TURBULENCE OF SHEAR FLOWS OF MAGNETIZED PLASMA

V.S. Mikhailenko[1,2], K.N. Stepanov[1,2], V.V. Mikhailenko[1], N.A. Azarenkov[1], D.V. Chibisov[1]

[1]V.N. Karazin Kharkov National University, Kharkov, Ukraine;
[2]NSC "Kharkov Institute of Physics and Technology", Kharkov, Ukraine

The presented report summarizes authors' recent results in the analytical investigation of turbulence of the magnetized plasma shear flows. The emphasis are made on low frequency turbulence of drift type in flows with strongly inhomogeneous shear with application to improved confinement regimes in toroidal plasma and on ion cyclotron turbulence in shear flows along the ambient magnetic field with applications to anomalous ions heating in ionosphere.
PACS: 52.35.-g , 52.35.Kt

1. INTRODUCTION

We present our new results of the analytical investigations of the temporal evolution of the drift-resistive instability and of the related anomalous transport under condition of strong inhomogeneous flow shear with application to the H-mode regime [1] of tokamak operation (Sec. 2). Also the results of the investigations of the nonlinear evolution and saturation of the ion cyclotron turbulence in shear flows along the ambient magnetic field with applications to ionosphere conditions are presented (Sec. 3).

2. ANOMALOUS TRANSPORT IN PLASMA FLOW WITH STRONG INHOMOGENEOUS VELOCITY SHEAR

Our investigations of the plasma shear flow at the boundary of tokamaks are based on the simple and extensively studied model for resistive drift-wave turbulence proposed by Hasegawa–Wakatani [2]. We consider regions far away from the X-point and divertor and use a slab geometry with the mapping $(r,\theta,\varphi)\rightarrow(x,y,z)$, where r,θ,φ are the radial, poloidal and toroidal coordinates, respectively, of the toroidal coordinate system. In this geometry the linearized Hasegawa-Wakatani system of equations for the dimensionless density $n=\tilde{n}/n_e$, potential $\phi=e\varphi/T_e$, (where perturbations n_e is the electron background density, T_e is the electron temperature) is

$$\rho_s^2\left(\frac{\partial}{\partial t}+\mathbf{v}_0(x)\cdot\nabla\right)\nabla^2\phi=a\frac{\partial^2}{\partial z^2}(n-\phi),$$
$$\left(\frac{\partial}{\partial t}+\mathbf{v}_0(x)\cdot\nabla\right)n+v_{de}\frac{\partial\phi}{\partial y}=a\frac{\partial^2}{\partial z^2}(n-\phi), \quad (1)$$

where $\mathbf{v}_0(x)$ is the drift velocity of the non-uniformly sheared flow, for which $\mathrm{v}''_0(x)\neq 0$, $a=T_e/n_0e^2\eta_{\parallel}$, $\eta_{\parallel}$ is the resistivity parallel to the homogeneous magnetic field, ρ_s is the ion Larmor radius at electron temperature T_e, $\mathrm{v}_{de}=cT_e/eBL_n$ is the diamagnetic drift velocity, $\mathrm{v}_*=\mathrm{v}_{de}+\rho_s^2\mathrm{v}''_0(\xi)$, $S=l\mathrm{v}_*/\mathrm{v}'_0$, $L_n^{-1}=-d\ln n_{0e}(x)/dx$, $C=ak_z^2/\rho_s^2l^2\mathrm{v}'_0=T_ek_z^2/\rho_s^2l^2\mathrm{v}'_0n_0e^2\eta_{\parallel}$. We use the non-modal approach that begins with a transformation to the convective coordinates

$$\xi=x,\qquad \eta=y-\mathrm{v}_0(x)t,\qquad z=z.$$

With these coordinates the system (1) may be combined into the following equation for the potential $\phi(t,\xi,l,k_z)$ (Fourier-transformed over coordinate η):

$$\frac{S^2(\xi)}{C}\frac{\partial^2}{\partial\tau^2}\left(\tilde{\Delta}_{\perp c}\phi\right)+S(\xi)l^2\rho_s^2\frac{\partial}{\partial\tau}\left(\tilde{\Delta}_{\perp c}\phi\right)-$$
$$-S(\xi)\frac{\partial\phi}{\partial\tau}-i\frac{l\mathrm{v}''_0(\xi)}{l\mathrm{v}'_0(\xi)}\frac{S}{C}\frac{\partial\phi}{\partial\tau}-iS\phi=0. \quad (2)$$

In Eq. (2) τ is the dimensionless time variable, defined by $t=T_0\tau$, where T_0 is the time scale of interest, and Laplassian Δ in convective coordinates is equal to

$$\tilde{\Delta}(\tau,l,\xi)=\frac{1}{l^2}\frac{\partial^2}{\partial\xi^2}-\frac{2i}{l}\frac{\tau}{S(\xi)}\frac{\partial}{\partial\xi}-$$
$$-i\frac{\mathrm{v}''_0(\xi)}{l\mathrm{v}'_0(\xi)}\frac{\tau}{S(\xi)}-1-\frac{\tau^2}{S^2(\xi)}.$$

Presence of small parameter $\left(\mathrm{v}'_0(\xi)T_0\right)^{-1}=S(\xi)<1$ in Eq. (2) in the case of strong flow shear permits to obtain the initial value problem solution for the perturbed potential ϕ in terms of a power series in this parameter:

$$\phi(\tau,\xi,l)=\phi_0(\tau,\xi,l)+\left(\mathrm{v}'_0(\xi)T_0\right)^{-1}\phi_1(\tau,\xi,l)+\ldots$$
$$=\left\{\frac{\phi(\xi,l,0)}{\left(1+l^2\rho_s^2\left(\mathrm{v}'_0(\xi)T_0\right)^2\tau^2\right)}-\frac{l^2\rho_s^2\left(\mathrm{v}'_0(\xi)T_0\right)}{\left(1+l^2\rho_s^2\left(\mathrm{v}'_0(\xi)T_0\right)\tau^2\right)^2}\times\right.$$
$$\times\frac{i\tau}{l}\frac{\mathrm{v}''_0(\xi)}{\mathrm{v}'_0(\xi)}\left[2\frac{\partial\phi(\xi,l,0)}{\partial\xi}+\phi(\xi,l,0)\left(1+\right.\right.$$
$$+\frac{2i}{l\rho_s\mathrm{v}'_0(\xi)T_0}tan^{-1}\left(l\rho_s\mathrm{v}'_0(\xi)T_0\tau\right)- \quad (3)$$
$$\left.\left.\left.-\frac{2i+4l^2\rho_s^2\left(\mathrm{v}'_0(\xi)T_0\right)^2\tau^2}{\left(1+l^2\rho_s^2\left(\mathrm{v}'_0(\xi)T_0\right)^2\tau^2\right)}\right)\right]\right\}\times$$
$$\times\exp\left[-\frac{i}{l\rho_s\mathrm{v}'_0(\xi)T_0}tan^{-1}\left(l\rho_s\mathrm{v}'_0(\xi)T_0\tau\right)\right]+O\left(\tau^{-5}\right).$$

It is interesting to note that in convective coordinates spatial derivatives with respect to ξ are absent in the equation for ϕ_0 as well as for ϕ_1 and the spatial variable ξ only enters solutions for ϕ_0 and ϕ_1 as a parameter. The specific spatial ξ dependence of solution (3) is determined entirely by the ξ-dependence of the flow velocity $\mathrm{v}_0(\xi)$ and by the initial condition $\phi(\xi,l,0)$.

Therefore, the main effect, which determines the temporal evolution of the initial disturbances in plasma flow with strong velocity shear is convective deformation of the initial perturbation along the shear flow. The solution (3) shows that in times $\tau \geq \left(\mathrm{v}'_0(\xi) T_0\right)^{-1}$ the terms reflecting flow curvature $\left(\mathrm{v}''_0(\xi)\right)$ decay more rapidly with time (as $O\left(\tau^{-3}\right)$). Therefore, the effect of the non-uniformity of the strong flow shear in Hasegawa-Wakatani system of equations appears to be subdominant.

The relevant quasi-linear flux Γ_x of electrons (ions) in the x-direction, resulting from the non-modal drift perturbations, is $\Gamma_x = \langle n \mathrm{v}_x \rangle = -D_x \dfrac{dn_0}{dx}$ with

$$D_x \sim \frac{c^2}{B_0^2} \int dl l^2 \, |\varphi_0|^2 \, \frac{t}{\left(\mathrm{v}'_0 t l \rho_s\right)^4} \sim t^{-3}, . \tag{4}$$

It follows that the anomalous particles flux, resulted from scattering of ions by non-modal drift perturbations (3), exhibits a subdiffusive behavior with a diffusion coefficient reducing with time as t^{-3}.

3. RENORMALIZED THEORY OF THE ION CYCLOTRON TURBULENCE OF THE MAGNETIC FIELD-ALIGNED SHEAR FLOW

The field-aligned currents, ion beams or relative streaming between ion species have been proposed to provide the free energy for the development of ion cyclotron (IC) instabilities. There are in situ ionospheric observations that support the development of the classical current driven electrostatic IC instability [3] in ionosphere. However, generally [4] the levels of field-aligned current at which IC waves and transverse ion heating were observed in ionosphere were subcritical for the development of this instability, which has the lowest threshold for current density in the ionospheric plasma environment [5]. The comprehensive investigation of the IC instabilities of magnetic-field aligned plasma shear flow was undertaken in [6]. It was shown analytically that shear flow along the magnetic field does not only modify the frequency, growth rate and the threshold of the known current driven IC instability, but it is a source of the development of the kinetic and hydrodynamic shear-flow-driven IC instabilities at the levels of field-aligned current which are subcritical for the development of the current driven IC instability. Shear flow along the magnetic field leads to the splitting of the separate IC mode, existing in the plasma without shear flow, into two IC modes, which correspond to different kinds of the IC instabilities with frequencies $\omega_{1,2}(\mathbf{k}) = k_z V_{0i}(x) + n\omega_{ci} + \delta\omega_{1,2}(\mathbf{k})$. At the case of weak flow shear, for which

$$\left(n\omega_{ci}\right)^2 A_{in}\left(k_\perp^2 \rho_i^2\right) > 4 \frac{k_y V'_{0i}}{k_z \omega_{ci}} k_z^2 v_{Ti}^2 \left(1 - A_{i0}\left(k_\perp^2 \rho_i^2\right) + \tau\right), \tag{5}$$

two kinetic IC instabilities develop. The first IC mode with

$$\mathrm{Re}\,\delta\omega_1(\mathbf{k}) = \delta\omega_{01}(\mathbf{k}) \approx \frac{n\omega_{ci} A_{in}\left(k_\perp^2 \rho_i^2\right)}{\left(1 - A_{i0}\left(k_\perp^2 \rho_i^2\right) + \tau\right)}, \tag{6}$$

and

$$\mathrm{Im}\,\delta\omega_1(\mathbf{k}) \approx \gamma_{e1} = -\frac{\left(\delta\omega_{01}(\mathbf{k})\right)^2}{\Omega_n} \tau \sqrt{\frac{\pi}{2}} \times \times \frac{\left(n\omega_{ci} - k_z\left(V_{0e} - V_{0i}\right)\right)}{|k_z| v_{Te}} \exp\left(-z_e^2\right), \tag{7}$$

is the current driven IC instability[3], modified by flow shear [6]. Here:

$$A_{\alpha n}\left(k_\perp^2 \rho_\alpha^2\right) = I_n\left(k_\perp^2 \rho_\alpha^2\right) e^{-k_\perp^2 \rho_\alpha^2}, \text{ and}$$

$$\Omega_n^2 = \left(n\omega_{ci}\right)^2 A_{in}^2 - 4 \frac{k_y V'_{0i}}{k_z \omega_{ci}} k_z^2 v_{Ti}^2 A_{in} \left(1 - A_{i0}\left(k_\perp^2 \rho_i^2\right) + \tau\right). \tag{8}$$

This instability develops due to inverse electron Landau damping, when $k_z\left(V_{0e} - V_{0i}\right) > n\omega_{ci}$, under condition $|\delta\omega_1(\mathbf{k}) / k_z \mathrm{v}_{Ti}| ? \; 1$ of negligible IC damping of the mode $\omega_1(\mathbf{k})$.

The second IC mode, $\omega_2(\mathbf{k})$ with

$$\mathrm{Re}\,\delta\omega_2(\mathbf{k}) = \delta\omega_{02}(\mathbf{k}) \approx \frac{k_y V'_{0i}}{k_z \omega_{ci}} \frac{k_z^2 v_{Ti}^2}{n\omega_{ci}}, \tag{9}$$

$$\mathrm{Im}\,\delta\omega_2(\mathbf{k}) \approx \gamma_{e2} = \frac{\left(\delta\omega_{02}(\mathbf{k})\right)^2}{\Omega_n} \tau \sqrt{\frac{\pi}{2}} \times \times \frac{\left(n\omega_{ci} - k_z\left(V_{0e} - V_{0i}\right)\right)}{|k_z| v_{Te}} \exp\left(-z_e^2\right) \tag{10}$$

becomes unstable for any values of $k_\perp \rho_i$ due to inverse electron Landau damping, when the velocity of the relative drift between ions and electrons is *below* the critical value V_{0c} [6], roughly estimated as $V_{0e}^{(c)} = V_{0i} + n\omega_{ci}/k_z$, i.e. under conditions at which the current driven IC instability modified by shear flow does not developed. Under conditions of strong flow shear, for which condition opposite to (5) met, two shear flow driven kinetic IC instabilities may be developed with frequency

$$\delta\omega_{(+,-)} \approx \pm k_z \mathrm{v}_{Ti} \left(\frac{k_y V'_{0i}}{k_z \omega_{ci}} \frac{A_{in}\left(k_\perp^2 \rho_i^2\right)}{\left(1 - A_{i0}\left(k_\perp^2 \rho_i^2\right) + \tau\right)}\right)^{1/2} \tag{11}$$

and the growth rate (10). Also in this case when $\Omega_n^2 < 0$ the shear flow driven IC instability of the hydrodynamic (reactive) type is excited with frequency $\delta\omega_{(H)}$ and growth rate $\gamma_{(H)}$ approximately equal to

$$\delta\omega_{(H)} ; \; \frac{1}{2} \frac{n\omega_{ci} A_{in}\left(k_\perp^2 \rho_i^2\right)}{\left(1 - A_{i0}\left(k_\perp^2 \rho_i^2\right) + \tau\right)}, \tag{12}$$

$$\gamma_{(H)} ; \; \frac{\left(\dfrac{k_y}{k_z} \dfrac{V'_{0i}}{\omega_{ci}} k_z^2 v_{Ti}^2 A_{in}\left(k_\perp^2 \rho_i^2\right)(1+\tau) - \dfrac{n\omega_{ci}}{4} A_{in}^2\left(k_\perp^2 \rho_i^2\right)\right)^{1/2}}{\left(1 - A_{i0}\left(k_\perp^2 \rho_i^2\right) + \tau\right)}.$$

An important, but still absent, element in the studies of IC instabilities of plasmas with parallel shear flow is an understanding of the processes of their nonlinear evolution and saturation. In this report we have presented the renormalized theory of the IC turbulence in the magnetic field-aligned plasma shear flow. The developed theory extends the earlier studies [7] of the renormalized theory of the IC turbulence by including a new combined effect of plasma turbulence and shear flow, which consists in turbulent scattering of ions across the shear flow into the regions with a greater or smaller flow velocity and their convection by shear flow, enhancing by this means transport of ions along shear flow. Analytically it manifests in nonlinear broadening of IC resonances. Our analysis bases on the renormalized solution of the Vlasov-Poisson system of equations. We use leading center coordinates for ions $X = x + (v_\perp/\omega_{ci})\sin\phi$, $Y = y - (v_\perp/\omega_{ci})\cos\phi$, where x, y, z are usual local particle coordinates with z-axis directed along the magnetic field $\mathbf{B}$, and where $v_\perp$ is velocity and ϕ is gyrophase angle of the gyromotion of ion. We find it suitable to use instead of z and ϕ new variables $z_1 = z - \int^t v_z(\tau)\,d\tau, \phi_1 = \phi + \omega_{ci}t$. With these variables the governing Vlasov equation describing the perturbation f_i by the self-consistent electrostatic potential Φ of the ion distribution function F_i, $F_i = F_{i0} + f_i$, where F_{i0} is the equilibrium function of the distribution of ions, is:

$$\frac{\partial f_i}{\partial t} + \frac{e}{m_i\omega_{ci}}\left(\frac{\partial\Phi}{\partial X}\frac{\partial f_i}{\partial Y} - \frac{\partial\Phi}{\partial Y}\frac{\partial f_i}{\partial X}\right) + \frac{e}{m_i}\frac{\omega_{ci}}{v_\perp}\left(\frac{\partial\Phi}{\partial\phi}\frac{\partial f_i}{\partial v_\perp} - \frac{\partial\Phi}{\partial v_\perp}\frac{\partial f_i}{\partial\phi}\right) -$$

$$-\frac{e}{m_i}\frac{\partial\Phi}{\partial z_1}\frac{\partial f_i}{\partial v_z} = \frac{e}{m_i\omega_{ci}}\frac{\partial\Phi}{\partial Y}\frac{\partial F_{i0}}{\partial X} - \frac{e}{m_i}\frac{\omega_{ci}}{v_\perp}\frac{\partial\Phi}{\partial\phi_1}\frac{\partial F_{i0}}{\partial v_\perp} + \frac{e}{m_i}\frac{\partial\Phi}{\partial z_1}\frac{\partial F_{i0}}{\partial v_z}.$$

This form of the Vlasov equation we consider as the most efficient for the deriving the renormalized solution for f_i. Using the system of equations for characteristics for the above presented Vlasov equation

$$dt = \frac{dX}{-\frac{e}{m_i\omega_{ci}}\frac{\partial\Phi}{\partial Y_1}} = \frac{dY}{\frac{e}{m_i\omega_{ci}}\frac{\partial\Phi}{\partial X_1}} = \frac{dv_\perp}{\frac{e}{m_i}\frac{\omega_{ci}}{v_\perp}\frac{\partial\Phi}{\partial\phi_1}} =$$

$$= \frac{d\phi_1}{-\frac{e}{m_i}\frac{\omega_{ci}}{v_\perp}\frac{\partial\Phi}{\partial v_\perp}} = \frac{dv_z}{-\frac{e}{m_i}\frac{\partial\Phi}{\partial z_1}} = \tag{13}$$

$$= \frac{df_i}{\frac{e}{m_i\omega_{ci}}\frac{\partial\Phi}{\partial Y}\frac{\partial F_{i0}}{\partial X} - \frac{e}{m_i}\frac{\omega_{ci}}{v_\perp}\frac{\partial\Phi}{\partial\phi_1}\frac{\partial F_{i0}}{\partial v_\perp} + \frac{e}{m_i}\frac{\partial\Phi}{\partial z_1}\frac{\partial F_{i0}}{\partial v_z}},$$

the following nonlinear solution for f_i with known F_{i0} is obtained:

$$f_i = \frac{e}{m}\int\left[\frac{1}{\omega_{ci}}\frac{\partial\Phi}{\partial Y}\frac{\partial F_{i0}}{\partial X} - \frac{\omega_{ci}}{v_\perp}\frac{\partial\Phi}{\partial\phi_1}\frac{\partial F_{i0}}{\partial v_\perp} + \frac{\partial\Phi}{\partial z_1}\frac{\partial F_{i0}}{\partial v_z}\right]dt' \tag{14}$$

Supposing that the particle orbit disturbance δX due to the electrostatic plasma turbulence is sufficiently small, we find the solutions for system (13) in the form:

$$X = \bar{X} + \delta X, \qquad \delta X = -\frac{e}{m_i\omega_{ci}}\int\frac{\partial\Phi}{\partial\bar{Y}}dt_1,$$

$$Y = \bar{Y} + \delta Y, \qquad \delta Y = \frac{e}{m_i\omega_{ci}}\int\frac{\partial\Phi}{\partial\bar{X}}dt_1,$$

$$v_\perp = \bar{v}_\perp + \delta v_\perp, \qquad \delta v_\perp = \frac{e}{m_i}\frac{\omega_{ci}}{v_\perp}\int\frac{\partial\Phi}{\partial\bar{\phi}}dt_1, \tag{15}$$

$$\phi = \bar{\phi} + \delta\phi, \qquad \delta\phi = -\frac{e}{m_i}\frac{\omega_{ci}}{v_\perp}\int\frac{\partial\Phi}{\partial\bar{v}_\perp}dt_1,$$

$$v_z = \bar{v}_z + \delta v_z, \qquad \delta v_z = -\frac{e}{m}\int_0^t\frac{\partial\Phi}{\partial\bar{z}}dt_1,$$

where $\bar{X}$ and $\bar{Y}$ are the guiding center coordinates averaged over the turbulent pulsations. In these solutions the perturbed electrostatic potential Φ is defined in variables $\bar{X}$, $\bar{Y}$, $\bar{v}_\perp$, $\bar{\phi}$, $\bar{v}_z$ and δX, δY, $\delta v_\perp$, $\delta\phi$, δv_z as

$$\Phi(\mathbf{r},t) = \sum_{n=-\infty}^{\infty}\int d\mathbf{k}J_n\left(\frac{k_\perp\bar{v}_\perp}{\omega_{ci}}\right)\exp[ik_x\bar{X} + ik_y\bar{Y} + ik_z\bar{z} +$$

$$+ik_z\left(\bar{v}_z + V_{i0}(\bar{X})\right)t - in\left(\bar{\phi} - \omega_{ci}t - \theta\right)]\exp\left(i\mathbf{k}\delta\mathbf{r}(t)\right)\times \tag{16}$$

$$\times\int d\omega\Phi(\mathbf{k},\omega)\exp(-i\omega t),$$

where the perturbations of the ions orbits due to wave-ion interactions,

$$\mathbf{k}\delta\mathbf{r}(t) = k_x\delta X(t) + k_y\delta Y(t) + k_z\delta z(t) -$$

$$-\frac{k_\perp\delta v_\perp(t)}{\omega_{ci}}\sin(\phi - \theta) - \frac{k_\perp\bar{v}_\perp}{\omega_{ci}}\cos(\phi - \theta)\delta\phi(t), \tag{17}$$

with

$$\delta z(t) = \int\delta v_z(t_1)\,dt_1 + V_0'(\bar{X})\int\delta X(t_1)\,dt_1$$

are included. The Fourier transformed Poisson's equation,

$$\Phi(\mathbf{k},\omega) = -\frac{4\pi}{k^2}\sum_{\alpha=i,e}e_\alpha\delta n_\alpha(\mathbf{k},\omega) = -\frac{4\pi}{k^2}\sum_{\alpha=i,e}e_\alpha\int f_\alpha d\mathbf{v},$$

and Eq.(14) in which the potential $\Phi(\mathbf{r},t)$ is related with $\Phi(\mathbf{k},\omega)$ by Eq.(16) with $\delta\mathbf{r}$ determined by Eq.(17), compose the system of nonlinear integral equations for $\Phi(\mathbf{k},\omega)$ and f_α. Assuming the equilibrium distribution function $F_{\alpha 0}$ to be a drifting Maxwellian

$$F_{i0} = \frac{n_{i0}(\bar{X})}{\left(2\pi v_{Ti}^2\right)^{3/2}}\exp\left(-\frac{v_\perp^2}{v_{Ti}^2} - \frac{\left(v_z - V_{i0}(\bar{X})\right)^2}{v_{Ti}^2}\right),$$

we obtain from this system the nonlinear dispersion equation which accounted for the scattering of ions by turbulence in shear flow

$$1 + \varepsilon_{0i}(\mathbf{k},\omega) + \varepsilon_e(\mathbf{k},\omega) + \varepsilon_{sh}(\mathbf{k},\omega) = 0,. \tag{18}$$

In Eq.(18) $\varepsilon_{0i}(\mathbf{k},\omega)$ and $\varepsilon_e(\mathbf{k},\omega)$ are known [6] linear permittivities of ion and electron components of parallel plasma shear flow, and $\varepsilon_{sh}(\mathbf{k},\omega)$ is nonlinear ion permittivity, which accounted for combined effect of ions scattering by IC electrostatic turbulence with fluctuating potential $\Phi(\mathbf{k}_1)$ and their convection by shear flow

$$\varepsilon_{sh}(\mathbf{k},\omega) = \frac{2}{k^2\lambda_{Di}^2}\sum_{n=-\infty}^{\infty} -\frac{n\omega_{ci}}{(\delta\omega)^3}A_{in}\left(C_2 + 3i\frac{C_3}{\delta\omega}\right) -$$

$$-\frac{6}{k^2\lambda_{Di}^2}\left(1 - \frac{k_y V_0'}{k_z\omega_{ci}}\right)\sum_{n=-\infty}^{\infty}\frac{k_z^2 v_{Ti}^2}{(\delta\omega)^4}A_{in}\left(C_2 + 4i\frac{C_3}{\delta\omega}\right). \tag{19}$$

In Eq.(19) terms C_2 and C_3 are determined by nonlinear equations:

$$C_2 = \frac{e^2}{2m_i^2\omega_{ci}} \mathrm{Re} \sum_{n_1=-\infty}^{\infty} \int d\mathbf{k}_1 \,|\,\Phi(\mathbf{k}_1)\,|^2 \, e^{-k_\perp^2\rho_i^2} \times$$
$$\times k_z k_{1z}\left(1+\frac{k_{1y}V'_{i0}}{k_{1z}\omega_{ci}}\right)\left(k_x k_{1y} - k_{1x}k_y\right)\times \tag{20}$$
$$\times I_{n_1}\left(k_\perp^2\rho_i^2\right)\int_0^\infty d\tau \exp\Big[-i\left(\omega - n\omega_c - k_z V_{i0}\left(\bar{X}\right)\right)\tau -$$
$$-\frac{1}{2}\langle(\mathbf{k}_1\delta\mathbf{r}(\tau))^2\rangle\Big],$$

$$C_3 = \frac{e^2}{3m_i^2} \mathrm{Re} \sum_{n_1=-\infty}^{\infty} \int d\mathbf{k}_1 \,|\,\Phi(\mathbf{k}_1)\,|^2 \times$$
$$\times I_{n_1}\left(k_\perp^2\rho_i^2\right) e^{-k_\perp^2\rho_i^2} k_z^2 k_{1z}^2 \left(1+\frac{k_{1y}V'_{i0}}{k_{1z}\omega_{ci}}\right)^2 \times \tag{21}$$
$$\times\int_0^\infty d\tau \exp\Big[-i\left(\omega - n\omega_c - k_z V_{i0}\left(\bar{X}\right)\right)\tau - \frac{1}{2}\langle(\mathbf{k}_1\delta\mathbf{r}(\tau))^2\rangle\Big],$$

where $\bar{X}$ is the guiding center coordinate averaged over the turbulent pulsations and

$$\frac{1}{2}\left\langle(\mathbf{k}\cdot\delta\mathbf{r})^2\right\rangle = \frac{1}{2}\left\langle(\mathbf{k}\cdot\delta\mathbf{r})^2\right\rangle_0 + k_x k_z \left\langle \delta X \delta z\right\rangle +$$
$$+ k_y k_z \left\langle \delta Y \delta z\right\rangle + \frac{1}{2}k_z^2\left\langle(\delta z)^2\right\rangle, .$$

The term

$$\frac{1}{2}\left\langle(\mathbf{k}\cdot\delta\mathbf{r})^2\right\rangle_0 = k_x^2\left\langle(\delta X)^2\right\rangle + k_y^2\left\langle(\delta Y)^2\right\rangle + \tag{22}$$
$$+2k_x k_y\left\langle\delta X\delta Y\right\rangle + \frac{1}{2}\frac{k_\perp^2 \mathrm{v}_\perp^2}{\omega_c^2}\left\langle(\delta\phi)^2\right\rangle + \frac{1}{2}\frac{k_\perp^2}{\omega_c^2}\left\langle(\delta \mathrm{v}_\perp)^2\right\rangle = C_1 t$$

accounts for the turbulent scattering of the guiding center coordinates X and Y, as well as velocity coordinates $\mathrm{v}_\perp$ and ϕ of the ion Larmor orbit with [7]

$$C_1 = \frac{e^2}{2m_i^2\omega_{ci}^2} \mathrm{Re} \sum_{n_1=-\infty}^{\infty} \int d\mathbf{k}_1 \,|\,\Phi(\mathbf{k}_1)\,|^2 \times$$
$$\times e^{-k_\perp^2\rho_i^2}\Big[2\left(k_x k_{1y} - k_{1x}k_y\right)^2 I_{n_1}\left(k_\perp^2\rho_i^2\right) +$$
$$+\frac{k_\perp^2 k_{1\perp}^2}{2}\left(I_{n_1+1}\left(k_\perp^2\rho_i^2\right) + I_{n_1-1}\left(k_\perp^2\rho_i^2\right)\right)\Big]\times \tag{23}$$
$$\times\int_0^\infty d\tau \exp\Big[-i\left(\omega - n\omega_c - k_z V_{i0}\left(\bar{X}\right)\right)\tau - \frac{1}{2}\langle(\mathbf{k}_1\delta\mathbf{r}(\tau))^2\rangle\Big], .$$

The term C_1 exists in magnetized plasma without shear flow [7]. The terms C_2 and C_3, which are determined as

$$k_x k_z \left\langle \delta X \delta z\right\rangle + k_y k_z \left\langle \delta Y \delta z\right\rangle + \frac{1}{2}k_z^2\left\langle(\delta z)^2\right\rangle =$$
$$= C_2 t^2 + C_3 t^3,$$

with

$$\delta z(t) = \int \delta \mathrm{v}_z(t_1)\,dt_1 + V_0'\left(\bar{X}\right)\int \delta X(t_1)\,dt_1, \tag{24}$$

account for the known weak scattering of the ions along the magnetic field (first term in Eq.(24)) and much more stronger effect of the ehanced random convection of ions due to their scattering accross the flow the regions with a greater or smaller flow velocity (second term in Eq.(24)). We have derived the approximate solution to Eq. (19) in the form

$$\omega(\mathbf{k}) = \omega_0(\mathbf{k}) - iC_1 - \frac{\Delta\varepsilon_e(\mathbf{k},\omega) + \varepsilon_{sh}(\mathbf{k},\omega_0(\mathbf{k}))}{\dfrac{\partial\varepsilon_{0i}(\mathbf{k},\omega_0)}{\partial\omega_0}}, \tag{25}$$

where $\omega_0(\mathbf{k})$ is the solution of the equation $1+\varepsilon_{0i}(\mathbf{k},\omega) = 0$ and

$$\gamma(\mathbf{k}) = \mathrm{Im}\,\omega(\mathbf{k}) = \gamma_0(\mathbf{k}) - C_1 + \gamma_{sh}(\mathbf{k}), \tag{26}$$

with

$$.\,\gamma_0 = -\frac{\mathrm{Im}\Delta\varepsilon_e(\mathbf{k},\omega_0(\mathbf{k}))}{\dfrac{\partial\varepsilon_{0i}(\mathbf{k},\omega_0)}{\partial\omega_0}}, \quad \gamma_{sh} = -\frac{\mathrm{Im}\varepsilon_{sh}(\mathbf{k},\omega_0(\mathbf{k}))}{\dfrac{\partial\varepsilon_{0i}(\mathbf{k},\omega_0)}{\partial\omega_0}}.$$

We use this approximate solution for qualitative analysis of the role of the discovered effect in nonlinear evolution of the shear flow modified current driven IC instability and shear flow driven kinetic IC instabilities. We have considered the limits of weak and strong flow shear for short wavelength and long wavelength parts of the spectrum of IC waves separately and have arrived at the following conclusions.

4. CONCLUSIONS

1. The shear flow modified current driven IC instability, which develops under condition of weak flow shear (5), saturates as the ordinary IC instability driven by shearless current[6] on the high level of the IC turbulence in the steady state, $e\tilde{\Phi}/T_e$, where $\tilde{\Phi} = \left(\int|\,\Phi(\mathbf{k}_1)\,|^2\, d\mathbf{k}_1\right)^{1/2}$ is the root-mean-square (rms) magnitude of the perturbed electrostatic potential

$$\frac{e\tilde{\Phi}}{T_i} \sim 1 \tag{27}$$

in the long wavelength, $k\rho_i << 1$, part of the spectrum, and on a very low level

$$\frac{e\tilde{\Phi}}{T_i} \sim \frac{1}{\left(k_\perp\rho_i\right)^{5/2}} \propto \left(\frac{\mathrm{v}_{Ti}}{\mathrm{v}_{Te}}\frac{V'_{i0}}{\omega_{ci}}\right)^{5/4} \tag{28}$$

in the short wavelength, $k\rho_i >> 1$, part. The effect of the shear flow, determined by the term $\gamma_{sh}(\mathbf{k})$, is negligible on the saturation of this instability.

2. The long wavelength part of the spectrum of the shear flow driven IC instability under condition of weak flow shear saturates due to combined effect of shear flow and turbulent scattering of ions, determined by the nonlinear damping rate $\gamma_{sh}(\mathbf{k})$ in Eq.(26) with

$$\gamma_{sh}^{(2)} \sim -\frac{24\left(\tau + k_\perp^2\rho_i^2\right)C_3}{1+\tau}\frac{\omega_{ci}^2}{k_z^4 v_{Ti}^4},$$

where C_3 is determined by integral equation

$$C_3 + \frac{2e^2}{m_i^2}\sum_{n_1=-\infty}^{\infty}\int d\mathbf{k}_1\,|\,\Phi(\mathbf{k}_1)\,|^2\, A_{in}\left(k_{1\perp}^2\rho_i^2\right)\times$$
$$\times k_z^2 k_{1z}^2\left(\frac{k_{1y}V'_{i0}}{k_{1z}\omega_{ci}}\right)^2\frac{C_3}{\left(\delta\omega_{02}\right)^4}.$$

This instability saturates on levels

$$\frac{e\tilde{\Phi}}{T_i} \sim \left(\frac{\omega_{ci}}{V'_{i0}}\right)^2 \quad \text{or} \quad \frac{e\tilde{\Phi}}{T_i} \sim \frac{v_{Ti}}{v_{Te}}, \qquad (29)$$

depending on $\omega_{ci}/V'_{i0} < \left(v_{Ti}/v_{Te}\right)^{1/2}$ or opposite condition holds. These levels appear to be much lower then the corresponding level (27) for current driven IC instability modified by flow shear.

3. The saturation of the short wavelength spectrum subrange of the shear flow driven instability under conditions of weak (5), as well as strong (when condition opposite to (5) met) flow shear arises from the turbulent scattering of ions by IC turbulence as in shearless plasma and determined by term C_1 in Eq.(26}). It occurs at very low level in the range

$$\frac{v_{Ti}}{v_{Te}}\left(\frac{V'_{0i}}{\omega_{ci}}\right)^{3/2} \lesssim \frac{e\tilde{\Phi}}{T_i} \lesssim \left(\frac{v_{Ti}}{v_{Te}}\frac{V'_{0i}}{\omega_{ci}}\right)^{5/4}, \qquad (30)$$

in the case of weak flow shear and on level

$$\frac{e\tilde{\Phi}}{T_i} \sim \left(\frac{v_{Ti}V'_{0i}}{v_{Te}\omega_{ci}}\right)^{7/4} \qquad (31)$$

in the case of strong flow shear respectively. As a demonstration of level (30) we take the data detected in magnetopause by satellite Prognoz-8 [8], $B = 2\cdot 10^{-3}G$, $V'_{0i}/\omega_{ci} = 0.5$, $T_e \sim T_i = 100eV$, and obtain the estimate $\tilde{E} \approx 0.4\cdot 10^{-3}V/m$ at fundamental cyclotron mode, which is in good agreement with measured value[8] $\tilde{E} \sim 0.5\cdot 10^{-3}V/m$.

4. Nonlinear evolution of the long wavelength part of the IC turbulence spectrum developed by the shear flow driven instability under conditions of strong flow shear is determined by the $\gamma_{sh}(\mathbf{k})$ term in Eq.(26). In this case combined effect of shear flow and turbulent scattering of ions introduces a principally new effect into the nonlinear development of the IC turbulence, which is absent in shearless plasma flows. It is the shear flow driven nonlinear instability with growth rate

$$\gamma_{sh}(\mathbf{k}) \sim \frac{24}{1+\tau} C_3 A_{in}\left(k_\perp^2\rho_i^2\right)\frac{k_z^2 v_{Ti}^2}{\left(\delta\omega_+\right)^4}\left|\frac{k_y V'_{0i}}{k_z\omega_{ci}}\right|, \qquad (32)$$

where C_3 is determined by the integral equation

$$\begin{aligned} &C_3 + \frac{2e^2}{m_i^2}\sum_{n_1=-\infty}^{\infty}\int d\mathbf{k}_1 \,|\,\Phi(\mathbf{k}_1)|^2 \times \\ &\times A_{in}\left(k_{1\perp}^2\rho_i^2\right)k_z^2 k_{1z}^2\left(\frac{k_{1y}V'_{i0}}{k_{1z}\omega_{ci}}\right)^2\frac{C_3}{\left(\delta\omega_+\right)^4} = \\ &= \frac{e^2}{3m_i^2}\sum_{n_1=-\infty}^{\infty}\int d\mathbf{k}_1 \,|\,\Phi(\mathbf{k}_1)|^2 A_{in}\left(k_{1\perp}^2\rho_i^2\right)\times \\ &\times k_z^2 k_{1z}^2\left(\frac{k_{1y}V'_{i0}}{k_{1z}\omega_{ci}}\right)^2\frac{\gamma_{01}}{\left(\delta\omega_+\right)^2}. \end{aligned} \qquad (33)$$

Above level $e\tilde{\Phi}T_i \gtrsim \tau^{3/2}A_{in}^{1/2}$ growth rate (32) becomes greater than the corresponding linear growth rate $\gamma_{02}(\mathbf{k})$ determined by Eq.(10). It contrasts dramatically with effect of ion scattering by IC turbulence in shearless plasma flows [7], where it leads only to saturation of IC instability.

ACKNOWLEDGEMENTS

This work was supported in part by Fundamental Researches State Fund of Ukraine under Project No 25.2/153.

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Article received 27.11.08

ТУРБУЛЕНТНОСТЬ СДВИГОВЫХ ТЕЧЕНИЙ ПЛАЗМЫ В МАГНИТНОМ ПОЛЕ

В.С. Михайленко, К.Н. Степанов, В.В. Михайленко, Н.А. Азаренков, Д.В. Чибисов

Дан обзор новых результатов авторов по аналитическому исследованию турбулентности сдвиговых течений плазмы в магнитном поле. Рассмотрена дрейфовая турбулентность в течениях плазмы с сильно неоднородным градиентом скорости течения с приложениями к режимам улучшенного удержания тороидальной плазмы и ионная циклотронная турбулентность сдвиговых течений плазмы вдоль магнитного поля с приложениями к аномальному нагреву ионов в ионосфере.

ТУРБУЛЕНТНІСТЬ ЗСУВНИХ ТЕЧІЙ ПЛАЗМИ У МАГНІТНОМУ ПОЛІ

В.С. Михайленко, К.М. Степанов, В.В. Михайленко, М.О. Азарєнков, Д.В. Чибисов

Дано огляд нових результатів авторів з аналітичного дослідження турбулентності зсувних течій плазми у магнітному полі. Розглянуто дрейфову турбулентність плазми у течіях з сильно неоднорідним градієнтом швидкості течії з застосуваннями до режимів поліпшеного утримання тороїдальної плазми та іонну циклотронну турбулентність зсувних течій плазми вздовж магнітного поля із застосуванням до аномального нагрівання іонів у іоносфері.