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Stochasticity of the Magnetic Field Lines and High Z Ion Motion

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Stochastic layer of the magnetic field lines is numerically analyzed for the realistic magnetic field of HELIAS reactor with use of the flux coordinate system. Analytical expressions for the stochastic diffusion coefficients are obtained for the simplified magnetic field model.

1. INTRODUCTION

Magnetic islands are the part of the divertor configuration in the modern fusion devices and future reactor systems. The examples are the operating devices Wendelstein-7AS and heliotron Large Helical Device, the stellarator under construction Wendelstein-7X and stellarator reactor system HELIAS. Overlapping of the adjacent magnetic islands leads to the creation of the stochastic layer. The properties of the stochastic layer are usually analyzed by numerical calculations of the magnetic field lines with the use of Biot-Savart law in cylindrical coordinate system [1]. In this paper the properties of the stochastic layer are studied by numerical calculations of the drift motion equations with the use of the expansion of magnetic field in the series depending on coordinates. This approach also allows considering the particle motion and MHD model for the impurity ion dynamics in finite b plasma. Numerical calculations are provided with use of the flux coordinates for the reactor system HELIAS.

2. ANALYTICAL TREATMENT

In the reactor system HELIAS the magnetic field configuration with five islands is realized. To create the model of such configuration the main magnetic field $\mathbf{B}_m$ with the field perturbation $\mathbf{B}_1$ are taken. To model stochasticity of the magnetic island surfaces it is necessary to use one more magnetic field perturbation $\mathbf{B}_2$. Taking into account all this it is possible to write the total magnetic field in the following form

$$\mathbf{B} = \mathbf{B}_\Sigma + \mathbf{B}_2 , \tag{2.1}$$

where

$$\mathbf{B}_\Sigma = \mathbf{B}_m + \mathbf{B}_1 . \tag{2.2}$$

Let us consider the toroidal magnetic flux y, which concerns to the total magnetic field and is a function of the coordinates. Using quasi-cylindrical coordinate system one can write

$$\frac{dy}{dj} = \frac{\partial y}{\partial j} + \frac{\partial y}{\partial r}\frac{\partial r}{\partial j} + \frac{\partial y}{\partial J}\frac{\partial J}{\partial j} , \tag{2.3}$$

where

$$y = y_\Sigma + y_2 . \tag{2.4}$$

Here y_Σ is related to $\mathbf{B}_\Sigma$, and y_2 is related to the additional magnetic field perturbation which is presented by $\mathbf{B}_2$.

It is necessary to note that for the magnetic force line equation (2.3) can be rewritten as follows

$$\frac{dy}{dj} = \frac{\partial y}{\partial j} + R\frac{\partial y}{\partial r}\frac{B_r}{B_j} + \frac{R}{r}\frac{\partial y}{\partial J}\frac{B_J}{B_j} . \tag{2.5}$$

In the small perturbation approximation the toroidal flux can be represented by expansion in Taylor series

$$y = y_\Sigma + \Delta r\frac{\partial y_\Sigma}{\partial r} + r\Delta J\frac{\partial y_\Sigma}{r\partial J} + R\Delta j\frac{\partial y_\Sigma}{R\partial j} . \tag{2.6}$$

Substituting expansion (2.6) in to (2.5) one can obtain

$$\frac{\partial y_\Sigma}{\partial j} + A_{21}\frac{\partial y_\Sigma}{\partial r} + A_{31}\frac{\partial y_\Sigma}{\partial J} + D_{rr}\frac{\partial^2 y_\Sigma}{\partial r^2} + \tag{2.7}$$

$$+ D_{JJ}\frac{\partial^2 y_\Sigma}{\partial J^2} + D_{rJ}\frac{\partial^2 y_\Sigma}{\partial r\partial J} = 0 .$$

This is the equation, which describes "diffusion" of the magnetic field line in $j = const$ cross section. Coefficients D_{rr}, D_{JJ}, D_{rJ} are interpreted as diffusion coefficients [2,3] and have following form

$$D_{rr} = \frac{\Delta r\left(B_{\Sigma r} + B_{2r}\right)}{A_1} , \tag{2.8}$$

$$D_{JJ} = \frac{r\Delta J\frac{1}{r^2}\left(B_{\Sigma J} + B_{2J}\right)}{A_1} , \tag{2.9}$$

$$D_{rJ} = \frac{r\Delta J\frac{1}{r}\left(B_{\Sigma r} + B_{2r}\right) + \frac{1}{r}\Delta r\left(B_{\Sigma J} + B_{2J}\right)}{A_1} , \tag{2.10}$$

where

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$$A_1 = \frac{B_{\Sigma r}}{R}\left(\frac{1}{R}\frac{\partial}{\partial j}(R\Delta j) + R\Delta j\ \frac{\partial}{\partial j}\left(\frac{1}{R}\right)\right) +$$
$$+\left(B_{\Sigma r} + B_{2r}\right)\left(\frac{1}{R}\frac{\partial}{\partial r}(R\Delta j) + R\Delta j\ \frac{\partial}{\partial r}\left(\frac{1}{R}\right)\right) + \qquad (2.11)$$
$$+\frac{1}{r}\left(B_{\Sigma J} + B_{2J}\right)\left(\frac{1}{R}\frac{\partial}{\partial J}(R\Delta j) + R\Delta j\ \frac{\partial}{\partial J}\left(\frac{1}{R}\right)\right).$$

These analytical expressions for the stochastic diffusion coefficients can be used for the optimization of the magnetic field perturbation parameters that leads to creation of the effective divertor configurations.

3. NUMERICAL STUDIES

As was mentioned above for the numerical studies of the magnetic field properties HELIAS reactor system was chosen. Parameters of HELIAS reactor system are the following: large torus radius $R_0 = 2200\ cm$, plasma radius $a_p = 180\ cm$, and magnetic field at the circular axis of the torus $B_0 = 5T$. Finite b case is considering ($b = 3\%$). The main magnetic field is written as an expansion with the use of the flux coordinate system [4]

$$\frac{B}{B_0} = 1 + \sum_{k=0}^{\infty} b_{0,k}\left(|\mathbf{r}_p|\right)\cos(Mkz) +$$
$$+\sum_{l=1}^{\infty}\sum_{k=-\infty}^{\infty} b_{l,k}\left(|\mathbf{r}_p|\right)\cos(Mkz - lJ). \qquad (3.1)$$

Here B is magnitude of the magnetic field at the point with coordinates (r_p, z, J), $|\mathbf{r}_p|$ is the radial coordinate of the force line, z is the coordinate along the torus, J is the angle between the equatorial plane of the torus and vector $\mathbf{r}_p$. Now introduce magnetic field perturbations of the form [5,6]

$$d\mathbf{B}_i = \nabla \times a_i \mathbf{B}. \qquad (3.2)$$

Perturbation parameter is taken as a harmonic function of coordinates, perturbation frequency and phase d

$$a_i = a\,|\mathbf{r}_p|^m \sin(nz - mJ - wt + d). \qquad (3.3)$$

Here a is an amplitude of perturbation, n, m are the 'wave' numbers. There is the sense to use a perturbation frequency w in the case, when the perturbation depends on time. In this work static perturbations are considered ($w = 0$). The equations of drift motion are written in Hamiltonian form [5, 6]

$$\dot{P}_z = -\frac{\partial H}{\partial z},\ \dot{P}_J = -\frac{\partial H}{\partial J}, \qquad (3.4)$$

$$\dot{z} = \frac{\partial H}{\partial P_z},\ \dot{j} = \frac{\partial H}{\partial P_J}, \qquad (3.5)$$

$$\dot{y} = \frac{\partial y}{\partial P_J}\dot{P}_J + \frac{\partial y}{\partial P_z}\dot{P}_z, \qquad (3.6)$$

$$\dot{r}_{\parallel} = \frac{\partial r_c}{\partial P_J}\dot{P}_J + \frac{\partial r_c}{\partial P_z}\dot{P}_z -$$
$$-\frac{\partial a}{\partial y}\left(\frac{\partial y}{\partial P_J}\dot{P}_J + \frac{\partial y}{\partial P_z}\dot{P}_z\right) - \frac{\partial a}{\partial J}\dot{j} - \frac{\partial a}{\partial z}\dot{z}. \qquad (3.7)$$

Relationship $r_{\parallel} = r_c - a(y, J, z)$ is also very important to be introduced. The Hamiltonian for the drift motion is

$$H = \frac{1}{2}r_{\parallel}^2 B^2 + mB + \Phi, \qquad (3.8)$$

where $r_{\parallel}$ is a normalized parallel gyroradius, m is a normalized magnetic moment and Φ is an electric potential. On Fig.1 the model of the magnetic field configuration that is realized as a main magnetic field configuration in HELIAS reactor is presented.

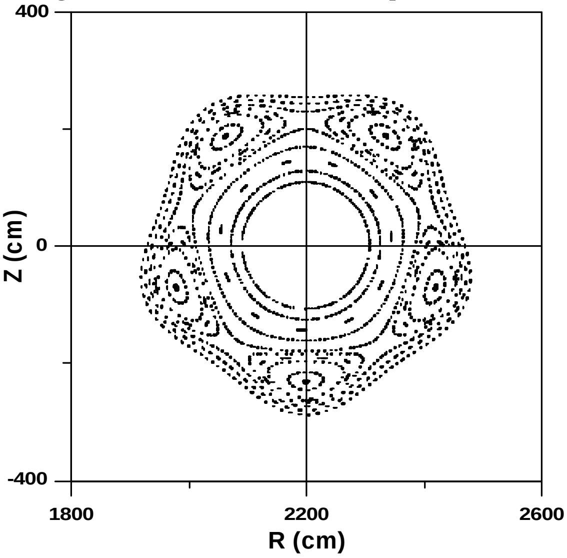

Fig.1 Vertical cross section of the main magnetic field configuration in HELIAS reactor system represented in flux coordinates.

In a real device five magnetic islands have to appear without any additional perturbation. But for the analytical and numerical calculations these islands, which have to be crossed by divertor plates, are modeled by magnetic field perturbation term $\mathbf{B}_1$ with 'wave' numbers $m = 5, n = 5$. Divertor plates are the strong source of high charged tungsten ions and other types of heavy ions. Such ions can be considered as impurity ions, which can appear at outside magnetic surfaces of the confinement volume (Fig.2).

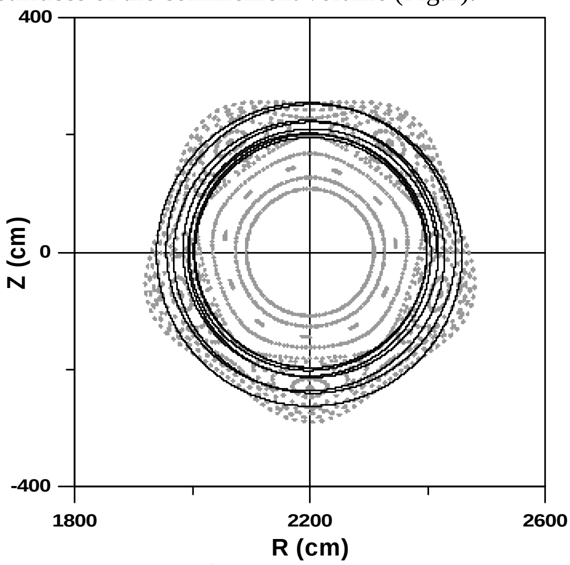

Fig.2 Trajectory of highly charged tungsten ion in the main magnetic field configuration (vertical cross section).

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As the test particle the tungsten ion with charge number Z=30, the energy W=1keV and $V_{\parallel}/V = 0.5$ is taken.

If stochasticity of outside magnetic surfaces and island magnetic surfaces is caused by some effect, specific conditions can be created that leads to escape of impurity ions from outside magnetic surfaces and confinement volume. As was mentioned in section 2, such stochasticity of the magnetic field is modeled by using additional perturbation term $\mathbf{B}_2$ with 'wave' numbers $m = 9, n = 10$ (Fig.3).

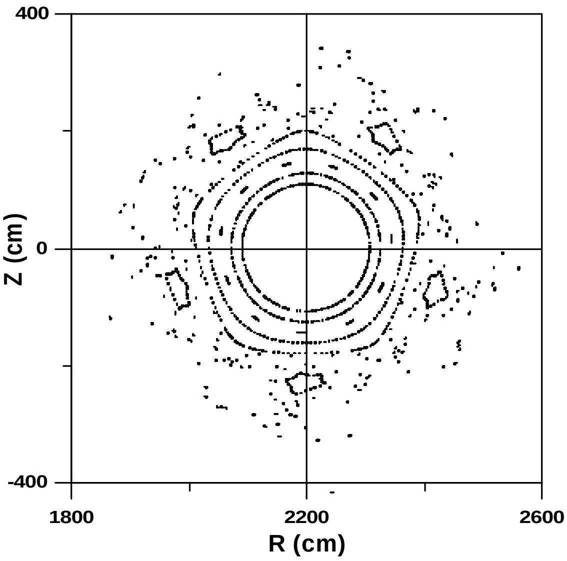

Fig.3 Vertical cross section of the magnetic field configuration with stochastic layer caused by additional perturbation with 'wave' numbers (m=9,n=10).

On Fig.4 the trajectory of the tungsten ion in the stochastic magnetic field configuration is shown.

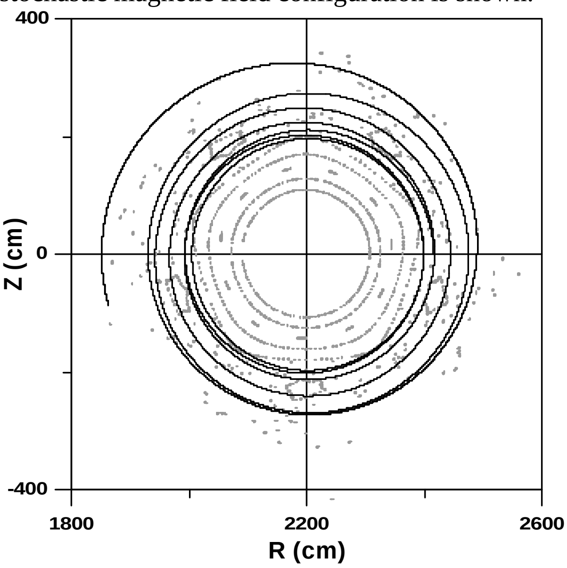

Fig.4 Trajectory of highly charged tungsten ion in magnetic field configuration with stochastic layer (vertical cross section).

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As one can see after several rounds along the small radius of the torus the tungsten ion follows the resonance structure and stochastic magnetic field lines in the divertor region and moves outside from the last close magnetic surface. Such effect can be used to remove impurity ions from the edge of plasma back to the divertor plates and to the wall of the vacuum vessel.

CONCLUSIONS

1. Analytical expressions for the stochastic diffusion coefficients that can be used for the optimization of the magnetic perturbations parameters (amplitudes, "wave" numbers, phases) to create effective divertor configurations are obtained.
2. It is shown that the high Z impurity ion follows the resonance structure (magnetic islands) and stochastic magnetic field lines in divertor region of HELIAS configuration.

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