TRANSITION RADIATION OF A MODULATED ELECTRON BEAM IN PLASMA WITH CONDUCTIVE DUST PARTICLES

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Transition radiation of a modulated electron beam in plasma with randomly distributed conductive dust particles is studied theoretically. The problem is treated for the simplest geometry, i.e. thin electron beam of limited length is injected into cold isotropic unbounded plasma. Dust particles are considered to be ideal conductive spheres with the sizes of the order of Debye length. In this case the appearance of transition radioemission is a result of diffraction scattering of electromagnetic field of electron beam on the dust particles. The solution for scattering of a single plane wave from the spectrum of beam's electromagnetic field on a single dust particle is derived.

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1. INTRODUCTION

The use of Mie scattering induced by the laser light illumination is the elementary and typical method for the detection of dust particles. Nowadays the development of new diagnostic methods for dusty plasma is widely discussed [1].

It is also well-known the investigations of influence of beam-excited electromagnetic radiation on the background plasma caused by injection of a modulated electron beam [2]. The transition radiation of the modulated electron beam on plasma fluctuations induced by dust grains is one of the possible mechanisms of radioemission. The dust particles perturb the background plasma density leading to the appearance of random plasma inhomogeneities.

In our previous work [3], the non-resonant transition radiation was calculated for small plasma fluctuations and simplified model of dusty plasma inhomogeneities described as a system of equal and randomly distributed spherical dust particles. The investigation of transition radioemission for complex structure of highly asymmetric dipole dust particles in the strong magnetic field was done in [4]. It was shown that effectiveness of transition radiation is determined by the beam current density and depends on dusty parameters of plasma.

Transition radiation in [2, 3] was based on scattering of electromagnetic beam's field on random spatial fluctuation of background plasma density induced by dust particles. In this work we assume the size of dust grain to be of the order of Debye length. In this case we propose to calculate transition radioemission as a result of diffraction scattering of beam's field on randomly distributed dust grains. Dust particles are assumed to be identical ideal conductive spheres. Total radiation can be determined by contributions from every scattering event, i.e. by a sum of scattered beam's field from each scattering center of full ensemble of dust particles. We consider single incoherent scattering as a simple approach. This is valid for small density of dust grains as compared to background plasma density.

2. MODEL DESCRIPTION AND INITIAL EQUATIONS

Let us consider cold collisionless isotropic plasma with the random distribution of dust particles. We assume the size of dust particles are of about Debye length. This allows us to examine just the problem of diffraction scattering of electromagnetic beam's field on the dust particles. For the model simplification the dust particles are considered to be the same kind, i.e. ideal conductive spheres of the same radius a_p.

The cylindrical modulated electron beam of the limited length L is injected into background plasma from the plane $z=0$ along the axis Oz. Its current density can be predetermined as a given function:

$$\vec{j}(\vec{r}) = \begin{cases} \vec{e}_z j_m \exp[-i\omega t + i\kappa z] & 0 < r \le a, 0 < z \le L \\ 0 & r > a, z > L \end{cases} . \quad (1)$$

Here, a is the beam's radius; $\kappa=\omega/v_0$, ω is the modulation frequency and v_0 is the beam's velocity; $\omega>\omega_p$ – the modulation frequency exceeds the Langmuir frequency of background plasma. We consider that the beam is charge-compensated and it does not perturb the density of background plasma.

Let's calculate the self-field of modulated electron beam determined by the current density (1). The problem's solution will be built for the vector potential under condition of Coulomb gauge. From Maxwell equations we can get the following wave equation:

$$\Delta \vec{A} + k_0^2 \vec{A} = -\frac{4\pi}{c}\vec{j} . \quad (2)$$

Here, $k_0^2=\varepsilon_0\omega^2/c^2$, ε_0 is dielectric permittivity of background plasma.

The electromagnetic field of electron beam can be expanded into plane modes by applying Fourier transform for (2) in rectangular coordinates:

$$A_i(k_x, k_y, k_z)\exp[-i\omega t + i(k_x x + k_y y + k_z z)] , \; i=\{x,y,z\}, \quad (3)$$

where the respective spectrum amplitudes of vector potential field can be determined via spectrum of current density (1) as follows:

$$A_x(k_x,k_y,k_z) = -\frac{4\pi}{ck_0^2}\frac{k_x k_z}{k_x^2+k_y^2+k_z^2-k_0^2} j(k_x,k_y,k_z) ,$$

$$A_y(k_x,k_y,k_z) = -\frac{4\pi}{ck_0^2}\frac{k_y k_z}{k_x^2+k_y^2+k_z^2-k_0^2} j(k_x,k_y,k_z),$$

$$A_z(k_x,k_y,k_z) = -\frac{4\pi}{ck_0^2}\frac{k_z^2-k_0^2}{k_x^2+k_y^2+k_z^2-k_0^2} j(k_x,k_y,k_z),$$

$$j(k_x,k_y,k_z) = ij_m a \frac{1-\exp[i(\kappa-k_z)L]}{\kappa-k_z}\frac{J_1(\sqrt{k_x^2+k_y^2}a)}{\sqrt{k_x^2+k_y^2}}.$$

Next, we consider scattering of plane wave (3) on a separate dust particle.

3. SCATERRING OF A PLANE WAVE ON A DUST PARTICLE

Let's move to the new coordinate system $O'x'y'z'$ related with dust particle where axis $O'z'$ is oriented along fixed propagation vector $\vec{k} = \{k_x, k_y, k_z\}$. We can choose direction of axis $O'x'$ to be perpendicular to $\vec{A}$, so that $\vec{A}$ lies in the plane $O'y'z'$. The rotation matrix from old to the new coordinate system has the following form:

$$K = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix}, \qquad (4)$$

where

$$\alpha_1 = \frac{k_y}{\sqrt{k_x^2+k_y^2}},\ \alpha_2 = -\frac{k_x}{\sqrt{k_x^2+k_y^2}},\ \alpha_3 = 0,$$

$$\beta_1 = \frac{k_x k_z}{\sqrt{k_x^2+k_y^2}\sqrt{k_x^2+k_y^2+k_z^2}},$$

$$\beta_2 = \frac{k_y k_z}{\sqrt{k_x^2+k_y^2}\sqrt{k_x^2+k_y^2+k_z^2}},$$

$$\beta_3 = -\frac{\sqrt{k_x^2+k_y^2}}{\sqrt{k_x^2+k_y^2+k_z^2}},$$

$$\gamma_1 = \frac{k_x}{\sqrt{k_x^2+k_y^2+k_z^2}},$$

$$\gamma_2 = \frac{k_y}{\sqrt{k_x^2+k_y^2+k_z^2}},$$

$$\gamma_3 = \frac{k_z}{\sqrt{k_x^2+k_y^2+k_z^2}}.$$

The problem of a plane wave scattering

$$\vec{A}^{(i)} = K\vec{A} = \vec{A}'(k_x,k_y,k_z)\exp[-i\omega t + ikz'] \qquad (5)$$

on a dust particle can be examined by the following wave equation

$$\Box\vec{A} + k_0^2\vec{A} = 0, \qquad (6)$$

that describes propagation of electromagnetic waves in the background plasma without external charges and currents.

The solution of vector equation (6) can be expanded into series:

$$\vec{A} = \sum_n (a_n\vec{M} + b_n\vec{N})$$

via orthogonal characteristic functions $\vec{M}$ and $\vec{N}$ with the following properties:

$$\vec{M} = rot(\vec{r}\psi),\ \vec{N} = rot\vec{M}/k_0, \qquad (7)$$

where $\vec{M}$ and $\vec{N}$ are defined through scalar potential ψ. The solution for (6) can be derived from the similar scalar wave equation:

$$\Box\psi + k_0^2\psi = 0.$$

The scattered wave must propagate away from the origin of system and vanish at infinity. Basing on this, we can find out the following general solution for scattered field in spherical coordinate system $\{r, \theta, \varphi\}$, whose origin coincides with the center of dust grain:

$$\vec{A}^{(s)} = \sum_{n=0}^{\infty}\sum_{m=-n}^{n}(a_{emn}\vec{M}_{emn} + a_{omn}\vec{M}_{omn} + b_{emn}\vec{N}_{emn} + b_{omn}\vec{N}_{omn}), \qquad (8)$$

where

$$\psi_{emn} = h_n^{(1)}(k_0 r)P_n^m(\cos(\theta))\cos m\varphi,$$

$$\psi_{omn} = h_n^{(1)}(k_0 r)P_n^m(\cos(\theta))\sin m\varphi,$$

$$h_n^{(1)}(k_0 r) = \sqrt{\pi/2k_0 r}H_{n+1/2}^{(1)}(k_0 r),$$

and a_{emn}, a_{omn}, b_{emn}, b_{omn} are arbitrary coefficients.

4. BOUNDARY CONDITIONS

The components of incident field (5) can be expressed in the spherical coordinates as follows:

$$A_r^{(i)} = (A_{y'}\sin\theta\sin\varphi + A_{z'}\cos\theta)\exp[-i\omega t + ikr\cos\theta]$$

$$A_\theta^{(i)} = (A_{y'}\cos\theta\sin\varphi + A_{z'}\sin\theta)\exp[-i\omega t + ikr\cos\theta] \qquad (9)$$

$$A_\varphi^{(i)} = A_{y'}\cos\varphi\exp[-i\omega t + ikr\cos\theta].$$

In order to determine arbitrary coefficients in series (8) we need to expand angular dependence for θ in a series of a similar form for associated Legendre functions of the first kind $P_n^m(cos\theta)$ and apply the boundary conditions on the surface of ideal conductive dust particle with the radius a_p:

$$A_\theta^{(i)} + A_\theta^{(s)} = 0 \text{ and } A_\varphi^{(i)} + A_\varphi^{(s)} = 0, \text{ when } r = a_p. \qquad (10)$$

The axial dependence of incident field in (9) defines the form of solution (8), so we can leave terms in series with $m=0$ та $m=1$ only:

$$A_r^{(s)} = \sum_{n=0}^{\infty}(b_{o1n}N_{r,o1n} + b_{e0n}N_{r,e0n}),$$

$$A_\theta^{(s)} = \sum_{n=0}^{\infty}(a_{e1n}M_{\theta,e1n} + b_{o1n}N_{\theta,o1n} + b_{e0n}N_{\theta,e0n}), \qquad (11)$$

$$A_\varphi^{(s)} = \sum_{n=0}^{\infty}(a_{e1n}M_{\varphi,e1n} + b_{o1n}N_{\varphi,o1n}).$$

Since the terms in the series expansions (11) are independent of each other and the boundary conditions (10) must hold for each corresponding terms in the series, we can determine the following coefficients:

$$a_{e1n} = A_{y'}i^n\frac{2n+1}{n(n+1)}\frac{j_n(ka_p)}{h_n^{(1)}(k_0 a_p)},$$

$$b_{o1n} = A_{y'} i^{n+1} \frac{2n+1}{n(n+1)} \frac{k_0}{k} \frac{j_n(ka_p) + ka_p j_n{}'(ka_p)}{h_n^{(1)}(k_0 a_p) + k_0 a_p h_n^{(1)\prime}(k_0 a_p)}, \quad (12)$$

$$b_{e0n} = A_{z'} i^{n+1} \frac{k_0}{k} \frac{j_n(ka_p)}{h_n^{(1)}(k_0 a_p) + k_0 a_p h_n^{(1)\prime}(k_0 a_p)},$$

where

$$j_n(kr) = \sqrt{\pi/2kr}\, J_{n+1/2}(kr)$$

and primes near spherical functions denote argument derivative.

5. SOLUTION FOR SCATTERED WAVE

Finally, the components of the scattered field can be written as follows:

$$A_r^{(s)} = \frac{\exp[-i\omega t]}{k_0 r} \sum_{n=0}^{\infty} n(n+1) h_n^{(1)}(k_0 r) \times$$

$$\times \left[b_{o1n} P_n^1(\cos\theta) \sin\varphi + b_{e0n} P_n^0(\cos\theta) \right],$$

$$A_\theta^{(s)} = \exp[-i\omega t] \sum_{n=0}^{\infty} \left\{ \sin\varphi \left[-a_{e1n} h_n^{(1)}(k_0 r) \frac{P_n^1(\cos\theta)}{\sin\theta} + \right.\right.$$

$$\left. + b_{o1n} \frac{1}{k_0 r} \frac{d}{dr} \left(r h_n^{(1)}(k_0 r) \right) \frac{d}{d\theta} P_n^1(\cos\theta) \right] - \quad (13)$$

$$\left. - b_{e0n} \frac{1}{k_0 r} \frac{d}{dr} \left(r h_n^{(1)}(k_0 r) \right) P_n^1(\cos\theta) \right\},$$

$$A_\varphi^{(s)} = \exp[-i\omega t] \cos\varphi \sum_{n=0}^{\infty} \left[-a_{e1n} h_n^{(1)}(k_0 r) \frac{d}{d\theta} P_n^1(\cos\theta) + \right.$$

$$\left. + b_{o1n} \frac{1}{k_0 r} \frac{d}{dr} \left(r h_n^{(1)}(k_0 r) \right) \frac{P_n^1(\cos\theta)}{\sin\theta} \right].$$

These expressions describe scattering of a plane wave on a single particle. In order to find out transition radiation induced by a modulated electron beam we need to integrate (13) over all possible plane modes and to average the result for random spatial distribution of dust grains.

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ПЕРЕХОДНОЕ ИЗЛУЧЕНИЕ МОДУЛИРОВАННОГО ЭЛЕКТРОННОГО ПУЧКА В ПЛАЗМЕ С ПРОВОДЯЩИМИ ПЫЛЕВЫМИ ЧАСТИЦАМИ

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Теоретически изучено переходное излучение модулированного электронного пучка в плазме со случайно распределенными проводящими пылевыми частицами. Рассмотрена упрощенная геометрия задачи, когда тонкий электронный пучок конечной длины инжектируется в холодную изотропную безграничную плазму. Пылевые частицы считаются идеально проводящими сферами с размерами порядка длины Дебая. В этом случае появление переходного излучения является результатом дифракционного рассеяния собственного электромагнитного поля пучка на пылевых частицах. Получено решение для рассеяния единичной плоской волны из спектра электромагнитного поля пучка на единичной пылевой частице.

ПЕРЕХІДНЕ ВИПРОМІНЮВАННЯ МОДУЛЬОВАНОГО ЕЛЕКТРОННОГО ПУЧКА В ПЛАЗМІ З ПРОВІДНИМИ ПИЛОВИМИ ЧАСТИНКАМИ

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Теоретично вивчено перехідне випромінювання модульованого електронного пучка у плазмі з випадково розподіленими провідними пиловими частинками. Розглянуто спрощену геометрію задачі, коли тонкий електронний пучок скінченної довжини інжектується в холодну ізотропну безмежну плазму. Пилові частинки вважаються ідеально провідними сферами з розмірами порядку довжини Дебая. У цьому випадку поява перехідного випромінювання є результатом дифракційного розсіювання власного електромагнітного поля пучка на пилових частинках. Отримано розв’язок для розсіювання одиночної плоскої хвилі зі спектру електромагнітного поля пучка на одиночній пиловій частинці.