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THE METHOD AND ALGORITHMS FOR IDENTIFICATION OF DYNAMIC OBJECTS ON BASIS OF INTEGRAL EQUATIONS

In work questions of development of numerical algorithms of identification for operated dynamic objects on the basis of integrated models are considered.

Key words: *dynamic objects, models, identification algorithms, Volterra integral equations, automatic control systems.*

Application of mathematical models and computer means realized in information technologies is especially peculiar to processes of development and functioning the automated and automatic control systems by the technological processes and many views of industrial objects, etc. Modern methods and control facilities in many cases are created by electronic systems, mobile objects on the basis of use of mathematical models of management objects of management. One of the basic methods of reception the mathematical descriptions is construction models of processes by experimental data that corresponds to an identification problem.

Are the integrated equations which, possessing property of universality the effective mathematical device for modeling continuous objects, including for the decision of a problem of calculation parameters dynamic models of objects? They are especially effective when mathematical models are under construction on experimentally measured input and output signals. It speaks that at the decision many problems of numerical modeling it is possible to realize following advantages of integrated substitution: smoothing properties of integrated operators and high stability of numerical operations at integration. Therefore methods of identification of the models, based on application the integrated equations and their numerical realization, are effective for practical realization.

Integrated models. As dynamic model we shall understand mathematical model of dynamic objects (DO) or systems of a view

$$A[Y(x,t);F(x,t);Q(x,t)]=0, \tag{1}$$

where $t \in [0,T], x \in \Omega \in R^m$, $Y \in B_1$, $F \in B_2$, $Q \in B_3$, t — time, Ω — some compact set from R^m, $x = (x_1, x_2, ..., x_m)$ — a vector of spatial coordinates of model DO, $B_i, i = 1,2,3$ — functional spaces, A — any generally the unknown operator certain on the Cartesian product of spaces $B_1 \times B_2 \times B_3$, satisfying to a condition of existence of continuous implicit function $Y(F,Q)$, Y — a vector of output coordinates (signals) DO, F — a vector of input coordinates (signals) DO, Q — a vector, generally speaking, unknown parameters [1; 2], describing non-stationary DO with the concentrated parameters [7].

In case of integrated dynamic model (IDM) input or output variables enter under a sign on integral and thus there is no operation of differentiation of these signals (operator A — in (1) integrated). In other words model (1) we shall name IDM if the operator A is integrated.

Researches of last year's show, that in some cases it is expedient to consider more the general integrated dynamic models of view

$$A(t)y(t) + \int\limits_{G(t)} K(t,\tau)y(\tau)d\tau = F(t), \tau \in G(t), \tag{2}$$

where $A(t)$ and $K(t,\tau)$, functions a subject definition, $y(t)$ — a output signal $F(t) := F(f;t)$ — the known function defined through values of input signal f, G — some final or infinite set.

Integrated method of identification of dynamic objects. We shall consider the problem constructions on the basis of square-law formulas of computing algorithms of definition of parameters of linear integrated dynamic models of a view

$$\begin{gathered} A_1(x,t)y(x,t) + \int\limits_{D_1 \otimes G_1(t)} K[x,t,s,\tau,y(s,\tau)]dsd\tau + L_1(x,t) = \\ = A_2(x,t)f(x,t) + \int\limits_{D_2(x) \otimes G_2(t)} K_2[x,t,s,\tau,f(s,\tau)]dsd\tau + L_2(x,t), \end{gathered} \tag{3}$$

where y (x, t) and f (x, t) — accordingly vectors of entrance and output variables $A_i(x,t)$ $K_i(x,s,t,\tau)$, $L_i(x,t)$ — the functions-matrixes defined structural and physical properties of modeled objects or systems; x, s — spatial coordinates: $x, s \in D = D_1 \cup D_2 \subset R^m$ $(m = 1,2,...) t, \tau$ — time $t, \tau \in G_1(t) \cup G_2(t) \subset R^1$, D_i and $G_i(t)$, $i = 1,2$ — some sets from accordingly Euclidean spaces R^m and R^1.

For such class DO model (2) will become

$$a_1(t)y(t)+\int\limits_{G_1(t)} K_1(t,\tau)y(\tau)d\tau+L_1(t)=$$
$$=a_2(t)f(t)+\int\limits_{G_2(t)} K_2(t,\tau)f(\tau)d\tau+L_2(t), \tag{4}$$

where $a_i(t),\ K_i(t,\tau),\ L_i(t)\quad (i=1,2)$ parameters a subject definition, y and f — according to output and entrance signals, $G_i(t)$ variable, generally, unknown areas of integration. In the further for simplicity of a statement we shall believe, that $G_1(t)=G_2(t)=[0,T], t\in[0,T]$.

The simple algorithm for calculation of unknown parameters in (4) can be received, applying to calculation of integrals in (4) quadrature formulas of a view [3; 5]

$$\int\limits_0^{t_i} x(\tau)d\tau=\sum_{j=0}^{N_i} W_{ij}x(t_i)+r_i[x], N_i=\overline{1,N}, \tag{5}$$

where W_{ij} — weights, $r_i[x]$ — a residual member square-law formulas, t_i — units of a view

$$0\le t_1\le t_2\le ...<t_N\le T \tag{6}$$

with some errors

$$\tilde{f}(t_i)=f(t_i)+\delta_i,\ \max_{0\le i\le N}|\delta_i|=\delta, \tag{7}$$

$$\tilde{y}(t_i)=y(t_i)+\varepsilon_i,\ \max_{0\le i\le N}|\varepsilon_i|=\varepsilon, \tag{8}$$

where $\tilde{f}$ and $\tilde{y}$ — confidants, f and y — exact values of signals.

The essence of the quadrature algorithm of calculation of parameters of model (4) consists that settlement expressions in it are formed on the basis of digitization of integrals by means of square-law formulas of a view (4) with rejection of corresponding residual members. Digitizing thus model (4) in points t_i, $i=\overline{0,N}$ of a view (6), we receive following system from $N+1$-th linear algebraic equation concerning unknown parameters:

$$a_1(0)y(0)+L_1(0)=a_2(0)f(0)+L_2(0),$$
$$a_1(t_i)y(t_i)+\sum_{j=0}^{N_i} W_{ij}K_1(t_i,t_j)y(t_j)+L_1(t_i)=$$
$$=a_2(t_i)y(t_i)+\sum_{j=0}^{M_i} W_{ij}K_2(t_i,t_j)f(t_j)+L_2(t_i), \tag{9}$$
$$M_iN_i=\overline{1,N}, i=\overline{1,N}.$$

At absence of any additional information on unknown parameters $a_\nu(t_i), K_\nu(t_i, t_j), L_\nu(t_i), \nu = 1,2$ in system (9) will be, generally speaking, $2(N+1)(N+3)$ unknown persons, i.e. in this case there are known difficulties of the decision not predetermined SLAE. To avoid it is possible, for example, having assumed, that unknown parameters in (4) have polynomial a view, i.e.

$$a_\nu(t) = \sum_{k=1}^{m_\nu} \alpha_{\nu k} \beta_k^\nu(t), \tag{10}$$

$$L_\nu(t) = \sum_{k=1}^{l_\nu} \lambda_{\nu k} \gamma_k^\nu(t), \tag{11}$$

$$K_\nu(t,\tau) = \sum_{k=1}^{P_\nu} \sum_{s=1}^{Q_\nu} C_{\nu rs} \varphi_r^\nu(t) \psi_s^\nu(\tau), \tag{12}$$

where $\alpha_{\nu k}, \lambda_{\nu k}, C_{\nu rs}$ — unknown constant factors, and $\{\beta_k^\nu\}_{k=1}^{m_\nu}$, $\{\gamma_k^\nu\}_{k=1}^{l_\nu}$, $\{\varphi_r^\nu\}_{r=1}^{P_\nu}$, $\{\psi_s^\nu\}_{s=1}^{Q_\nu}$, — some systems of linearly independent functions $\nu = 1,2$.

It is obvious, that, if

$$N = p = m_1 + m_2 + l_1 + l_2 + p_1 Q_1 + p_2 Q_2 - 1, \tag{13}$$

that number of the equations in (9) will be equal to number of unknown persons. Certainly, thus there are opened, generally, questions of a choice of functions β_k^ν, γ_k^ν, φ_k^ν, ψ_k^ν; questions of existence and uniqueness of decision СЛАУ (9), and also a question on influence of errors δ_i both ε_i in (7) and in (8) on accuracy of calculation of unknown parameters. These questions in the certain degree can be solved in private, but important enough case when required parameters $a_\nu(t), L_\nu(t)$ and $K_\nu(t,s)$ are defined by following parities:

$$a_1(t) \equiv 1, a_2(t) \equiv 0, L_1(t) \equiv 0, \tag{14}$$

$$K_1(t,s) = K(t-s) = \sum_{j=1}^{m} q_j \frac{(t-s)^{j-1}}{(j-1)!}, m \in N, \tag{15}$$

$$K_2(t,s) = \frac{(t-s)^{m-1}}{(m-1)!}, \tag{16}$$

$$L_2(t) = \sum_{j=1}^{m-1} (c_j \frac{t_j}{j!} + q_j \sum_{k=0}^{m-j-1} c_k \frac{t^{k+j}}{(k+j)!}), \tag{17}$$

where q_i — unknown persons, and c_i — known constants.

It is easy to notice, that the equation (4) at the chosen values of parameters $a_v(t),\ L_v(t), K_v(t,s)$ will be equivalent to the differential equation of a view

$$y^{(m)}(t)+\sum_{j=1}^{m}q_j y^{(m-j)}(t)=f(t),\ \ y^{(k)}(0)=c_k,\ \ k=\overline{0,m-1}.$$

For formation of system of the linear algebraic equations concerning unknown factors $q_j, j=\overline{1,m}$ we shall transform the equation (9) in view of (10)–(17) to a view

$$\sum_{j=1}^{m}q_j\left[\int_0^t\frac{(t-s)^{j-1}}{(j-1)!}y(s)ds-\sum_{k=0}^{m-j-1}c_k\frac{t^{k+j}}{(k+j)!}\right]= \\ =\int_0^t\frac{(t-s)^{m-1}}{(m-1)!}f(s)ds+\sum_{j=0}^{m-1}c_j\frac{t_j}{j!}-y(t). \tag{18}$$

From here for points of fixing (measurement) $t_i\ (i=\overline{0,N})$ of a view (6), believing, that $m=n$ we receive system of the linear algebraic equations concerning unknown factors $q_i, j=1,m$

$$Aq=b\,, \tag{19}$$

where $q=(q_1,...,q_m)^T, b=(b_1,...,b_m)^T, A=[A_{ij}]_{i,j=1}^m$,

$$A_{ij}=\int_0^{t_i}\frac{(t_i-s)^{j-1}}{(j-1)!}y(s)ds+\sum_{k=0}^{m-j-1}c_k\frac{t_i^{k+j}}{(k+j)!}, \tag{20}$$

$$b_j=\int_0^{t_i}\frac{(t_i-s)^{m-1}}{(m-1)!}f(s)ds-y(t_i)+\sum_{\nu=0}^{m-1}c_\nu\frac{t_i^\nu}{\nu!}\,. \tag{21}$$

Apply calculation of integrals in (20) and (21) quadrature formulas of a view (5) which for any integrated on Remand functions $u(t)$, will become

$$\int_0^{t_i}(t_i-s)^\nu y(s)ds=\sum_{j=0}^{L_i}W_{ij}(t_i-t_j)^\nu y(t_j)+r_{i\nu}[n], \tag{22}$$

where $1\le L_i\le N,\ r_{i\nu}$ — residual members of this formula, and other sizes are certain in (5).

Rejecting residual members square-law formulas, accordingly $r_{ij}[y]$ and $r_{im}[f]$, considering that fact that according to (7) and (8) values of input and output signals are set experimentally with some errors, from system of the equation (19) we come to following system of the equations concerning the approached values a component of a vector $\tilde{q} = (\tilde{q}_1, ..., \tilde{q}_m)^T$:

$$\tilde{A}\tilde{q} = \tilde{b}, \qquad (21)$$

where

$$\tilde{A} = \left[A_{ij} \right]_{i,j=1}^{m}, \ \tilde{b} = \left[\tilde{b}_1, ..., \tilde{b}_m \right]^T, \qquad (22)$$

$$\tilde{A}_{ij} = \frac{1}{(j-1)!} \sum_{k=0}^{N_i} W_{ik} (t_i - t_k)^{j-1} \tilde{y}(t_k) - \sum_{l=0}^{m-j-1} c_l \frac{t_i^{l+j}}{(l+j)!}, 1 \le N_i \le N, \qquad (23)$$

$$\tilde{b}_i = \frac{1}{(m-1)!} \sum_{k=0}^{M_i} W_{ik} \left| t_i - t_k \right|^{m-1} \tilde{f}(t_k) + \sum_{v=0}^{m-1} c_v \frac{t_i^v}{v!} - \tilde{y}(t_i), \ 1 \le M_i \le N`. \qquad (24)$$

Thus, we have received final system for calculation of parameters $q_i (i = \overline{1, n})$. The block diagram quadrature algorithm is resulted on fig. 1.

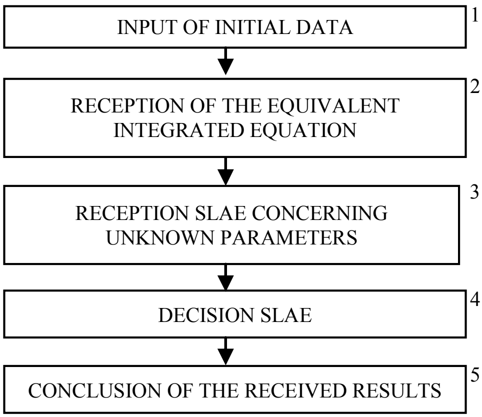

Fig. 1. *The block diagram quadrature algorithm*

The analysis of the operations entering into given algorithm, allows assume, that at calculation of parameters of dynamic models of a view (4) it possesses high speed and stability. Besides by virtue of the simplicity it allows to synthesize high-efficiency computers. To be convinced of working capacity of a method, we shall consider some examples of the decision of some test problems.

Example. Inputsignal: $u(t) = 1 - e^{-2t} \ t \in [0; 2] \ h = 0.01$.

Output signal: $f(t) = -14 e^{-2t} - 0.2$.

Entry conditions: $C_1 = 0, \ C_2 = 2, \ C_3 = -4, \ C_4 = 8, \ C_5 = -16$.

Problem: to define factors p_i of the equivalent differential equation

$$u^{(5)}(t)+p_1u^{(4)}(t)+p_2u^{(3)}(t)+p_3u^{(2)}(t)+p_4u^{(1)}(t)+p_5u(t)=f(t),\quad u^{(i-1)}(0)=C_i,\ i=\overline{1,5}. \tag{24}$$

The exact decision: $p_1=1.2, p_2=-2, p_3=3.1, p_4=0.7, p_5=-0.2$.

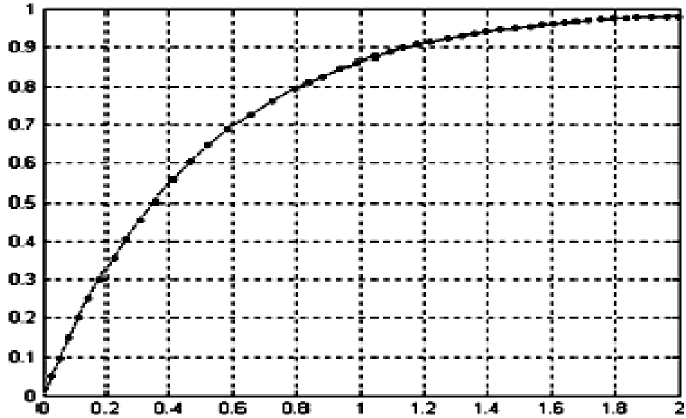

Fig. 2. *The decision of the equation (24)*

Using expressions (21)–(23) and quadrature the formula of trapezes for approximation of the integrals entering into expressions (22), (23), we receive SLAE concerning unknown factors p_i, and the given system is not joint. Applying to its decision a method of the least squares, we receive following values of required factors: $p_1=1.8930$, $p_2=-1.8640$, $p_3=3.0249$, $p_4=5.5493$, $p_5=-0.1997$.

On fig. 2 the decision of the equation (24) is presented accordingly at exact values of factors (—) and at factors received as a result of calculation (). A root-mean-square mistake $\Delta=3.4\times10^{-6}$.

Let's add to a output signal *f* the casual handicap distributed under the normal law. In table 1 the values of factors p_i received at various values of handicapes are presented:

Table 1

The values of factors received at various values of handicapes

Size of a handicap in % from a output signal	Root-mean-square mistake	p_1	p_2	p_3	p_4	p_5
1	4.0×10^{-5}	2.01	–2.19	5.16	12.09	–0.20
5	8.8×10^{-4}	–4.77	4.04	8.09	–61.30	–0.21
10	3.1×10^{-3}	41.17	–36.85	–15.41	422.95	–0.25

Results of the decision the considered examples (and many other things) about such properties of an integrated method of identification, as high enough stability, efficiency in sense of expenses of machine time and volume of calculations, simplicity of realization. Thus, the prospective method can be effectively used at the decision of problems of parametrical identification which are characterized by presence of an error in initial data.

In connection with that in practice, as a rule, the number of measurements (points of quantization) and number of unknown parameters $M := \sum_{i=0}^{N} m_i$ are equal, and also in connection with necessity of effective calculation of integrals in (18), we investigate a question of reception of normal systems of the linear algebraic equations rather $\{q_{ij}\}_{i=0,r}^{j=\overline{1,m_i}}$ on the basis of preliminary approximation of initial data $\tilde{y}(t_i)$ and $\tilde{f}(t_i), i=\overline{1,N}$ by means of adderly operators [4]. Not limiting a generality of reasoning's, we shall believe in the further, that in initial model (18) everything $y_k = y_k(0)$, $k=\overline{0,r-1}$, and functions $\varphi_j(t)$ in such, that integrals of a view

$$\int_0^t \tau^\nu \varphi_k^{(j)}(s)ds,\ \nu \in N$$

are calculated precisely.

Let's consider algorithm of calculation of parameters q_{ij} which consists in the following.

Representing by means of adderly the operator $U_{n,N}(t)$ entrance and output signals in the form of equation Volterra

$$A_0(t)y(t) + \int_0^t k(t,\tau)y(\tau)d\tau = F(t), \tag{25}$$

in which

$$K(t,\tau) = \sum_{j=1}^{r} (-1)^{j-1} \binom{r}{j} \frac{(t-s)^{j-1}}{(j-1)!} A_0^{(j)}(s) + \\ + \sum_{\tau=1}^{r} \sum_{j=0}^{r-\tau} (-1)^{j-1} \binom{r-\tau}{j} \frac{(t-s)^{\tau+j-1}}{(\tau+j-1)!} A_\tau^{(j)}(s), \tag{26}$$

$$\binom{p}{n} = \frac{p!}{(p-n!n!)},$$

$$F(t) = \sum_{\tau=0}^{r-1} \sum_{j=0}^{r-\tau-1} \frac{B_j^{(r-\tau)}(A_{r-\tau};y)}{(j+\tau)!} t^{j+\tau} + I_r(f;t); \tag{27}$$

$$B_j\begin{pmatrix} r-\tau \\ j \end{pmatrix}(A_{r-\tau};y)=\frac{(r-\tau)}{j!}\sum_{v=0}\begin{pmatrix} j \\ v \end{pmatrix}A_{r-\varepsilon}^{(v)}(0)\times$$
$$\times y_{j-v}\sum_{i=0}^{n}(-1)^{i}\frac{(j-i)!}{(r-\tau-v)!}\begin{pmatrix} v \\ i \end{pmatrix}. \tag{28}$$

Through $I_r(f;t)$ *r-th* integral from inputsignal is designated:

$$I_r(f;t)=\frac{1}{(r-1)!}\int_0^t (t-s)^{r-1}f(s)ds, \tag{29}$$

$$Y_k:=Y^{(k)}(0), k=\overline{0,r-1}, \tag{30}$$

we pass from the equation (25) to corresponding approached integrated model. In the further the given algorithm of calculation of parameters of dynamic model (25) we shall name integro-adderly (IA).

Scheme IA of algorithm for calculation of parameters of dynamic model (14)–(17) stationary DO same, and system (21) will become $A\cdot\tilde{q}=u,$ where

$$A=\left[\alpha_{ij}\right]_{i,j=1}^{m,m},\ q=\left[\tilde{q}_1,\tilde{q}_2,...,\tilde{q}_m\right]^T,\ U=[u_1,u_2,...,u_m]^T,$$

$$A_{ij}=\frac{1}{(j-1)!}\int_0^{t_i}(t_i-s)^{j-1}U_n(\tilde{y},s)ds-\sum_{v=0}^{m-j-1}c_v\frac{t_i^{v+j}}{(v+j)!},\ c_{-1}=0, \tag{31}$$

$$F_i=\frac{1}{(m-1)!}\int_0^{t_i}(t_i-s)^{m-1}U_n(\tilde{f};s)ds+\sum_{v=0}^{m-1}c_v\frac{t_i^v}{v!}-U_n(\tilde{y};t_i) \tag{32}$$

Question of realization IA of algorithm we shall consider on an example stationary DO described in the form of the differential equation of the second order

$$\begin{gathered} y''(t)+q_1y'(t)+q_2y=f(t), \\ y(0)=C_0, y'(0)=c_1. \end{gathered} \tag{33}$$

For the differential equation (33) equivalent integrated equation is

$$y(t)+\int_0^t[q_1+q_2(t-s)]y(s)ds=\int_0^t(t-s)f(s)ds+c_0+c_1t+q_1c_0t. \tag{34}$$

Let's write this equation in the convenient form for formation SLAE concerning unknown parameters

$$q_1(\int_0^t y(s)ds-c_0t)+q_2\int_0^t(t-s)y(s)ds=\int_0^t(t-s)f(s)ds+c_0+c_1t-y(t).$$

As adder the operator we shall take interpolation a cubic spline of a view

$$s_3(x;\tilde{y}) = d_1^j(x_j - x)^3 + d_2^j(x - x_{j-1})^3 + d_3^j(x_j - x) + d_4^j(x - x_{j-1}), \quad (35)$$

$$s_3(x;\tilde{f}) = a_1^j(x_j - x)^3 + a_2^j(x - x_{j-1})^3 + a_3^j(x_j - x) + a_4^j(x - x_{j-1}). \quad (36)$$

Let's substitute (35) and (36) in (34), we shall receive:

$$\begin{gathered} q_1\left(\sum_{j=1}^{i}\int_{t_{j-1}}^{t_j} s_3(s;\tilde{y})ds - c_0 t_j\right) + q_2\sum_{j=1}^{i}\int_{t_{j-1}}^{t_j}(s_j - s)s_3(s,\tilde{y})ds = \\ = \sum_{j=1}^{i}\int_{t_{j-1}}^{t_j}(t_j - s).s_3(s,\tilde{f})ds + c_0 + c_1 t_j - s_3(s;\tilde{y}_j),\ i = \overline{1,m}. \end{gathered} \quad (37)$$

For formation of system rather q_1 also q_2 it is necessary to calculate integrals of a view

$$\int_{t_{j-1}}^{t_j} d_1^j(s_j - s)^3 + d_2^j(s - s_{j-1})^3 + d_3^j(s_j - s) + d_4^j(s - s_{j-1})]ds, \quad (38)$$

$$\int_{t_{j-1}}^{t_j}(s_j - s)[d_i^j(s_j - s)^3 + d_2^j(s - s_{j-1})^3 + d_3^j(s_j - s) + d_4^j(s - s_{j-1})]ds. \quad (39)$$

First we shall calculate integral

$$\begin{gathered} \int_{t_{j-1}}^{t_i}\left[d_1^j(s_j - s)^3 + d_2^j(s - s_{j-1})^3 + d_3^j(s_j - s) + d_4^j(s - s_{j-1})\right]ds = \\ = d_1^j\left[(jh)^3 h - 3j^3h^4 + \frac{3}{2}j^2h^4 + 3j^3h^4 - 3j^2h^4 + jh^4 + j^3h^4 - \right. \\ \left. -\frac{3}{2}j^2h^4 + jh^4 - \frac{h^4}{4}\right] + d_2^j\left[j^3h^4 - \frac{3}{2}j^3h^4 - \frac{3}{2}j^2h^4 + jh^4 + \frac{h^4}{4} - \right. \\ -3j^3h^4 + 6j^2h^4 - 4jh^4 + 3j^3h^4 - \frac{15}{2}j^2h^4 - 6j^4h^4 + \frac{3}{2}h^4 - j^3h^4 + \\ \left. +3j^2h^4 + h^4\right] + d_3^j\left(j^2h^2 -^2 h^2 + jh^2 + \frac{j^2h^2}{2} - \frac{j^2h^2}{2} + jh^2 - \frac{h}{2}\right)^2 + \\ +d_4^j\left[\frac{j^2h^2}{2} - \frac{j^2h^2}{2} + jh^2 - \frac{h^2}{2} - (j^2 - j)h^2 + (j^2 - 2j + 1)h^2\right] = \\ = d_1^j\left(2j^3 - 3j^2 + 2j - \frac{1}{4}\right)h^4 + d_2^j\left(6j - 6j^4 + \frac{15}{4}\right)h^4 + \\ +d_3^j\left(2j - \frac{1}{2}\right)h^2 + d_4^j\frac{h^2}{2}. \end{gathered} \quad (39a)$$

Now we shall calculate integral (39), using (39a):

$$\int_{t_{i-1}}^{t_j} (s_j - s)\left[d_1^j (s_j - s)^3 + d_3^j (s_j - s) + d_4^j (s - s_{j-1}) \right] ds = s_j \left[d_1^j \left(2j^3 - \right.\right.$$

$$\left. -3j^2 + 2j - \frac{1}{4} \right) h^4 + d_2^j \left(6j - 6j^4 + \frac{15}{4} \right) h^4 + d_3^j (2j - \frac{1}{2}) h^2 + d_4^j \frac{h^2}{2} \Bigg] -$$

$$-\int_{t_{j-1}}^{t_j} s\left[d_1^j (s_j - s)^3 + d_2^j (s - s_{j-1})^3 + d_3^j (s_j - s) + d_4^j (s - s_{j-1}) \right].$$

Further we shall calculate integral

$$\int_{t_{j-1}}^{t_j} s\left[d_1^j (s_j - s)^3 + d_2^j (s - s_{j-1})^s + d_3^j (s_j - s) + d_4^j (s - s_{j-1}) \right] ds =$$

$$= \left(\frac{1}{4} h^5 - \frac{h^5}{5} \right) d_1^j + + \left(2j^3 + \frac{1}{4} j - \frac{1}{20} \right) h^5 \cdot d_2^j + \qquad (39')$$

$$+ (2j^2 h^3 - \frac{3}{2} jh^3 + \frac{h}{3})^3 d_3^j + \frac{h^2}{2} d_4^j .$$

and, substituting (39) in (39a), we shall receive

$$\int_{t_{j-1}}^{t_j} s(s_j - s)\left[d_1^j (s_j - s)^3 + d_2^j (s - s_{j-1})^3 + d_3^j (s_j - s) + d_4^j (s - s_{j-1}) \right] ds =$$

$$= \left[d_1^j \left(2j^4 - 3j^3 - 2j^2 - \frac{1}{2} j + \frac{1}{5} \right) h^5 + d_2^j \left(-2j^3 + 6j^2 + \right.\right.$$

$$\left.\left. + \frac{7}{2} j - 6j^5 + \frac{1}{20} \right) h^5 + d_3^j \left(j - \frac{1}{6} \right) h^3 + d_4^j \left(-j^2 + 2j - \frac{5}{6} \right) h^3 \right].$$

For calculation of integrals $\int_{x_{j-1}}^{x_j} (x_j - x) s_3(x, f) dx$ we shall take advantage of the previous expression

$$\int_{t_{j-1}}^{t_j} (s_j - s) s_3(s, \tilde{f}) ds = \int_{t_{j-1}}^{t_j} \left[(s_j - s)^3 + a_2 (s - s_{j-1})^3 + a_3^j (s_j - s) + \right.$$

$$\left. + a_4^j (s - s_{j-1}) \right] ds = a_1^j \left(2j^4 - 3j^3 + 2j^2 - \frac{1}{2} j + \frac{1}{5} \right) h^5 + a_2^j \left(-2j^3 + \right.$$

$$\left. + 6j^2 + \frac{7}{2} j - 6j^5 + \frac{1}{20} \right) h^5 + a_3^j \left(j - \frac{1}{6} \right) h^3 + a_4^j \left(-j^2 + 2j - \frac{5}{6} \right) h^3 .$$

Now for formation of system rather q_1 also q_2 we use expressions (38) and (39). Then (37) it is possible to write down in the form of

$$q_1\left[\sum_{j=1}^{i} d_1^j\left(2j^3-3j^2+2j-\frac{1}{4}\right)h^4+d_2^j\left(6j-6_j^4+\frac{15}{4}\right)h^4+d_3^j\left(2j-\frac{1}{2}\right)h^2+\right.$$

$$\left.+d_4^j\frac{h^2}{2}-C_0jh\right]+q_2\sum_{j=1}^{i}\left[d_1^j\left(2j^4-3j^3+2j^2-\frac{1}{2}j+\frac{1}{5}\right)h^5+d_2^j\left(-2j^3+\right.\right.$$

$$\left.\left.+6j^2+\frac{7}{2}j-6j^5+\frac{1}{20}\right)h^5+d_3^j\left(j-\frac{1}{3}\right)h^3+d_4^j\left(-j^2+2j-\frac{5}{6}h^3\right]-$$

$$-\sum_{j=1}^{i}\left[a_1^j\left(2j^4-3j^3+2j^2\right]-\frac{1}{2}j+\frac{1}{5}\right)h^5+a_2^j\left(-2j^3+6j^2+\frac{7}{2}j-\right.$$

$$\left.-6j^5+\frac{1}{20}\right)h^5+a_3^j\left(j-\frac{1}{3}\right)h^3+a_4^j\left(-j^2+2j-\frac{5}{6}\right)h^3\Bigg]+C_0+C_1jh-y_j.$$

So, final view SLAE rather q_1 also q_2 looks like

$$Aq=F\,, \tag{40}$$

where

$$A=\left\{A_{k1,}A_{k2}\right\}_{k=\overline{1,m}},q=(q_1,q_2)^T,F=(F_1,F_2,...,F_m),$$

$$A_{k1}=\sum_{j=1}^{k}\left[d_1^j\left(2j^3-3j^2+2j-\frac{1}{4}\right)h^4+d_2^j\left(6j+\frac{15}{4}-6j^4\right)h^4+\right.$$

$$+d_3^jh^2\left(2j-\frac{1}{2}\right)+d_4^j\frac{h^2}{2}-C_0jh],A_{k2}=\sum_{j=1}^{k}[d_1^j(2j^4-3j^3+2j^2-\frac{1}{2}j+\frac{1}{5})h^5+$$

$$\left.+d_2^j\left(-2j^3+6j^2+\frac{7}{2}j-6j^5+\frac{1}{20}\right)h^5+d_3^j\left(j-\frac{1}{3}\right)h^3+d_4^j\left(-j^2+2j-\frac{5}{6}\right)h^3\right],$$

$$F_k=\sum_{j=1}^{k}\left[a_1^k\left(2k^4-3k^3+2k^2-\frac{1}{2}k+\frac{1}{5}\right)h^5+a_2^k\left(-2k^3+6k^2+\frac{7}{2}k-\right.\right.$$

$$\left.\left.-6k^5+\frac{1}{20}\right)h^5+a^k\left(k-\frac{1}{3}\right)h^3+a_4^k\left(-k^2+2k-\frac{5}{6}\right)h^3\right].$$

So, the algorithm of calculation of parameters in case of when inputandoutput signal are approximated by cubic splines, consists in the following:

1) the task of initial data $t_j=jh,h,yi,fi,j=\overline{1,n},C_0,C_1$;
2) calculation of factors of a cubic spline on each site (x_{j-1},x_j), approximating entrance a signal $f(t)$ and a output signal $y(t)$;

3) formation of a matrix A;
4) formation of the right part F;
5) decision of system (40).

The block diagram a spline-integrated of algorithm of calculation of parameters of dynamic models DO is resulted on fig. 3. Application of splines allows:

1) to raise accuracy of calculation of parameters q on the order concerning a step h in comparison with quadrature algorithm on the basis of the formula of trapezes;
2) to receive in case of the incomplete initial information additional points for formation of normal systems concerning counted parameters.

Thus, the considered algorithm possesses a number of properties necessary for digital realization, in particular, it is noise proof concerning an error of initial data, and also suitable as a basis of the organization of software for the decision of problems of identification. Procedure of numerical realization of systems of the linear equations supposes representation of a matrix of system *And* in the form of the product of the top and bottom triangular matrix allowing parallelizing computing processes.

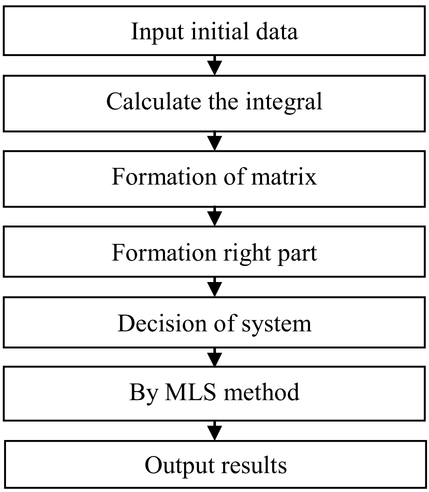

Fig. 3. *The block diagram spline-integrated of algorithm*

Consider calculation of parameters of T-models [8] of non-stationary objects which in effect adjoins an integrated method.

Consider initial mathematical model in the form of the equation

$$\sum_{i=0}^{n} a_i(t) y^{(i)}(t) = \sum_{j=0}^{s} b_j(t) x^{(j)}(t), \tag{41}$$

where $y(t)$ and $f(t)$ — output both entrance signals, $a_i(t)$ and $b_j(t)$ variable restored parameters.

As it has noted been, to number of the basic difficulties arising at calculation of parameters $a_i(t)$ and $b_j(t)$ model (41) at known with some errors of measurements values of entrance and output signals, the incorrectness of a problem of numerical differentiation of functions $y(t)$ concerns and $f(t)$.

One of effective ways of overcoming of the specified difficulty is transition from model (41) to integrated model Volterra equivalent to it by M-fold ($r = \max(n,s)$) integration of the left and right part (41) and repeated application thus of the formula of integration in parts.

In that specific case, for example, when $r = n = s$ to model (41) corresponds (to be convinced of it easily by analogy to the proof of the theorem of equivalence from [6]), in the assumption of sufficient smoothness of dependences $a_i(t)$ and $b_j(t)$, the following integrated model of a view equivalent to it

$$a_r(t)y(t) + \int_0^t K_1(t,\tau)y(\tau)dt = b_r(t)x(t) + \int_0^t K_2(t,\tau)x(\tau)d\tau + F(t)\,, \quad (42)$$

where

$$K_v(t,\tau) = \sum_{j=1}^{r}(-1)^{j-1}\binom{r}{j}\frac{(t-\tau)^{j-1}}{(j-1)!}\overset{0}{C}{}^{(j)}_{v0}(\tau) +$$
$$+\sum^{r}\sum_{j=0}^{r-l}(-1)^{j-1}\binom{r-l}{j}\frac{(t-\tau)^{l+j-1}}{(l+j-1)!}C^{(j)}_{vl}(t),\ v=1,2, \quad (43)$$

$$C_{1l}(t) := a_{r-l}(t), C_{2l}(t) := b_{r-l}(t),$$

$$F(t) = \sum_{l=0}^{r-1}\sum_{j=0}^{r-l-1}\frac{d_{jl}(x,y)}{(j+l)!}t^{j+s}, \quad (44)$$

$$d_{jl}(x,y) = \frac{(r-l)}{j!}\sum_{\mu=0}^{j}\begin{pmatrix} j \\ \mu \end{pmatrix}[a_l^{\mu}(0)y^{(j-\mu)}0 - b_l^{(\mu)}(0)x^{(j-\mu)}0]\times$$
$$\times\sum_{i=0}^{\mu}(-1)^i\,\frac{(j-i)}{(r-l-v)!}\begin{pmatrix} \mu \\ i \end{pmatrix}, \quad (45)$$

$$\begin{pmatrix} p \\ q \end{pmatrix} := \frac{p!}{(p-q)!q!}. \quad (46)$$

Conclusions. The offered method of identification of dynamic objects on the basis of integrated equations Volterra allows to receive a set square-law algorithms of calculation of parameters of mathematical models; the problem is reduced to the decision of algebraic systems, generally not joint which dimension is defined by character of initial experimental data and demanded accuracy of calculations. Research of algorithms allows drawing a conclusion on their high stability, efficiency in sense of expenses of machine time and volume of calculations, simplicity of realization.

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У роботі розглянуто питання створення чисельних алгоритмів ідентифікації для експлуатованих динамічних об'єктів на основі інтегрованих моделей.

Ключові слова: *динамічні об'єкти, моделі, алгоритми ідентифікації, інтегральні рівняння Вольтерра, автоматичні системи управління.*

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