INFLUENCE OF CASUAL DISTRIBUTIONS ON RESONANT PROPERTIES OF ENSEMBLES OF NONLINEAR OSCILLATORS

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It is shown, that properties of ensembles of nonlinear oscillators each of which makes chaotic movement, can essentially differ from properties separate oscillators. In particular, is shown, that dynamics of average values of ensembles can essentially depend on the statistical characteristics of separate oscillators.

PACS: 05.45.-a; 05.45.Jn

1. INTRODUCTION

Now essential influence even of weak noise on oscillatory systems does not cause doubts. Especially brightly role of noise is shown at realization of conditions of a stochastic resonance. In the simplest case the stochastic resonance is revealed in that fact that at action of noise on nonlinear bistable system there are transitions from one stable position to another. This transition occurs with Kramers time. If on the same bistable system acts the external regular signal, which period is close to Kramers time, then resonant interaction of this signal with bistable system can take place. This is one of possible demonstration of a stochastic resonance (see, for example, [1]). In the beginning the stochastic resonance was studied on systems with small number of degrees of freedom (on separate oscillators). Now similar effects are studied and for the distributed systems (see, for example, [2] and literature quoted there). In many cases, as well known, the dynamics of nonlinear oscillators can become chaotic. At that the noise force not acts on them. It is possible to assume, that dynamics of ensembles of such oscillators also will be essential depend from features of nonlinear dynamics of each of these oscillators. Below we shall show that really such dependence takes place. In this work the results of researches of dynamics of system connected nonlinear oscillators are reported. These oscillators are under action of external regular periodic forces. Dynamics of such oscillators are chaotic. Chaotic is caused or by presence of connections between oscillators or influence of external regular forces. The special attention is paid on two types of nonlinear oscillators - on system, which consist from N mathematical pendulums and on system of N Duffing oscillators. One of the most important results is the proof of that fact that the topology of phase space for the moments, for example for average value, can qualitatively differ from topology of phase space of separate oscillators. It, in particular, means that the oscillatory and resonant properties of dynamics of average value can be essentially differ from dynamics separate oscillators. And, this difference is caused by a kind of casual distributions, by their moments.

2. STATEMENT OF A TASK. THE BASIC EQUATIONS

We shall investigate the system representing ensemble of the connected nonlinear oscillators, on which the external regular periodic force acts. Hamiltonian of such system it is possible to present in the following form:

$$H = \sum_j \sum_{i=1}^{N} \left[\frac{\dot{x}_i^2}{2} + \Phi\left(x_i\right) + G\left(x_i, x_j\right) - \varepsilon\left(\tau\right) \cdot x_i \right]. \quad (1)$$

For Hamiltonian (1) the following system of the equations of the second order is corresponds:

$$\ddot{x}_i = F_0\left(x_i\right) + F_1\left(x_i, x_j\right) + \varepsilon\left(\tau\right). \quad (2)$$

If last two items in the right part of system (2) are absent, this system describes of dynamics of ensemble of independent from each other nonlinear oscillators. At that the character of nonlinearity is defined by function $F_0\left(x_i\right) = -\partial\Phi / \partial x_i$. The second item $F_1 = -\sum_i^N \partial G / \partial x_i$ describes the interaction between nonlinear oscillators. The third - describes external periodic force. Let, for definiteness, each of examined nonlinear oscillators represents the charged particle, which was placed in some nonlinear potential. Below we shall consider that as a result of interaction between oscillators or as a result of influence on them the external regular force their dynamics is chaotic. In this case the displacement of each of them is possible to present as:

$$x_i = \bar{x} + \delta_i , \quad (3)$$

where $\bar{x} = \lim_{N\to\infty} (\sum_{i=1}^{\infty} x_i) / N$ - mean coordinate of displacement; δ_i - casual displacement. At that, $\langle \delta_i \rangle = 0$. In this case mean value (on ensemble) from functions F_0 and F_1 are convenient to expand out in a series on the moments:

$$\left\langle F_0\left(\bar{x} + \delta_i\right)\right\rangle = F_0\left(\bar{x}\right) + \sum_{n=1}^{\infty} \frac{M_n}{n!} \left(\frac{d}{dx_i}\right)^n \cdot F_0 \mid_{\delta_i = 0}, \quad (4)$$

where $\mathrm{M}_n = \left\langle \left(\delta\right)^n \right\rangle$ - moments.

3. EXAMPLES

As the first example we shall consider ensemble each one of which represents a mathematical pendulum. In this case $F_0\left(x_i\right) = -\sin\left(x_i\right)$. The mean value of this function will look like:

$$\left\langle F_0\left(x_i\right)\right\rangle = -\left\langle \sin\left(x_i\right)\right\rangle = -\left[1 - \sum_{m=1}^{\infty} \frac{\mathrm{M}_{2m}}{(2m!)}\right] \sin\bar{x}. \quad (5)$$

Using formula (5) one can immediately to get one of the more significant results. This result consists in that that the characteristics of ensemble even of not interact mathematical pendulums can be essentially differ from the characteristics of separate pendulum. To illustrate this fact, we shall find (from system (2)) the equation, which describes dynamics of an average deviation. For simplicity of the external force and of the connection between pendulums we shall neglect.

$$\ddot{\bar{x}} + \left[1 - \sum_{m=1}^{\infty} \frac{M_{2m}}{(2m!)}\right] \sin \bar{x} = 0 . \qquad (6)$$

The formula (6) describes dynamics of a mathematical pendulum. However potential of this mathematical pendulum, and accordingly, the oscillatory characteristics of this pendulum essentially depend on the statistical characteristics of separate pendulums, which make up considered ensemble. Hence, the oscillatory properties of the ensemble even of independent nonlinear oscillators are depend from characteristics of chaotic dynamics of the separate pendulums. It is necessary, however, to mean, that the chaotic dynamics of separate pendulums will arise only in result or interaction between them, or, as result of external influence on these pendulums. These facts in an obvious kind in the formula (6) are not reflected. Let's pay attention that second item in square brackets in the formula (6) has a negative sign. It means that the randomness of dynamics of separate pendulums always lead to reduction of effective potential, in which the ensemble is placed. In particular, the frequency of small linear oscillation of such ensemble decreases also.

The second important example is the ensemble each component of which represents Duffing oscillator. For such oscillator $F_0(x_i) = \alpha \cdot x_i - \beta \cdot x_i^3$. And we shall consider that $\alpha > 0$ and $\beta > 0$. Such choice of parameters means, that we shall examine bistable Duffing oscillators. Acting similarly, how we did above for ensemble of mathematical pendulums, it is easy to get the equations describing dynamics of average displacement of this ensemble:

$$\ddot{\bar{x}} - (\alpha - 3 \cdot \beta \cdot M_2)\bar{x} + \beta \cdot \bar{x}^3 = 0 . \qquad (7)$$

In the equation (7) we, as well as in the equation (6), have omitted influence of connection between oscillators and also influence of external forces. Besides, Duffing oscillators, generally, have only three moments. As well as for mathematical pendulum, we took into account influence only of even moments. As one can see from (7), the value of effective potential, in which the ensemble is placed, can be significantly reduced, if dispersion (the second moment) will be enough large. The frequency of small linear oscillation of the ensemble significantly reduce too: $\Omega = \sqrt{2(\alpha - 3 \cdot \beta \cdot M_2)}$.

Above we have considered a case, when nonlinear oscillators are not connected and on them the external force does not act. However we assumed, that dynamics everyone oscillators is chaotic. Casual dynamics can be caused by either connection between oscillators or influence of external force. The analysis of a general case (when there are connection and the external forces) can be carried out only by numerical methods. Such researches were carried out. Some results of such researches for ensemble of mathematical pendulums are given below.

4. NUMERICAL RESULTS

For numerical researches the connection between oscillators was chosen in form, which corresponds to coulomb interaction between oscillators:

$$F_1(x_i, x_j) = -\mu \sum_{j=1}^{N} \frac{sign(x_i - x_j)}{(x_i - x_j - a)^2} . \qquad (8)$$

In the formula (8) value a characterize minimal distance between oscillators. External force is chosen in form: $\varepsilon(\tau) = A \cdot \cos \omega\tau$. The system of the equations (2) with such external force and with connection (8) describes ensemble of the charged particles, which are in external periodic potential and on which the external periodic force acts. Such system was studied by numerically. The main results of numerical researches are in the good qualitative consent with the described above picture, i.e. the presence of connection and external force lead to chaotic dynamics everyone oscillators and dynamics of the whole ensemble significantly depends on the statistical characteristics of individual oscillators. Below, in the Figs. 1- 4 some of the characteristic results are given.

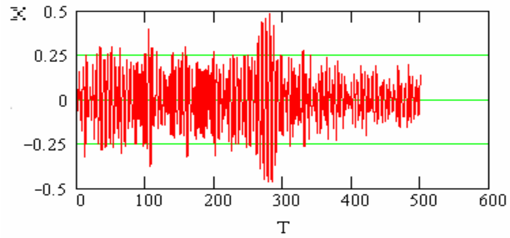

Fig.1. Dependence of the displacement of the individual pendulum from time

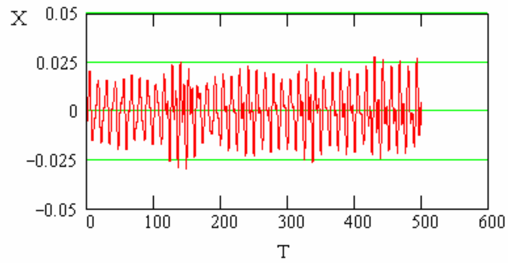

Fig.2. Dependence from time of the mean displacement of the ensemble

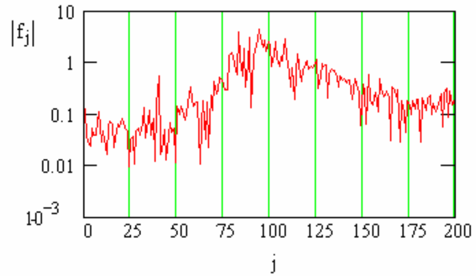

Fig.3. Characteristic form of the spectrum of individual pendulum

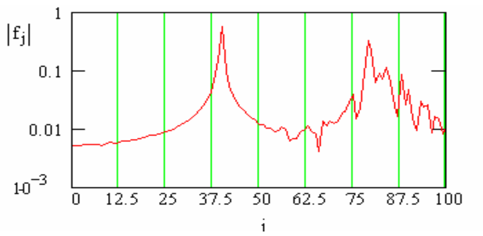

Fig.4. Spectrum of mean displacement of ensemble

The numerical results were got at such parameters: $a = 0.1$; $A = 0.01$; $\mu = 0.001$; $\Omega = 0.5$. So, in Figs. 1, 2 the dependence of displacement of individual pendulum (Fig. 1) and dependence of displacement of an mean position of ensemble from time are submitted. First of all, one can see that dynamics of individual pendulum is chaotic, and dynamics of ensemble is considerably more regular. One can see also, that the characteristic frequencies of oscillation of ensemble are much lower, than characteristic frequencies of individual pendulum. This fact is confirmed also by figures 3 and 4. On first of them the spectrum of individual pendulum is presented and on second - spectrum of ensemble is submitted. Except these characteristics, there were found the correlation functions and main Lyapunov indexes. It had been shown that the correlation functions of individual pendulum quickly fall down, and the correlation functions of ensemble change without damping. The main Lyapunov indexes for individual pendulum were positive practically on whole phase space. For ensemble these indexes practically were equal to zero.

CONCLUSIONS

As the main result of the carried out researches is in establish of fact that dynamics of some ensemble nonlinear oscillators essentially depend on the statistical characteristics of dynamics of individual oscillators. This feature is appeared even in that case, when oscillators are independent and do not interact with each other. The important peculiarity of this influence is that feature that the statistical moments of separate oscillators leads to reduction of effective potential, in which oscillate considered ensemble. The frequency of the small linear oscillation of this ensemble decreases also. It is necessary to note, that the reduction of effective potential can appear so significant, that the some particles (oscillators) can leave area of their capture.

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Article received 22.09.08

ВЛИЯНИЕ СЛУЧАЙНЫХ РАСПРЕДЕЛЕНИЙ НА РЕЗОНАНСНЫЕ СВОЙСТВА АНСАМБЛЕЙ НЕЛИНЕЙНЫХ ОСЦИЛЛЯТОРОВ

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Показано, что свойства ансамблей нелинейных осцилляторов, каждый из которых совершает хаотическое движение, может существенно отличаться от свойств отдельных осцилляторов. В частности, показано, что динамика средних величин ансамблей может существенно зависеть от статистических характеристик отдельных осцилляторов.

ВПЛИВ ВИПАДКОВИХ РОЗПОДІЛІВ НА РЕЗОНАНСНІ ВЛАСТИВОСТІ АНСАМБЛІВ НЕЛІНІЙНИХ ОСЦИЛЯТОРІВ

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Показано, що властивості ансамблів нелінійних осциляторів, кожен з яких здійснює хаотичний рух, може істотно відрізнятися від властивостей окремих осциляторів. Зокрема, показано, що динаміка середніх величин ансамблів може істотно залежати від статистичних характеристик окремих осциляторів.