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**V. V. Kurta**

# A Liouville comparison principle for solutions to semilinear parabolic second-order partial differential inequalities in the whole space

*(Presented by Corresponding Member of the NAS of Ukraine V. Ya. Gutlyanskii)*

*We obtain a new Liouville comparison principle for weak solutions $(u, v)$ to semilinear parabolic second-order partial differential inequalities of the form*

$$u_t - \mathcal{L}u - |u|^{q-1}u \geqslant v_t - \mathcal{L}v - |v|^{q-1}v \qquad (*)$$

*in the whole space $\mathbb{R} \times \mathbb{R}^n$. Here, $n \geqslant 1$, $q > 1$, and*

$$\mathcal{L} = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left[a_{ij}(t,x)\frac{\partial}{\partial x_j}\right],$$

*where $a_{ij}(t,x)$, $i$, $j = 1, \ldots, n$, are functions that are defined, measurable, and locally bounded in $\mathbb{R} \times \mathbb{R}^n$ and such that $a_{ij}(t,x) = a_{ji}(t,x)$ and*

$$\sum_{i,j=1}^{n} a_{ij}(t,x)\xi_i\xi_j \geqslant 0$$

*for almost all $(t,x) \in \mathbb{R} \times \mathbb{R}^n$ and all $\xi \in \mathbb{R}^n$. We show that the critical exponents in the Liouville comparison principle obtained, which are responsible for the non-existence of non-trivial (i. e., such that $u \not\equiv v$) weak solutions to $(*)$ in the whole space $\mathbb{R} \times \mathbb{R}^n$, depend on the behavior of the coefficients of the operator $\mathcal{L}$ at infinity and coincide with those obtained for solutions of $(*)$ in the half-space $\mathbb{R}_+ \times \mathbb{R}^n$. As direct corollaries, we obtain new Liouville-type theorems for non-negative weak solutions $u$ $(*)$ in the whole space $\mathbb{R} \times \mathbb{R}^n$ in the case where $v \equiv 0$. All the results obtained are new and sharp.*

**Introduction and preliminaries.** This work may be considered as a supplement to paper [1] and is devoted to a new Liouville comparison principle for weak solutions to parabolic inequalities of the form

$$u_t - \mathcal{L}u - |u|^{q-1}u \geqslant v_t - \mathcal{L}v - |v|^{q-1}v \qquad (1)$$

in the whole space $\mathbb{E} = \mathbb{R} \times \mathbb{R}^n$, where $n \geqslant 1$ is a natural number, $q > 1$ is a real number, and $\mathcal{L}$ is a linear second-order partial differential operator in the divergence form defined by the relation

$$\mathcal{L} = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left[a_{ij}(t,x)\frac{\partial}{\partial x_j}\right] \qquad (2)$$

for all $(t,x) \in \mathbb{E}$. We assume that the coefficients $a_{ij}(t,x)$, $i, j = 1, \ldots, n$, of the operator $\mathcal{L}$ are functions that are defined, measurable, and locally bounded in $\mathbb{E}$. We also assume that

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$a_{ij}(t,x) = a_{ji}(t,x)$, $i,j = 1,\ldots,n$, for almost all $(t,x) \in \mathbb{E}$, and the corresponding quadratic form satisfies the conditions

$$0 \leqslant \sum_{i,j=1}^{n} a_{ij}(t,x)\xi_i\xi_j \leqslant A(t,x)|\xi|^2$$

for all $\xi = (\xi_1,\ldots,\xi_n) \in \mathbb{R}^n$ and almost all $(t,x) \in \mathbb{E}$, where $A(t,x)$ is a function that is defined, measurable, non-negative, and locally bounded in $\mathbb{E}$.

It is worth to note that if $u = u(t,x)$ satisfies the inequality

$$u_t \geqslant \mathcal{L}u + |u|^{q-1}u, \tag{3}$$

and $v = v(t,x)$ satisfies the inequality

$$v_t \leqslant \mathcal{L}v + |v|^{q-1}v, \tag{4}$$

then the pair $(u,v)$ satisfies inequality (1). Thus, all the results obtained in this paper for solutions to (1) are valid for the corresponding solutions to inequalities (3) and (4).

The results obtained in [1] for solutions to inequality (1) in the half-space $\mathbb{S} = (0,+\infty) \times \mathbb{R}^n$, $n \geqslant 1$, show that the behavior of the coefficients $a_{ij}(t,x)$ of the operator $\mathcal{L}$ as $|x| \to +\infty$ manifests itself in Liouville-type results; namely, the critical exponents in the Liouville comparison principle for weak solutions to (1) in the half-space $\mathbb{S}$, which are responsible for the non-existence of non-trivial (i. e., such that $u \not\equiv v$) weak solutions to inequality (1) in $\mathbb{S}$, depend essentially on the behavior of the coefficients of the operator $\mathcal{L}$ as $|x| \to +\infty$.

The main goal of the present work is to show that similar critical exponents in the Liouville comparison principle for weak solutions to (1) in the whole space $\mathbb{E}$ also exist and, what is more intriguing, coincide with those obtained in [1] for solutions to (1) in the half-space $\mathbb{S}$. In this connection, it is important to note that the latter, generally speaking, is not the case for solutions to the equations corresponding to inequalities (1), (3), and (4). To make certain of this, it is enough to compare the famous Fujita critical blow-up exponent $q_F = 1 + 2/n$ for non-negative classical solutions to the equation

$$u_t = \Delta u + |u|^{q-1}u \tag{5}$$

in the half-space $\mathbb{S}$ obtained in [2–4] with the blow-up exponent for non-negative classical solutions to equation (5) in the whole space $\mathbb{E}$ obtained in [5, 6], which is equal to

$$q_B = \begin{cases} \dfrac{n(n+2)}{(n-1)^2}, & \text{if} \quad n \geqslant 2, \\ +\infty, & \text{if} \quad n = 1. \end{cases}$$

In order to trace the relation between the behavior of the coefficients $a_{ij}(t,x)$ of the operator $\mathcal{L}$ as $|x| \to +\infty$ and the critical exponents that are responsible for the non-existence of non-trivial weak solutions to inequality (1) in the whole space $\mathbb{E}$, we consider the quantity

$$\mathcal{A}(R) = \operatorname{ess\,sup}_{(t,x)\in(-\infty,+\infty)\times\{R/2<|x|<R\}} A(t,x)$$

for any $R > 0$ and assume that the coefficients of the operator $\mathcal{L}$ satisfy the condition

$$\mathcal{A}(R) \leqslant cR^{2-\alpha} \tag{6}$$

with some real constant $\alpha$ and some real positive constant $c$, for all $R > 1$. It is clear that if $\alpha < 2$, then the coefficients of the operator $\mathcal{L}$ may be unbounded in $\mathbb{E}$; if $\alpha = 2$, they are globally bounded in $\mathbb{E}$; and if $\alpha > 2$, they must vanish as $|x| \to +\infty$.

We also introduce a special function space $W^{\mathcal{L},q}(\mathbb{E})$, which is directly associated with the linear partial differential operator $\mathcal{P} = \partial/\partial t - \mathcal{L}$, and assume that the weak solutions to inequalities (1), (3), and (4) belong to this space only locally in $\mathbb{E}$.

**Definition 1.** Let $n \geqslant 1$ and $q > 1$, let $\mathcal{L}$ be a differential operator defined by (2) in the whole space $\mathbb{E}$, and let $\Omega$ be an arbitrary domain in $\mathbb{E}$. By $W^{\mathcal{L},q}(\Omega)$, we denote the completion of the function space $C^\infty(\Omega)$ with respect to the norm

$$\|w\|_{W^{\mathcal{L},q}(\Omega)} = \int\limits_{\Omega} |w_t| dtdx + \left[\int\limits_{\Omega} \sum_{i,j=1}^{n} a_{ij}(t,x) \frac{\partial w}{\partial x_i} \frac{\partial w}{\partial x_j} dtdx\right]^{1/2} + \left[\int\limits_{\Omega} |w|^q dtdx\right]^{1/q},$$

where $C^\infty(\Omega)$ is the space of all functions defined and infinitely differentiable in $\Omega$.

**Definition 2.** Let $n \geqslant 1$ and $q > 1$, and let $\mathcal{L}$ be a differential operator defined by (2) in the whole space $\mathbb{E}$. A function $w = w(t,x)$ belongs to the function space $W^{\mathcal{L},q}_{\mathrm{loc}}(\mathbb{E})$ if $w$ belongs to $W^{\mathcal{L},q}(\Omega)$ for any bounded domain $\Omega$ in $\mathbb{E}$.

**Definition 3.** Let $n \geqslant 1$ and $q > 1$, and let $\mathcal{L}$ be a differential operator defined by (2) in the whole space $\mathbb{E}$. A pair $(u,v)$ of functions $u = u(t,x)$ and $v = v(t,x)$ is called a weak solution to inequality (1) in $\mathbb{E}$, if these functions are defined and measurable in $\mathbb{E}$, belong to the function space $W^{\mathcal{L},q}_{\mathrm{loc}}(\mathbb{E})$, and satisfy the integral inequality

$$\int\limits_{\mathbb{E}} \left[u_t\varphi + \sum_{i,j=1}^{n} a_{ij}(t,x) \frac{\partial \varphi}{\partial x_i} \frac{\partial u}{\partial x_j} - |u|^{q-1}u\varphi\right] dtdx \geqslant$$

$$\geqslant \int\limits_{\mathbb{E}} \left[v_t\varphi + \sum_{i,j=1}^{n} a_{ij}(t,x) \frac{\partial \varphi}{\partial x_i} \frac{\partial v}{\partial x_j} - |v|^{q-1}v\varphi\right] dtdx \tag{7}$$

for every function $\varphi \in C^\infty(\mathbb{E})$ with compact support in $\mathbb{E}$, where $C^\infty(\mathbb{S})$ is the space of all functions defined and infinitely differentiable in $\mathbb{E}$.

**Remark 1.** We understand inequality (7) in the meaning discussed, e. g., in [7].

Analogous definitions of the solutions to inequality (3) and inequality (4) in $\mathbb{E}$, as special cases of inequality (1) in $\mathbb{E}$ for $v \equiv 0$ or $u \equiv 0$, follow immediately from Definition 3.

**Theorem 1.** *Let $n \geqslant 1$, $\alpha > 0$, and $1 < q \leqslant 1 + \alpha/n$, let $\mathcal{L}$ be a differential operator defined by* (2) *in the whole space $\mathbb{E}$, whose coefficients satisfy condition* (6) *with the given $\alpha$ and some $c > 0$, and let $(u,v)$ be a weak solution to inequality* (1) *in $\mathbb{E}$ such that $u \geqslant v$. Then $u = v$ in $\mathbb{E}$.*

As we have observed above, since any pair of solutions $u = u(t,x)$, $v = v(t,x)$ to inequalities (3) and (4) in $\mathbb{E}$ is a solution $(u,v)$ to inequality (1) in $\mathbb{E}$, the following statement is a direct corollary of Theorem 1.

**Theorem 2.** *Let $n \geqslant 1$, $\alpha > 0$, and $1 < q \leqslant 1 + \alpha/n$, let $\mathcal{L}$ be a differential operator defined by* (2) *in the whole space $\mathbb{E}$, whose coefficients satisfy condition* (6) *with the given $\alpha$ and some $c > 0$, let $u = u(t,x)$ be a weak solution to inequality* (3), *and let $v = v(t,x)$ be a weak solution to inequality* (4) *in $\mathbb{E}$ such that $u \geqslant v$. Then $u = v$ in $\mathbb{E}$.*

Each of the results in Theorems 1 and 2, which obviously have the character of a comparison principle, we term a Liouville-type comparison principle, since, in particular cases where either

$u \equiv 0$ or $v \equiv 0$, it becomes a Liouville-type theorem for solutions to (4) or (3), respectively. We formulate here only the case where $v \equiv 0$.

**Theorem 3.** *Let $n \geqslant 1$, $\alpha > 0$, and $1 < q \leqslant 1 + \alpha/n$, let $\mathcal{L}$ be a differential operator defined by* (2) *in the whole space $\mathbb{E}$, whose coefficients satisfy condition* (6) *with the given $\alpha$ and some $c > 0$, and let $u = u(t, x)$ be a non-negative weak solution to inequality* (3) *in $\mathbb{E}$. Then $u = 0$ in $\mathbb{E}$.*

Note that all the results in Theorems 1–3, including the partial case where $\mathcal{L}$ is the Laplacian operator, are new and sharp. (We demonstrate their sharpness below by Examples 1–2). Thus, as we have already mentioned above, the critical exponents in Theorems 1–3, which are responsible for the non-existence of non-trivial weak solutions to inequalities (1), (3), and (4) in the whole space $\mathbb{E}$, coincide with those obtained in Theorems 1–3 in [1] for weak solutions to the correspondi-ng inequalities in the half-space $\mathbb{S}$. In the particular case where $\alpha = 2$, the critical exponent in Theorems 1–3 coincides with the well-known Fujita critical blow-up exponent obtained in [2–4].

**Example 1.** Let $n \geqslant 1$, $\alpha > 0$, and $q > 1 + \alpha/n$. Consider the operator $\mathcal{L}$ defined by (2) in the whole space $\mathbb{E}$ with the coefficients given by the expression

$$a_{ij}(t, x) = (1 + |x|^2)^{(2-\alpha)/2}\delta_{ij} \tag{8}$$

for all $(t, x) \in \mathbb{E}$, where $\delta_{ij}$ are Kronecker's symbols, and $i, j = 1, \ldots, n$. It is easy to see that condition (6) is fulfilled for these coefficients with the given $\alpha$ and some $c > 0$. Further, for the given $\alpha$, let

$$u(t, x) = \begin{cases} \kappa t^{-\beta}\mathcal{E}(t, x), & \text{if} \quad t > 0, \quad x \in \mathbb{R}^n, \\ 0, & \text{if} \quad t \leqslant 0, \quad x \in \mathbb{R}^n, \end{cases} \tag{9}$$

where $\mathcal{E}(t, x) = \exp(-\gamma(1 + |x|^2)^{\alpha/2}/t)$ for all $t > 0$ and $x \in \mathbb{R}^n$, and the positive constants $\beta$, $\gamma$, and $\kappa$ will be chosen below.

First, since the function $u = u(t, x)$ of the form (9) with any fixed positive constants $\alpha$, $\beta$, $\gamma$, and $\kappa$ is infinitely differentiable in the whole space $\mathbb{E}$ and vanishes, along with all its derivatives, for all $t \leqslant 0$ and $x \in \mathbb{R}^n$, it is clear that $u = u(t, x)$ is a classical solution to inequality (3) for all $t \leqslant 0$ and $x \in \mathbb{R}^n$.

Now, consider the case where $t > 0$ and $x \in \mathbb{R}^n$. Making necessary calculations, we have

$$u_t = -\kappa\beta t^{-\beta-1}\mathcal{E}(t, x) + \kappa\gamma t^{-\beta-2}(1 + |x|^2)^{\alpha/2}\mathcal{E}(t, x),$$

$$\frac{\partial u}{\partial x_i} = -\alpha\kappa\gamma t^{-\beta-1}(1 + |x|^2)^{\alpha/2-1}\mathcal{E}(t, x)x_i$$

and

$$\frac{\partial}{\partial x_i}\left(a_{ii}(t, x)\frac{\partial u}{\partial x_i}\right) = -\alpha\kappa\gamma t^{-\beta-1}\mathcal{E}(t, x) + \alpha^2\kappa\gamma^2 t^{-\beta-2}(1 + |x|^2)^{\alpha/2}\frac{x_i^2}{1 + |x|^2}\mathcal{E}(t, x)$$

for all $t > 0$ and $x \in \mathbb{R}^n$, where the coefficients $a_{ii}(t, x)$ are given by (8), and $i = 1, \ldots, n$. Further, it is also easy to calculate that

$$u_t - \mathcal{L}u = (\alpha\kappa n\gamma - \kappa\beta)t^{-\beta-1}\mathcal{E}(t, x) + \left(\kappa\gamma - \alpha^2\kappa\gamma^2\frac{|x|^2}{1 + |x|^2}\right)t^{-\beta-2}(1 + |x|^2)^{\alpha/2}\mathcal{E}(t, x)$$

and

$$|u|^{q-1}u = \kappa^q t^{-\beta q}\mathcal{E}^q(t, x).$$

As a result, inequality (3) with $u = u(t,x)$ given by (9) takes the form

$$(\alpha\kappa n\gamma - \kappa\beta)t^{-\beta-1}\mathcal{E}(t,x) + \\ + \left(\kappa\gamma - \alpha^2\kappa\gamma^2\frac{|x|^2}{1+|x|^2}\right)t^{-\beta-2}(1+|x|^2)^{\alpha/2}\mathcal{E}(t,x) \geqslant \kappa^q t^{-\beta q}\mathcal{E}^q(t,x) \qquad (10)$$

for all $t > 0$ and $x \in \mathbb{R}^n$. Now, choosing the constants $\beta$, $\gamma$, and $\kappa$ such that

$$\beta = \frac{1}{q-1}, \qquad \frac{1}{\alpha n(q-1)} < \gamma \leqslant \left(\frac{1}{\alpha}\right)^2, \qquad 0 < \kappa \leqslant \left(\alpha n\left(\gamma - \frac{1}{\alpha n(q-1)}\right)\right)^{1/(q-1)} \qquad (11)$$

and taking into account that $\mathcal{E}(t,x) \leqslant 1$ for all $t > 0$ and $x \in \mathbb{R}^n$, it is not difficult to verify that inequality (10) holds for all $t > 0$ and $x \in \mathbb{R}^n$. Therefore, a function $u = u(t,x)$ of the form (9) with the given $\alpha$ and $q$ and with the constants $\beta$, $\gamma$, and $\kappa$ satisfying conditions (11) is a positive classical solution to inequality (3) for all $t > 0$ and $x \in \mathbb{R}^n$.

Thus, we may conclude that the function $u = u(t,x)$ of the form (9) with the given $\alpha$ and $q$ and with the constants $\beta$, $\gamma$, and $\kappa$ satisfying conditions (11) is indeed a non-trivial non-negative classical solution to inequality (3) in the whole space $\mathbb{E}$ with $a_{ij}(t,x)$ that are the coefficients of the operator $\mathcal{L}$ defined by (8). It is clear that the function $v = -u(t,x)$ is a non-trivial non-positive classical solution to inequality (4) in the whole space $\mathbb{E}$ with $a_{ij}(t,x)$ in (2) defined by (8). Thus, the pair of functions $u = u(t,x)$ and $v = v(t,x)$ is a non-trivial classical solution to system (3), (4), and, therefore, $(u,v)$ is a non-trivial classical solution to inequality (1) in the whole space $\mathbb{E}$ such that $u(t,x) \geqslant v(t,x)$, with $a_{ij}(t,x)$ in (2) defined by (8).

Note that the non-negative classical supersolutions to linear uniformly parabolic equations with globally bounded coefficients in the non-divergence form in the whole space $\mathbb{E}$ except the origin of coordinates in a form close to that given by relation (9) with $\alpha = 2$ were constructed in [8, p. 122]. Note also that the positive classical supersolutions to equation (5) in the half-space $\mathbb{S}$ in a form close to that given by relation (9) with $\alpha = 2$ were constructed in [9, p. 283].

**Example 2.** Let $n \geqslant 1$, $\alpha \leqslant 0$, $q > 1 + \alpha/n$, and $q > 1$, and let $\widehat{\alpha}$ be any positive number such that $q > 1 + \widehat{\alpha}/n$. Consider the operator $\mathcal{L}$ defined by (2) in the whole space $\mathbb{E}$ with the coefficients given by the relation

$$a_{ij}(t,x) = (1+|x|^2)^{(2-\widehat{\alpha})/2}\delta_{ij} \qquad (12)$$

for all $(t,x) \in \mathbb{E}$, where $\delta_{ij}$ are Kronecker's symbols, $i$, $j = 1,\ldots,n$. As in Example 2, it is easy to see that $\mathcal{A}(R) \leqslant CR^{2-\widehat{\alpha}}$ for all $R > 1$, where $C$ is some positive constant which depends, possibly, on $\widehat{\alpha}$ and $n$. Therefore, condition (6) is fulfilled for these coefficients with the given $\alpha$ and some $c > 0$. For the given $\widehat{\alpha}$ and $q$, let $\beta = 1/(q-1)$, $1/(\widehat{\alpha}n(q-1)) < \gamma \leqslant 1/\widehat{\alpha}^2$, $0 < \kappa \leqslant (\widehat{\alpha}n(\gamma - 1/(\widehat{\alpha}n(q-1))))^{1/(q-1)}$, and

$$u(t,x) = \begin{cases} \kappa t^{-\beta}\exp\left(-\gamma\dfrac{(1+|x|^2)^{\widehat{\alpha}/2}}{t}\right), & \text{if } \ t > 0, \ \ x \in \mathbb{R}^n, \\ 0, & \text{if } \ t \leqslant 0, \ \ x \in \mathbb{R}^n. \end{cases} \qquad (13)$$

Again, as in Example 2, it is not difficult to verify that the function $u = u(t,x)$ defined by expression (13) is a nontrivial non-negative classical solution to inequality (3) in the whole space $\mathbb{E}$ with $a_{ij}(t,x)$ in (2) defined by (12). It is clear that the function $v = -u(t,x)$ is a nontrivial non-positive classical solution to inequality (4) in the whole space $\mathbb{E}$ with $a_{ij}(t,x)$ defined by (12). Thus, the pair of functions $u = u(t,x)$ and $v = v(t,x)$ is a non-trivial classical solution to system (3), (4), and, therefore, $(u,v)$ is a non-trivial classical solution to inequality (1) in the whole space $\mathbb{E}$ such that $u(t,x) \geqslant v(t,x)$ with $a_{ij}(t,x)$ in (2) defined by (12).

Note that Examples 1–2 are constructed on the basis of those in [1].


1. *Kurta V. V.* A Liouville comparison principle for solutions of semilinear parabolic second-order partial differential inequalities // Asymptotic Analysis. – 2013. – **83**. – P. 83–99.
2. *Fujita H.* On the blowing up of solutions of the Cauchy problem for $u_t = \Delta u + u^{1+\alpha}$ // J. Fac. Sci. Univ. Tokyo, Sect. I. – 1966. – **13**. – P. 109–124.
3. *Hayakawa K.* On nonexistence of global solutions of some semilinear parabolic differential equations // Proc. Japan Acad. – 1973. – **49**. – P. 503–505.
4. *Kobayashi K., Sirao T., Tanaka H.* On the growing up problem for semilinear heat equations // J. Math. Soc. Japan. – 1977. – **29**, No 3. – P. 407–424.
5. *Bidaut-Véron M.-F.* Initial blow-up for the solutions of a semilinear parabolic equation with source term // Équations aux Dérivées Partielles et Applications. – Paris: Gauthier-Villars, 1998. – P. 189–198.
6. *Poláčik P., Quittner P., Souplet Ph.* Singularity and decay estimates in superlinear problems via Liouville-type theorems. Part II: Parabolic equations // Indiana Univ. Math. J. – 2007. – **56**, No 2. – P. 879–908.
7. *Oleinik O. A., Radkevich E. V.* Second order equations with nonnegative characteristic form. – New York: Plenum Press, 1973. – 259 p.
8. *Landis E. M.* Second order equations of elliptic and parabolic type. – Providence, RI: Amer. Math. Soc., 1998. – 203 p.
9. *Samarskii A. A., Galaktionov V. A., Kurdyumov S. P., Mikhailov A. P.* Blow-up in quasi-linear parabolic equations. – Berlin: Walter de Gruyter, 1995. – 535 p.



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**В. В. Курта**


## Принцип порівняння Ліувілля для розв'язків напівлінійних параболічних нерівностей другого порядку в частинних похідних у всьому просторі


*Встановлено принцип порівняння Ліувілля для слабких розв'язків $(u,v)$ напівлінійних параболічних нерівностей другого порядку в частинних похідних виду*

$$u_t - \mathcal{L}u - |u|^{q-1}u \geqslant v_t - \mathcal{L}v - |v|^{q-1}v \tag{$*$}$$

*у всьому просторі $\mathbb{R} \times \mathbb{R}^n$. Тут $n \geqslant 1$, $q > 1$ й*

$$\mathcal{L} = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left[a_{ij}(t,x)\frac{\partial}{\partial x_j}\right],$$

*де $a_{ij}(t,x)$ — вимірні, локально обмежені в просторі $\mathbb{R} \times \mathbb{R}^n$ функції такі, що $a_{ij}(t,x) = a_{ji}(t,x)$ й*

$$\sum_{i,j=1}^{n} a_{ij}(t,x)\xi_i\xi_j \geqslant 0$$

*для майже всіх* $(t, x) \in \mathbb{R} \times \mathbb{R}^n$ *та всіх* $\xi \in \mathbb{R}^n$*. Показано, що критичні показники в одержаному принципі порівняння Ліувілля, які відповідають за неіснування нетривіальних (тобто таких, що* $u \not\equiv v$*) слабких розв'язків нерівності* $(*)$ *у всьому просторі* $\mathbb{R} \times \mathbb{R}^n$ *залежать від поводження коефіцієнтів оператора* $\mathcal{L}$ *на нескінченності і збігаються з критичними показниками, отриманими для розв'язків нерівності* $(*)$ *у напівпросторі* $\mathbb{R}_+ \times \mathbb{R}^n$*. Як прямі наслідки одержано нові теореми Ліувілля для невід'ємних, слабких розв'язків и нерівності* $(*)$ *у всьому просторі* $\mathbb{R} \times \mathbb{R}^n$ *у випадку, коли* $v \equiv 0$*. Всі здобуті результати є новими й точними.*

**В. В. Курта**

## Принцип сравнения Лиувилля для решений полулинейных параболических неравенств второго порядка в частных производных во всем пространстве

*Установлен принцип сравнения Лиувилля для слабых решений* $(u, v)$ *полулинейных параболических неравенств второго порядка в частных производных вида*

$$u_t - \mathcal{L}u - |u|^{q-1}u \geqslant v_t - \mathcal{L}v - |v|^{q-1}v \qquad (*)$$

*во всем пространстве* $\mathbb{R} \times \mathbb{R}^n$*. Здесь* $n \geqslant 1$, $q > 1$ *и*

$$\mathcal{L} = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left[a_{ij}(t,x)\frac{\partial}{\partial x_j}\right],$$

*где* $a_{ij}(t, x)$ *— измеримые, локально ограниченные в пространстве* $\mathbb{R} \times \mathbb{R}^n$ *функции такие, что* $a_{ij}(t, x) = a_{ji}(t, x)$ *и*

$$\sum_{i,j=1}^{n} a_{ij}(t,x)\xi_i\xi_j \geqslant 0$$

*для почти всех* $(t, x) \in \mathbb{R} \times \mathbb{R}^n$ *и всех* $\xi \in \mathbb{R}^n$*. Показано, что критические показатели в полученном принципе сравнения Лиувилля, которые отвечают за несуществование нетривиальных (т. е. таких, что* $u \not\equiv v$*) слабых решений неравенства* $(*)$ *во всем пространстве* $\mathbb{R} \times \mathbb{R}^n$ *зависят от поведения коэффициентов оператора* $\mathcal{L}$ *на бесконечности и совпадают с критическими показателями, полученными для решений неравенства* $(*)$ *в полупространстве* $\mathbb{R}_+ \times \mathbb{R}^n$*. В качестве прямых следствий получены новые теоремы Лиувилля для неотрицательных, слабых решений и неравенства* $(*)$ *во всем пространстве* $\mathbb{R} \times \mathbb{R}^n$ *в случае, когда* $v \equiv 0$*. Все полученные результаты являются новыми и точными.*