KINK MACROINSTABILITIES AND RESISTIVE LAYER STRUCTURE OF INTERNAL KINKS IN CYLINDRICAL PLASMA

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Eigen-frequencies and shapes of the non-local magnetic kink-modes m=1, n>1 in a field reversal pinch are obtained in cylindrical force-free approximation by shooting method. The mode profile and the resistive layer containing the resonant surface of the internal kinks are calculated numerically. Instability is revealed, both for external and internal modes, which is not effected by resistive layer in plasmas with high conductivity, it is determined by the current gradient due to the high paramagnetic parameter in the pinch core.

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1. INTRODUCTION

At present, within the framework of quasi-single helicity (QSH) modern conception for the reversed-field pinch (RFP) laboratory plasma [1], the task is set to explain the nature of the frequency spectrum and to describe the space structure of dominant modes in QSH states. These modes turn out to be internal ones and answer to the condition of $F = kB_z - mB_\theta/r = 0$ on a resonant surface r_s, $0<r_s<a$ (a is the plasma column radius). Therefore, the tearing instability of resonant modes is considered commonly as a main basis for description of mode stability. However, the tearing theory does not determine the real frequencies of oscillations observed in usual kHz MHD spectra giving only increments of aperiodic instability which are low in the MHD scale. Moreover, the theory provides solutions localized close to the resonant surface, whereas the modes of the QSH spectrum don't demonstrate any real link to resonant surfaces. Instead, they show the result of non-local mechanism of mode excitation. In the theory of z-pinches the question remains: what total profiles of real modes actually occur with the presence of resonant surfaces inside pinches?

In this report, the radial profiles and frequencies of MHD modes, both external and internal ones, are presented on basis of general MHD theory taking into calculations Hall effect and the high plasma conductivity. Eigen-value boundary problem is described and a few shooting method results are presented.

2. THEORETICAL MODEL

We study the stability of a FRP configuration taking into account the Hall-effect under arbitrary parameter $\Pi = 4\pi e^2a^2N_0/Mc^2$ (the "linear ion" introduced by Braginskii as provided by ratios $\omega_h:\omega_A:\omega_{Bi} = 1:\Pi^{1/2}:\Pi$), where a, M, N are pinch radius, ion mass and plasma density correspondently, ω_h is "helicon frequency", $\omega_h = cB_0/4\pi ea^2N_0$, ω_A and ω_{Bi} are commonly used Alfven and ion cyclotron frequencies. In cylindrical coordinates (r,θ,z), when ω_h, a, $N_0(0)$, $B_0(0)$ are assumed as an units for ω, r, $N(r)$, $\mathbf{B}=\mathbf{B}_0(r)+ \delta\mathbf{B}(r)e^{-i\omega t+i\zeta}$ ($\zeta= kz-m\theta$ is the helical phase), oscillations are governed by equations:

$$\delta\mathbf{B} = \mathrm{rot}(\xi\times\mathbf{B}_0 + i\omega\xi/\Pi - i\eta\,\mathrm{rot}\mathbf{B}/\omega),$$
$$\omega^2N_0\,\xi = \Pi\{(\delta\mathbf{B}\times\mathrm{rot}\mathbf{B}_0) + (\mathbf{B}_0\times\mathrm{rot}\delta\mathbf{B})\}. \qquad (1)$$

Here ξ is a plasma displacement, $\eta = (c^2/4\pi\sigma\omega_h a^2) = \nu_e/\omega_{Be}N$, where ν_e and ω_{Be} are electron collisional and cyclotron frequencies, σ is a plasma conductivity. Fluctuations δN do not arrive in the set of equations (1) because the plasma convection is not taken into account and the "resistivity" η is assumed invariable. Also Eq. (1) disregard the plasma pressure but take into account effects of sharpened gradients of magnetic pressure in RFP.

In reality, in the PRP plasma core, the equilibrium is very close to the force-free one, $\mathrm{rot}\mathbf{B}_0\times\mathbf{B}_0 = 0$. We use only cylindrical force-free configuration, $\mathrm{rot}\mathbf{B}_0 = \lambda\mathbf{B}_0$, where $\lambda(r)$ is compatible uniquely with the real radial distribution of the safety factor $q(r)=arB_z/RB_\theta$ [1] under any choice of the aspect ratio R/a (R is major toroidal radius). In our model of magnetic configuration $\lambda(0) = 4$ thus the considerable paramagnetic pinch effect is taken into consideration, which is close to reality [1]. The radial distributions of B_z and B_θ used in our calculations are plotted in Fig. 1 jointly with $\lambda(r)/4$, $q(r)$ and $F(r)$.

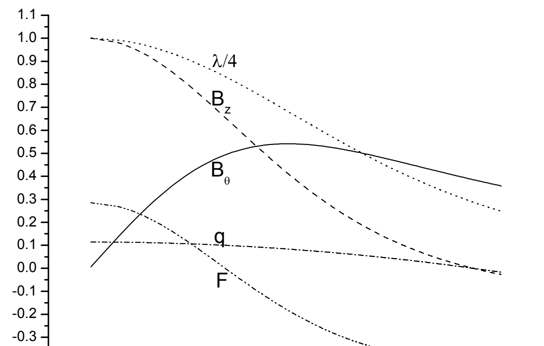

Fig.1. .Magnetic configuration $B_z(r)$, $B_\theta(r)$ and shapes of $\lambda(r)/4$, $q(r)$, $F(r)$ for internal mode m=1, n=10 (Π=40)

To consider the situation with discrete toroidal modes one can set $k=n/R$ (n is major toroidal number) and write the factor F through the safety factor: $F= (kRB_\theta/r)(q-m/n)$. Under our choice of magnetic configuration the value k=2 determines the boundary between external kinks m=1: k<2, $F(r)$<0 (0<r<1), n=9, 8, 7,…., or internal ones: k>2, $F(r_s)$=0 (0<r_s<1), n=10, 11, 12,… The case a/R=0.2285, k=2.285 chosen in Fig.1 corresponds to the internal mode with toroidal number $n=kR=10$. The resonant surface is given by equality $F(r)=0$ at $r=r_s=0.254$.

The differential problem (1) may be reduced to the more convenient form by means of non-divergent representation of **B** most suitable for helical analysis:

$$\mathbf{B} = \nabla\chi\times\mathbf{s} + B_s\mathbf{s},$$
$$\mathbf{s} = rs^2\,\nabla\zeta\times\mathbf{e}_r. \qquad (2)$$

Owing to the resistivity smallness, $\eta \ll 1$, the ideal consideration can be assumed for external modes, as well as for internal ones outsides resonant layers. By setting $\eta=0$, one can reduce the set of equations (1) to the ordinary two-order differential equation,

$$\frac{1}{rs^2}\frac{d}{dr}\left(rs^2 A\frac{d}{dr}\delta\chi\right) - \frac{A}{r^2s^2}\delta\chi = Q\delta\chi, \qquad (3)$$

where A and Q are an rational functions of ω with coefficients depending on r through the magnetic field components, plasma density N and their r-derivatives. Eq. (1) introduce the two point value problem under conditions $\delta\chi = r^{|m|}$ at $r\to 0$ and $\delta\chi(1) = 0$.

In a vicinity of the resonance $r=r_s$ the condition $\eta > 0$ must be taken into account. Eqs. (1) need more analysis in order to be reduced to the standard form suitable for numerical solution. The transformation is achieved by introducing the additional variable:

$$\delta\Xi = B_{s0}\left(\xi_r - \frac{i\omega}{\Pi}\frac{\vec{\xi}\vec{B}_0^*}{B_0^2}\right) - \frac{i\eta}{\omega}\frac{d}{dr}\delta B_s. \qquad (4)$$

Here $\mathbf{B}_0^*=\mathbf{B}_0\times\mathbf{e}_r$. Then eqs. (1) can be transcribed as a general ordinary differential problem for four-component vector-function $\mathbf{y}=(\delta\Psi, \delta\Psi', \delta B_s, \delta\Xi)$,

$$\mathbf{y}'(x) = \hat{\mathbf{A}}\mathbf{y}, \qquad (5)$$

where $x=r-r_s$. $\hat{\mathbf{A}}$ is a 4×4 matrix. The limit $\eta=0$ introduces the singularity into the problem imposing the unphysical requirement of wave reflection at the resonance surface: $\delta\chi(r_s) = 0$ if $\eta=0$. In reality, $\eta = 10^{-4}$–10^{-7}, so this zero constrain disappears but very small resistivity allows only very low values of complex magnitudes $\delta\chi(r_s)$ to be possible inside resistive layer.

The smallness of η introduces high stiffness into the set of equations (5) so to obtain solution inside the resistive layer we engage in calculations the *stiff* program from collection [2]. Being irreversible the *stiff* algorithm need positive definition of the matrix $\hat{\mathbf{A}}$. Unfortunately the $\hat{\mathbf{A}}$ loses its positive definition at $x=0$, and thus to avoid numerical errors we use two-way shooting method continuously sewing together the oppositely directed ideal trajectories $\mathbf{y}(x)$ outside some vicinity of the resonance at points $x=-\Delta x$ and $x=\Delta x$ with the stiff solutions inside the interval $(-\Delta x, \Delta x)$. The calculations show no noticeable effects of the Δx on the shape $\delta\chi(r)$, they do show effect of the η on the deepness of the profile dip inside resistive layer. The presented results are obtained for $\eta = 10^{-4}$.

3. RESULTS

Figs.2a-d illustrate radial distributions of absolute values of the complex amplitude $|\delta B_r(r)| = |\delta\chi(r)|/r$ for different kinks calculated under the parameter $\Pi=40$ according to the actual experimental data. They relate to the class of non-local perturbations which cover entire plasma volume bound by high conductive walls.

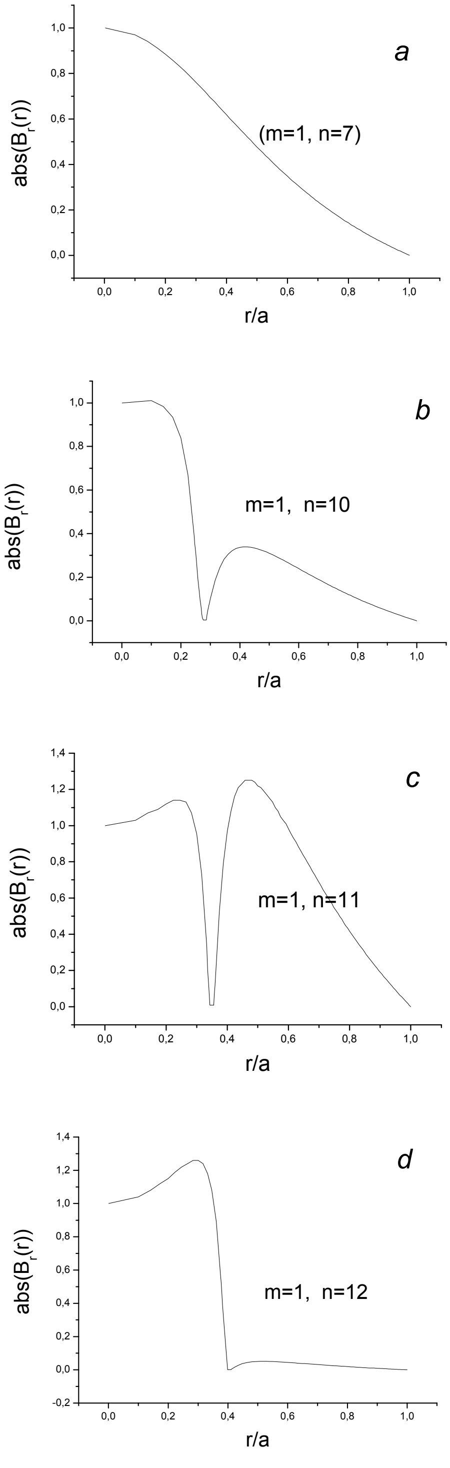

Fig.2. The shape $|\delta B_r(r)|$ of external kink: a) m=1, n=7; b) m=1, n=10; c) m=1, n=11 ; d) m=1, n=12 (Π=40)

All shapes are normalized to the value $\delta B_r(0) = 1$. In Fig. 2a the typical shape of external non-local mode (m=1, n=7) is plotted which has no nulls or dips within interval $(0<r<1)$. In contrast to the simple case of external modes, in the case of internal ones, Fig. 2b-d, displays complicated non-monotonous behavior of the radial distribution $|\delta B_r(r)|$ of internal modes. Resonant dips arrive against respective r_s for every internal mode. The dips, shown in Fig. 2b-d, locate at the r_s: b — r_s = 0.265; c — r_s = 0.364; d — r_s = 0.394.

The modes considered in the frame of our model turn out unstable. The complex eigen-frequencies, ω+iγ, calculated by the shooting method are characterized by the increments γ of order of 1 in the Hall scale, whereas frequencies ω turn out of order 0.01. The eigen-frequencies of basic internal modes are given in the Table.

Eigen-frequencies ω+iγ of internal modes (m,n)

(m,n)	*(1,10)*	*(1,11)*	*(1,12)*
ω+iγ	0.010+*i*1.69	0,0047+*i*1.71	0.0037+*i*1.53

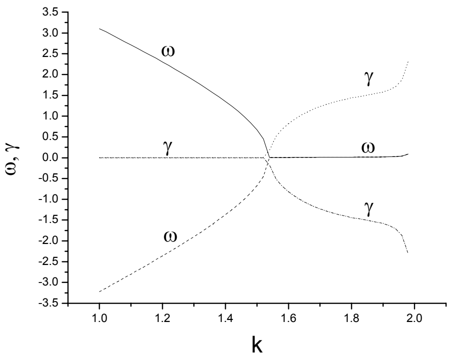

*Fig. 3. The dispersion of complex frequency ω+iγ for external modes, 1<k<2, (*Π*=40)*

The instability occurs under k<2 as far back as k=0,156 in the case of external modes as it follows from Fig. 3.

CONCLUSIONS

We ensured that the non-local kink-modes are unstable, i.e. be capable of self-excitation in the RFP,. in the frame of the weakly non-ideal model (1). The increments are found to be of order of ω_h. They are introduced by the magnetic gradient including second radial derivatives of magnetic components $\mathbf{B}_0(r)$, i.e. by high current gradients in paramagnetic RFP plasmas, independently from presence of resistive layers. This conclusion can be analytically confirmed by integration of (2) in the limit $\Pi\to\infty$. However, in this case the real frequencies ω vanish, and therefore calculated and observed 10-100 kHz kink spectra can be entirely linked with the Hall term in Eq. (1). Without doubt, plasma convection (rotation and radial transport) must be involved into consideration to explain the actually observed stable spectra as well as a smaller importance of the resonant peculiarities than when it is obtained in the frame of the basic MHD description (1).

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УСТОЙЧИВОСТЬ ВИНТОВЫХ КОЛЕБАНИЙ И СТРУКТУРА РЕЗИСТИВНОГО СЛОЯ ВНУТРЕННИХ МОД В ЦИЛИНДРИЧЕСКОЙ ПЛАЗМЕ

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Сформулирована краевая задача и численным методом стрельбы определены собственные частоты и профили винтовых кинк-мод m=1, n>1 в пинче с обращенным полем. В цилиндрическом приближении рассчитаны профили нелокальных мод, включая резистивные слои в случае внутренних мод. Показана неустойчивость мод в плазме с высокой проводимостью, которая определяется градиентом тока и не связана непосредственно с наличием резистивных слоев внутренних мод в плазме с высокой проводимостью.

СТАБІЛЬНІСТЬ ГВИНТОВИХ КОЛИВАНЬ ТА СТРУКТУРА РЕЗИСТИВНОГО ШАРУ ВНУТРІШНІХ КІНКІВ В ЦИЛІНДРИЧНІЙ ПЛАЗМІ

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Сформульована крайова задача та чисельним методом стрільби визначені власні частоти й профілі гвинтових кінк-мод m=1, n>1 в пінчі з оберненим полем. В циліндричному наближенні розраховані профілі нелокальних мод, включно з резистивними шарами в разі внутрішніх мод. Доведена нестійкість мод в плазмі з великою провідністю, яка визначається градієнтом струму й не пов’язана безпосередньо з наявністю резистивного шару внутрішніх мод у плазмі з великою провідністю.