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THE CONTINUUM APPROACH IN A GROUTING MODEL

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Анотація. *Отримано значення максимального розміру пори, при якому континуальний підхід все ще можна застосовувати в моделюванні поширення цементу в насиченому піску під час цементації, що не руйнує структуру ґрунту.*
Ключові слова: *континуальний підхід, елементарний об'єм, цементація ґрунту, похибка числового розрахунку, ущільнення ґрунту.*

Аннотация. *Получено значение максимального размера поры, при котором континуальный подход всё ещё можно применять в моделировании распространения цемента в насыщенном песке при цементации, которая не разрушает структуру грунта.*
Ключевые слова: *континуальный подход, элементарный объём, цементация грунта, погрешность численного расчёта, уплотнение грунта.*

Abstract. *The value of the maximal pore size at which the continuum approach can still be adopted for modelling cement grout propagation in saturated sand during permeation grouting is obtained.*
Keywords: *continuum approach, elementary volume, permeation grouting, numerical calculation error, soil compaction.*

1. Introduction

Before tunnelling construction in a weak soil, permeation grouting is used to increase the soil stand up time. Since chemical grouts are hazardous to environment [1], a cement grout is used as an infiltrate in this technique. A regime of permeation grouting is determined by the evolution of the cement concentration distribution in space [2]. Therefore, mathematical modelling of this evolution is important. The cement grout consists of particles. If they are large enough, they can get trapped in small pore throats. Otherwise, they can only deposit on walls of these throats and pores [3]. Therefore, the mathematical description of cement grout propagation in a porous medium is cumbersome. In [1, 2], to shed light on various issues that arise during the construction of this description a standard laboratory test is modelled. In this test, cement grout is injected in a vertical tube opened at the top and filled with water saturated sand. The injection point is at the bottom of the tube. In the papers [4, 5], to do the same, problem set ups that correspond to in situ grouting are considered. In [1], the injection pressure reaches 8 bars and it is assumed that at such the pressure the structure of the grouted sand is not ruined. The fact that results of numerical calculations according to the model [1] of permeation grouting coincide with the results of respective laboratory measurements verifies this assumption. Moreover, during grouting of this type the injection pressure can be as high as 12 bars [6]. In [7], Demchuk argues that permeation grouting performed at such the values of injection pressure can be modelled by a problem with a free moving boundary. Demchuk [8] presents grouting models of this class and shows that the continuum approach is properly adopted in them. As for the models [1, 2] and [4, 5], they can be used only when a size of an elementary volume over which the averaging is performed in the continuum approach is much smaller than a characteristic length of a domain in which modelling is performed. The greater uncertainties in parameters that characterize a porous medium are, the smaller the elementary volume is [9]. Since an inherent error is a part of a total calculation error, maximal allowed uncertainty in a parameter that characterizes soil is determined by the total calculation error. In [1, 2] and [4, 5], errors of numerical calculations were not estimated because the solutions have regions of high gradients which positions depend on time and are not known in advance. One of the main drawbacks of the models [1, 2] and [4, 5] is that calculations according to them require significant computer resources. It can be explained by the fact that they are systems of differential equations supplemented with boundary and initial conditions that do not conform to each other [10]. Demchuk [10] presents the model of the standard laboratory test [11], in which this drawback is absent. Demchuk and Saiyouri [11] propose the method of uncertainty uniformity principle realization in calculations according to the model [10]. Demchuk [12] estimates the errors of these calculations. The aim of this work is to check whether the continuum approach was properly adopted in [12]. Since productivity of tunnelling construction in the weak soil highly depends on quality of stabilization of this soil, this research responds to the urgent problem in the time of the global recession.

2. Elementary Volume Size Estimation

Estimating a size of an elementary volume Bear and Bachmat assume that this volume is a cube divided into N^3 equal cubic parts [9]. Demchuk [8] shows that if the uncertainty in the porosity m is equal to δm, then the minimal characteristic size s of the elementary volume can be estimated as

$$s = N \cdot d \tag{1}$$

where

$$d = 4 \cdot \tilde{d}_0 \cdot (1-m) ln\left(m \cdot (1-m) \Big/ \left|\delta m \cdot (1-2m) - (\delta m)^2\right|\right) \Big/ 3, \tag{2}$$

$\tilde{d}_0$ is the average diameter of a pore, and N is the solution of the following equation:

$$0{,}32 \cdot (\delta m)^2 = m \cdot (1-m) \left[N^3 + \sum_{p=1}^{N^3} \sum_{q=1,\, p \neq q}^{N^3} e^{-3 \cdot h_{pq} / 2 / \tilde{d}_0 / (1-m)} \right] \Big/ N^6 \tag{3}$$

where h_{pq} is the distance between the centres of the above mentioned cubic parts with numbers p and q.

3. Results of numerical calculations

A total error can be estimated as the square root of the sum of squares of errors from different sources [13]. Therefore, to estimate a minimal size of an elementary volume according to Eq. (1), in what follows we assume that maximal allowed uncertainty in porosity is determined by the condition that uncertainty in a calculated value due to uncertainty in the porosity is three times smaller than the total error of the calculation of this value. In the standard laboratory test [11], the porosity of the sand in the tube has such the value $m = 0{,}335$ and the injection front is detected at the moments of time $t_1 = 100$ sec, $t_2 = 250$ sec, and $t_3 = 400$ sec at distances from the injection point respectively equal to $0{,}2$ m, $0{,}4$ m, and $0{,}6$ m. The respective values of the cement concentration in the fluid phase calculated in [12] coincide within the limits of the total calculation error bars. Performing the numerical analysis similar to the one presented in [12], we obtain that if the uncertainty in the porosity δm_1 is equal to 0,014, then the uncertainties in these concentration values due to the uncertainty in the value of the porosity are three times smaller than the respective total calculation errors obtained in [12]. The dependence of the injection pressure upon the time calculated in [12] coincides with the one measured in [11] within the error bar limits. Performing the numerical analysis similar to the one presented in [12], we obtain that if the uncertainty in the porosity has such the value $\delta m_2 = 0{,}0463$, then the uncertainty in the injection pressure due to the uncertainty in the value of the porosity is three times smaller than the total error of the respective calculation obtained in [12]. Since $\delta m_2 > \delta m_1$, in what follows we assume that the maximal allowed uncertainty in the value of m is equal to δm_1. Substituting δm_1 for δm in Eqs. (2) and (3) we obtain that

$$d = 3{,}48 \cdot \tilde{d}_0, \quad 15 < N < 16. \tag{4}$$

The 1-dimensional model used in [11, 12] is derived from the 3-dimensional one in [10]. Therefore, the characteristic length of the domain in which the numerical modelling [12] is performed is equal to the diameter of the tube which is the following: $l_0 = 0{,}08$ m [11]. We assume that the continuum approach is properly adopted in [12] if $s < l_0/10$ where s is given by Eq. (1). Therefore, it follows from (1) and (4) that the continuum approach is properly adopted in [12] if $\tilde{d}_0 < 1{,}5 \cdot 10^{-4}$ m. Pores in sand are mesopores. Their diameters range from $1{,}0 \cdot 10^{-5}$ m to $1{,}0 \cdot 10^{-3}$ m [14]. Since in the laboratory test [11] the sand in the tube was compacted, we can assert that it is likely that the continuum approach was properly adopted in [12].

4. Conclusion

The degrees of uncertainties in the diameters of the tubes in the standard laboratory tests [1, 2], and [11] are approximately the same. Demchuk [12] shows that the main contributions to the errors of calculated values come from uncertainties in these values due to uncertainties in the diameter of the tube and concludes that in the recent research [1, 2] as well as in [12] the comparisons of model calculations with laboratory measurements provide small amounts of information. Sizes of pores in sand range from $1{,}0 \cdot 10^{-5}$ m to $1{,}0 \cdot 10^{-3}$ m [14]. In this work, we have shown that the continuum approach was properly adopted in [12] only if the average pore size in the compacted sand was smaller $1{,}5 \cdot 10^{-4}$ m. Thus, we can conclude that to improve the quality of the comparisons of model calculations with laboratory measurements in the recent research [1, 2], and [12] it is necessary not only to increase the accuracy of the measurements of the diameters of the tubes but also to ensure strong enough compactions of the sands.

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