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ELECTROMAGNETIC SIGNAL TRANSFORMATION IN NONSTATIONARY PLASMA AT TEMPORARY JUMP OF EXTERNAL MAGNETIC FIELD

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Electromagnetic wave transformation in plasma with a slowly changing external magnetic field has been considered in reference [1] and in suddenly formed plasma in the steady-state external magnetic field in [2,3]. In this work the electromagnetic transformation with changing in time both the external magnetic field and plasma density have been considered. Temporary changes of the magnetic field and plasma density are approximated by a succession of step functions. The field transformation for each temporal step is exactly defined by Volterra's integral equation solution describing the magnetic field in magnetized plasma.

INTRODUCTION

The electromagnetic field in uniform boundless plasma being in the homogenous external unlimited magnetic field satisfies the Volterra equation of the second kind [4,5]:

$$\vec{E}^{(n)} = \vec{F}^{(n)}(t,\vec{r}) + \frac{\omega_e^2}{4\pi}\int_{t_n}^{t} dt' \int_{\infty} d\vec{r}' K(t,t',\vec{r},\vec{r}') \int_{t_n}^{t'} \alpha^{(n)}(t'-t'')\vec{E}^{(n)}(t'',\vec{r}')dt'' \tag{1}$$

where $K = \left(\nabla\nabla - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\frac{\delta\left(t-t'-\frac{1}{c}|\vec{r}-\vec{r}'|\right)}{|\vec{r}-\vec{r}'|}$,

c is the light velocity, $\delta(t)$ is the delta function, t_n is the moment of the n-th jump of the magnetic field, ω_e is the plasma frequency. The tensor of polarization $\alpha^{(n)}$ has components as follows [6]:

$$\alpha^{(n)}ij(t) = \frac{1}{\Omega_n}\sin\Omega_n t\delta_{ij} + \frac{1}{\Omega_n}(\cos\Omega_n t - 1)e_{ikj}b_k^{(n)} + \left(1 - \frac{1}{\Omega_n}\sin\Omega_n t\right)b_i^{(n)}b_j^{(n)} \tag{2}$$

where Ω_n is the Larmor frequency, $b^{(n)}$ is the unit vector in the direction of the external magnetic field after the n-th jump, δ_{ij} is the Kronecher symbol e_{ijk} is the antisymmetric tensor in the third' dimention.

The free term of Eq. (1)

$$\vec{F}^{(n)}(t,\vec{r}) = \vec{F}^{(0)}(t,\vec{r}) + \\ + \sum_{k=1}^{n-1}\int_{t_k}^{t_{k+1}} dt' \int_{\infty} d\vec{r}' K(t,t',\vec{r},\vec{r}')\vec{P}^{(k)}(t',\vec{r}') + \int_{t_n}^{t} dt' \int_{\infty} d\vec{r}' K(t,t',\vec{r},\vec{r}')\vec{P}_r^{(k)}(t',\vec{r}') \tag{3}$$

takes into account as the pre-history of interaction of the electromagnetic field with plasma before changing of the external magnetic field

$$\vec{F}^{(0)}(t,\vec{r}) = \int_{-\infty}^{0} dt' \int_{\infty} dr' K(t,t',\vec{r},\vec{r}') \times \frac{\omega_e^2}{4\pi}\int_{-\infty}^{t'} dt''(t'-t'')\vec{E}(t'',\vec{r}'), \tag{4}$$

as well the effect of a jump of the external magnetic field on preliminary steps.

Let the external magnetic field after inclusion in zero moment abruptly changes in time, but remaining constant at each step. Then the expression of a polarization vector $P^{(n)}$ after the n-th jump of the external magnetic field obtained from equations of motion for plasma particles is expressed by the ratio

$$\vec{P}^{(n)}(t)=\vec{P}_r^{(n)}(t)+\frac{w_e^2}{4p}\int_{t_n}^{t}a^{(n)}(t-t')\vec{E}^{(n)}(t')dt',$$
$$\vec{P}_r^{(n)}(t)=\vec{P}^{(n-1)}(t_n)+a^{(n)}(t-t_n)\frac{d\vec{P}^{(n-1)}(t_n)}{dt} \tag{5}$$

where $P_r^{(n)}$ in the remainder of the plasma polarization. Polarization to the zero time momentum $(t_1=0)$ is defined by the expression

$$\vec{P}^{(0)}(t)=\frac{w_e^2}{4p}\int_{-\infty}^{t}(t-t')\vec{E}(t')dt', P_r^{(n)}=0 \tag{6}$$

The equation (1) solution is realized by the resolventa method. For its consideration let us studied one step of the external magnetic field. In this case $(n=1)$ of the field equations (1) resolventa is external magnetic field direction is along

$$R^{(1)}(t,t',\vec{r},\vec{r}')=\frac{1}{(2p)^4 i}\int_{-i\infty}^{i\infty}dp\int_{\infty}d\vec{k}T^{(1)}(p,\vec{k})e^{p(t-t')+i\vec{k}(r-r')} \tag{7}$$

Then it may be supposed, that $z,b^{(1)}=\{0,0,1\}$. Then in (7)

$$T^{(1)}(p,\vec{k})=\frac{-w_e^2c^2}{G(p,\vec{k})}j_e\left(T_1^{(1)}+p\Omega_1T_2^{(1)}+\frac{p^2}{c^2}T_3^{(1)}+\frac{p^2}{c^2}(p^2+w_e^2)I\right) \tag{8}$$

where $j_e=p^2+c^2k^2+w_e^2$, I - unit matrix, and $T_j^{(1)}$ matrices are:

$$T_1^{(1)}=p^2\begin{pmatrix} k_1^2 & k_{1Á}k_2 & \left(1+\frac{\Omega_1^2 j}{p^2 j_e}\right)k_1k_3 \\ k_2k_1 & k_2^2 & \left(1+\frac{\Omega_1^2 j}{p^2 j_e}\right)k_2k_3 \\ k_3k_1 & k_3k_2 & \left(1+\frac{\Omega_1^2 j}{p^2 j_e}\right){k_3}^2 \end{pmatrix} \tag{9}$$

$$T_2^{(1)}=\begin{pmatrix} -k_1k_2 & k_2^2+\frac{w_e^2}{j_e}k_3^2 & -\frac{w_e^2}{j_e}k_2k_3 \\ -k_2^2-\frac{w_e^2}{j_e}k_3^2 & k_2^2 & \frac{w_e^2}{j_e}k_3^2k_3 \\ -\frac{j}{j_e}k_3k_2 & \frac{j}{j_e}k_3k_1 & 0 \end{pmatrix},\quad T_3^{(1)}=\begin{pmatrix} 0 & p\Omega_1 & 0 \\ -p\Omega_1 & 0 & 0 \\ 0 & 0 & \Omega_1^2\frac{j}{j_e} \end{pmatrix}$$

$j=p^2+c^2k^2$. The hyromagnetic polynomial in (8) has the following form

$$G^{(1)}(p,k)=p^2(p^2+w_e^2)(p^2+w_e^2+c^2k^2)^2+\Omega_1^2(p^2+c^2k^2)\left[p^2(p^2+w_e^2+c^2k^2)+w_e^2c^2{k_3}^2\right]$$

In case of stepwise changing of the external magnetic field it is necessary to transfer gradually the initial temporary moment from step to step. The field for each step is defined by the same resolventa (7) for which it is necessary only to substitute the value of the Larmor frequency Ω_n. It is necessary to modify the free term of (3) in the equation (1) adding to it after each step the integral for the previous time integral.

Thus, after the n-th jump of the external magnetic field the formula for the electrical field in magnetoactive plasma will be of the form

$$\vec{E}^{(n)}(t,\vec{r}) = \vec{F}^{(n)}(t,\vec{r}) + \int_0^t dt' \int_\infty d\vec{r}' R^{(n)}(t,t',\vec{r},\vec{r}')\vec{F}^{(n)}(t',\vec{r}') \tag{10}$$

TRANSFORMATION OF ELECTROMAGNETIC OSCILLATIONS

1) Transformation of a plane wave.

Let before initial changing of the external magnetic field in plasma there is only one of proper waves e.g. the plane wave is $\vec{E}_0(t,\vec{r}) = \vec{E}_0 \exp\left[i\left(\omega t - \vec{s}\vec{r}\right)\right]$, where $s = c^{-1}\left(\omega^2 - \omega_e^2\right)^{1/2}$. From (4), (2) and (10) we find the expression of the transformed electrical field for the external magnetic field arbitrarily oriented:

$$\vec{E}^{(1)}(t,\vec{r}) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\left(I + T^{(1)}(p,-\vec{s})\right)\vec{O}^{(1)}(p)e^{pt - i\vec{s}\vec{r}}dp \tag{11}$$

where $\vec{O}^{(1)}(p) = \dfrac{-i}{\omega\left(p^2 + c^2 s^2\right)}\left[\left(c^2 s^2 - i\omega p\right)\vec{E}_0 + p^2 A^{(1)}\vec{E}_0 + c^2\vec{s}\left(\vec{s}A^{(1)}\vec{E}_0\right)\right]$,

$$A^{(1)} = \frac{1}{p^2\left(p^2 + \Omega_1^2\right)}\begin{pmatrix} p^2 & p\Omega_1 & 0 \\ -p\Omega_1 & p^2 & 0 \\ 0 & 0 & p^2 + \Omega_1^2 \end{pmatrix}.$$

a) If the including magnetic field is oriented along the wave vector of the initial wave $\vec{b}^{(1)} \| \vec{s} = \{0,0,s\}$, $\vec{E}_0 = \{E_0,0,0\}$, then the transformed field contains three couples of waves with frequencies p_l those are the roots of the polynomial

$$H^{(1)}(p) = p^2\left(p^2 + \omega^2\right)^2 + \Omega_1^2\left(p^2 + c^2 s^2\right)^2, \tag{12}$$

and vector amplitudes

$$\frac{1}{\omega\dfrac{dH^{(1)}(p_l)}{dp}}\left\{\omega\left(p_l + i\omega\right)Q^{(1)}(p_l) - i\omega_e^2\Omega_1^2\left(p^2{}_l + c^2 s^2\right), -i\omega_e^2\Omega_1 p_l^2\left(p_l + i\omega\right), 0\right\} \tag{13}$$

where $Q^{(1)}(p) = p^4 + \left(\omega^2 + \Omega_1^2\right)p^2 + \Omega_1^2 c^2 s^2$. In each couple of waves transverse waves have the same wave vectors as the initial ones and propagate in opposite directions.

â) If the external magnetic field is perpendienlar to the wave vector i.e., $\vec{s} = \{s,0,0\}$, $\vec{E}_0 = \{0,E_2,E_3\}$, then the transformed electrical field is determined by

$$\vec{E}^{(1)}(t,\vec{r}) = \vec{E}_{\|}^{(1)}\exp\left[i\left(\omega t - \vec{s}\vec{r}\right)\right] + \sum_{m=0}^{1}\sum_{l=1}^{2} E_{\perp ml}^{(1)}\exp\left[(-1)^m i p_l t - i\vec{s}\vec{r}\right] \tag{14}$$

where $\vec{E}_{\|}^{(1)} = \{0,0,E_3\}$,

$$\vec{E}_{\perp mn}^{(1)} = \left\{-\Omega_1\omega_e\left[(-1)^m p_l + \omega\right],\quad i\omega\left[(-1)^m p_l + \omega\right]\left(-p_l^2 + \omega_e^2 + \Omega_1^2\right) - i\Omega_1^2\omega_e^2,\quad 0\right\}E_2$$

and new frequencies equal to

$$p_l = 2^{-\frac{1}{2}}\left[\omega^2 + \omega_e^2 + \Omega_1^2 + (-1)^l\sqrt{\left(\omega^2 + \omega_t^2 + \Omega_1^2\right)^2 + 4\omega_e^2\Omega_1^2}\right]^{\frac{1}{2}}.$$

In this case the component of the initial wave parallel to the vector $\vec{b}^{(1)}$ does not change. And the component perpendicular to $\vec{b}^{(1)}$,forms two couples of waves. These waves have transverse and longitudinal components and correspond to rapid and slow unusual waves [5].

It should be noted that in all of the cases the sharp jump of the external magnetic field transforms a linear spectrum of the initial electromagnetic field into linear that.

2) Transformation of plasma oscillations.

Having been substituted $\omega = \omega_e$ in formulas obtained above we find the expression for transformation of plasma oscillations for including of the magnetic field. If $\vec{b}^{(1)}$ is perpendicular to the electrical field of oscillations the latter ones are transformed into two elliptically polarized oscillations:

$$\vec{E}^{(1)}(t)=\sum_{l=1}^{2}\frac{E_0}{2\left(w_e^2-w_l^2\right)+\Omega_1^2}\left\{\left(w_e^2-w_l^2+\Omega_1^2\right)\cos w_l t,\quad \Omega_1\frac{w_e^2}{w_l}\sin w_l t,\ 0\right\} \tag{15}$$

where $w_l^2=\frac{1}{2}\left(2w_e^2+\Omega_1^2+(-1)^{l-1}\Omega_1\sqrt{4w_e^2+\Omega_1^2}\right)$.

In case of the weak magnetic field $\Omega_1 << w_e$ both oscillations have small different frequencies, $w_l^2\approx w_e^2\pm w_e\Omega_1$ and nearly circular polarization:

$$\vec{E}^{(1)}(t)=\sum_{l=1}^{2}\frac{E_0}{(-1)^{l-1}2w_e+\Omega_1}\times\left\{(-1)^{l-1}\cos w_l t,\quad \left[1+(-1)^{l-1}\frac{\Omega_1}{w_e}\right]\sin w_l t,\ 0\right\}$$

In case of a strong magnetic field $\Omega_1 >> w_e$ oscillation swith the frequency $w_1\approx\Omega_1$ will be cyclotron ones with circular polarization and a small amplitude equal $\approx\left(w_e/\Omega_1\right)^2$. The second oscillation has the plasma frequency of $w_2\approx w_e$ and linear polarization. When the external magnetic field is parallel to the electrical one of oscillations those do not change.

CONCLUSION

The plane electromagnetic wave and also plasma oscillation transformation the temporary jump of the arbitrary oriented external magnetic field were considered. It is shown that if the turning magnetic field is directed to the primary wave propagation then latter is transformed into three couples of waves with different frequencies, with that in each couple waves remain transverse ones conserve the wave vector and propagate in opposite directions. In case of the perpendicular orientation of the magnetic field waves have both transverse and longitudinal components and corresponds to rapid and slow unusual waves [7].*

Plasma oscillations in case of inclusion of magnetized plasma perpendicular to the electric field are transformed in two elliptically polarized oscillations with different frequencies. In case of the strong magnetic field one of three oscillation has almost circular polarization and frequency near cyclotron one, the second oscillations has almost linear polarization and frequency near plasma that. Inclusion of the magnetic field parallel to the electrical one of oscillations does not effect on the latter. In all of these cases transformed field in the result of inclusion of magnetization has the discrete spectrum.

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