waves with axial wave numbers lower than critical can not propagate in such structure under given conditions. When external magnetic field is rather strong ($j > 4$) its further growth has not influence on the critical wave number value and on the frequency μ. The increase of the j value leads to the essential decrease of the attenuation coefficient α especially in the region of large axial wave numbers (Fig. 3).

The outer waveguide metal wall strongly influences on the wave dispersion in the region of the moderate $x \approx 2$ and on the coefficient α in the region of short wavelength. When vacuum gap thickness value is rather large ($R_3/R_2 > 2$) its further growth does not essentially influences on the dispersion and attenuation of the wave.

The influence of radial plasma density non-uniformity on the wave attenuation is presented on the Fig. 4. It was obtained that the coefficient α take on some minimum value when $\delta \neq 0$. Such dependence is determined by the fact that under given parameters of waveguide structure and plasma density radial profile the electromagnetic-field strength culminates its maximum in the region where plasma density tends to zero. Further growth of the δ value leads to the convergence of the radial positions of the maximum values of plasma density and wave field amplitude. This feature causes the increase of spatial wave attenuation as a result of Joule wave energy losses under plasma density non-uniformity parameter growth.

It was obtained that the coefficient α is not too large in the region of small axial wave numbers ($x \leq 2.5$). But for rather small wavelengths one can observe some critical x_{cr} value. In the neighborhood of this x_{cr} value the spatial attenuation sharply growths up. Waves with x value larger than this critical value cannot propagate under the given set of problem parameters (see Fig. 4).

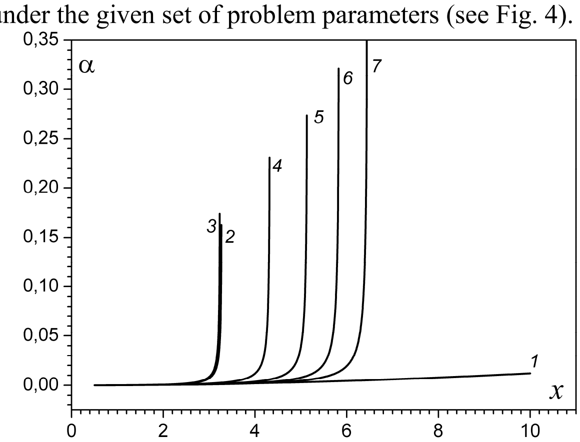

Fig. 4. The dependence of coefficient α on the wave number x. Numbers 1-7 correspond to δ values: 0.0, 0.061, 0.19, 0.4, 0.6, 0.8, 1.0. *Other parameters are the same as for the fig. 2, except* $j = 4.0$

The dispersion equation (4) was analyzed for the cases of symmetric ($m = 0$) and dipolar ($m = 1$) waves. Main attention was focused to the study of the properties of the symmetric waves. The solution of the dispersion equation in the case of dipolar waves is more complicated problem because the equation possesses the set of eigen solutions. These solutions essentially differ from each other by the dispersion characteristics and by the spatial attenuation coefficient.

4. CONCLUSIONS

It was studied the influence of the effective collision frequency, the value of direct current, geometric parameters of waveguide system and plasma density radial profile on dispersion characteristics and attenuation coefficient of electromagnetic waves that propagates along coaxial waveguide structure. It was shown that it is possible to control effectively the dispersion properties and spatial attenuation of the considered waves by varying the value of mentioned parameters.

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СИММЕТРИЧНЫЕ И ДИПОЛЬНЫЕ ЭЛЕКТРОМАГНИТНЫЕ ВОЛНЫ В КОАКСИАЛЬНОЙ СТРУКТУРЕ, ЗАПОЛНЕННОЙ НЕОДНОРОДНОЙ ДИССИПАТИВНОЙ ПЛАЗМОЙ С АЗИМУТАЛЬНЫМ МАГНИТНЫМ ПОЛЕМ

В.П. Олефир, А.Е. Споров

Приведено исследование влияния эффективной частоты столкновений электронов, силы постоянного тока, протекающего по центральному проводнику коаксиальной волноводной структуры, геометрических параметров волновода и радиального профиля плотности плазмы на дисперсию, пространственное затухание и радиальную структуру поля высокочастотных электромагнитных волн. Показано, что, изменяя указанные параметры, можно эффективно управлять дисперсионными характеристиками и коэффициентом пространственного затухания рассматриваемых волн.

СИМЕТРИЧНІ ТА ДИПОЛЬНІ ЕЛЕКТРОМАГНИТНІ ХВИЛІ В КОАКСІАЛЬНІЙ СТРУКТУРІ, ЩО ЗАПОВНЕНА НЕОДНОРІДНОЮ ДИСИПАТИВНОЮ ПЛАЗМОЮ З АЗИМУТАЛЬНИМ МАГНІТНИМ ПОЛЕМ

В.П. Олефір, О.Є. Споров

Наведено дослідження впливу ефективної частоти зіткнень електронів, сили постійного струму, що протікає по центральному провіднику коаксіальної хвилеводної структури, геометричних параметрів хвилеводу та радіального профілю густини плазми на дисперсію, просторове загасання та радіальну структуру поля високочастотних електромагнітних хвиль. Показано, що, змінюючи вказані параметри, можливо ефективно керувати дисперсійними характеристиками та просторовим загасанням хвиль, що розглядаються.