TWO FEATURES OF DYNAMICS OF THE CHARGED PARTICLES IN EXTERNAL MAGNETIC FIELD AND IN FIELD OF ELECTROMAGNETIC WAVE

V. A. Buts

National Science Center “Kharkov Institute of Physics and Technology”, 61108, Kharkov, Ukraine, e-mail: vbuts@kipt.kharkov.ua

It has shown that regimes with superdiffuse in space of velocity are inherent for moving of charged particles in external magnetic field and in fields of electromagnetic waves. The regimes with dynamic tunneling were finding too.

PACS: 52.20.-j; 05.45.-a

1.INTRODUCTION

The main tendency of the development of high frequency electronics is contain in using more and more intense electromagnetic fields. The dynamic of charged particles in such fields can significantly change. Some peculiarities of this dynamic were described in the works [1,2]. Below we shall show that at enough large strength of the fields the regimes with super-diffuse are more character regimes for moving charged particles in external magnetic field and in field of electromagnetic wave. Except it, below, we shall show that there are regimes with dynamic tunneling. At these regimes the super-diffuse take place. At the dynamic tunneling the particles very quickly overpass through stochastic layers. It is necessary to note that at usual conditions the stochastic layers possess property of the “stickiness”. It means that the time which the particles spend into these layers or in their vicinity significantly large then the time that these particles spend in another phase space. The regime of the dynamic tunneling can very useful at the cascade acceleration of the charged particles.

2. BASIC EQUATIONS

Let’s look at the charged particle which is moving in external magnetic field H_0, which is directed along z axis and in field of the electromagnetic field with arbitrary polarization. This wave has following components:

$$\vec{\varepsilon} = \mathrm{Re}\, \mathrm{E} \exp\left(i\vec{k}\vec{r} - i\omega t\right);$$
$$\vec{E} \equiv \left\{E_0\left(\alpha_x, i\alpha_y, \alpha_z\right)\right\} \qquad (1)$$
$$\vec{H} = \mathrm{Re}\frac{c}{\omega}\left[\vec{k}\vec{E}\right]\exp\left(i\vec{k}\vec{r} - i\omega t\right),$$

where $\vec{\alpha} \equiv \left\{\alpha_x, i\alpha_y, \alpha_z\right\}$- polarization vector.

Without any restriction we can consider that the wave vector k has only two components k_x and k_z.

We shall measure the time in ω^{-1}, velocities in c, wave vector in ω / c, impulse in mc. Except it, it is useful to introduce dimensionless amplitude $\varepsilon = eE_0 / mc\omega$. The equations of motion in these variables will look as:

$$\dot{\vec{P}} = \left(1 - \frac{\vec{k}\vec{p}}{\gamma}\right)\mathrm{Re}\left(\vec{\varepsilon}\, e^{i\Psi}\right) + \frac{\omega_H}{\gamma}\left[\vec{p}\vec{e}\right] + \frac{\vec{k}}{\gamma}\mathrm{Re}\left(\vec{p}\vec{\varepsilon}\right)e^{i\psi};$$
$$\dot{\vec{r}} = \vec{p}/\gamma; \quad \dot{\psi} = \vec{k}\vec{p}/\gamma - 1, \qquad (2)$$

where $\tau \equiv \omega t, \quad \vec{e} \equiv \vec{H} / H_0; \ \omega_H \equiv eH_0 / mc\omega;$
$\psi = \vec{k}\vec{r} - \tau$.

The equations (2) have following integral of movement:

$$\vec{p} - \mathrm{Re}\left(i\vec{\varepsilon}\, e^{i\psi}\right) + \omega_H\left[\vec{r}\vec{e}\right] - \vec{k}\gamma = const\,. \qquad (3)$$

For further it is convenient to introduce new dependent variables $p_\perp$, $p_\parallel$, θ, ξ и η:

$$p_x = p_\perp \cos\theta,\, p_y = p_\perp \sin\theta,\, p_z$$
$$= p_\parallel,\, x = \xi - \frac{p_\perp}{\omega_H}\sin, \quad y = \eta + \frac{p_\perp}{\omega_H}\cos\theta\,. \qquad (4)$$

3. SUPER-DIFFUSE

The system (2) was studied in works [3, 4] for the cases when amplitude of the wave was enough small ($\varepsilon << 1$) and for relativistic case ($p_\perp >> 1$). The main peculiarity of the slow dynamic of the charged particles is fact that this motion are described by mathematical pendulum. If the amplitude of the wave is enough large the investigation of the system (2) can be fulfilled only by using numerical methods. Such investigations were fulfilled. The significant result of these investigations is that all nonlinear cyclotron resonances are strongly overlapped. The dynamic of the charged particles are chaotically. It’s very significantly that all moments from second moment are increasing in the time. Moreover the higher moments are always larger and grow more quickly than the more low moments.

It’s well known that for unrestricted large second moments are correspond on microscopic level to the processes type “Levi flight”. In this case the usual definition of the diffuse coefficients is not correct because this coefficient is proportional to second moment. In this case it’s necessary to receive equation for distribution function. This equation must be or integro-differential or differential with fractional derivatives. We first of all should like to know expression for distribution function for large velocities (expression for tails of the distribution function) and asymptotic behavior of this function at a large time.

In this section we'll use these restrictions. It will permit us to receive not only equation for function of distribution but permit to receive its analytical expression. Let's the moving of the charged particle in external electromagnetic fields will define by function $f(v)$. This function determines a density of probability for particles to displace from initially located place in the space of velocity on the value v. The value v is determining dimensionless velocity, which is normalized at initial value of the velocity ($v \equiv v/v_0$). This function obeys such expression:

$$\int_{-\infty}^{\infty} f(v)dv = 1 \ . \tag{5}$$

The fact that the all moments from second moment are infinitely can be expressed by expression:

$$\langle v^n \rangle = \int_{-\infty}^{\infty} v^n f(v)dv = \infty \quad , \qquad n \geq 2 \, . \tag{6}$$

We shall consider that a casual jump in velocity space has equal probability for direction along velocity axis. It means that the function $f(v)$ must be even. Namely the tails of the distribution function define super-diffuse behavior. The details of the function $f(v)$ at small value velocity don't play significant role. The wide class of the functions $f(v)$ will give the same asymptotic dynamic. That is why we shall choice this function in the form that permits us to receive analytical results with smaller efforts. Using these reasons, we write this function in the form:

$$f(v) = \frac{\Gamma(\alpha+1/2)}{\sqrt{\pi}\,\Gamma(\alpha)} \frac{1}{\left(1+v^2\right)^{\alpha+1/2}} \quad . \qquad 0 < \alpha < 1 \tag{7}$$

It's easy to see that this function obey all our requirements. The combination of the gamma-function Euler is chosen only for simplicity of the further calculation. Let's notice that similar form for the density of the probability was used in work [5]. Using the usual ideology for receiving kinetic equations (see, for example, [6]) we can write:

$$n(v,n+1) - n(v,n) = \int_{-\infty}^{\infty} \left[n(v-v',n) - n(v,n) \right] f(v')dv' \, , \tag{8}$$

where $n(v,n)$ - density of the particles.

If the moments are restricted then we can resolve the expression inside integral and being limited only by second moments, we shall receive usual diffuse equation with diffusion coefficients $D = \langle v^2 \rangle / 2$. If the second moment is unlimited then the equation will remain integro-differential. In common cases to solve these equations are difficult problem. But we are interested with expressions for the density of the particles only for large value of velocity. In this case we can to receive enough simple equations. Except it we shall find their analytical solution. For this purpose, using (8), we shall receive the equation for Fourier-images of density of particles:

$$\frac{\partial n_k}{\partial t} = (f_k - 1) n_k \, , \tag{9}$$

where $f_k = \dfrac{2^{1-\alpha}}{\Gamma(\alpha)} k^\alpha \cdot K_\alpha(k)$; $K_\alpha(k)$ - Macdonald function.

We are interested with large value of the velocity ($v >> 1$). In Fourier-images it corresponds for small «wave numbers» ($k << 1$). In this case we use the asymptotic expression for Macdonald function. The equation (9) became significantly simple:

$$\frac{\partial n_k}{\partial t} = -\frac{\Gamma(1-\alpha)|k|^{2\alpha}}{\Gamma(1+\alpha)2^{2\alpha}} n_k \, . \tag{10}$$

The solving of this equation looks like:

$$n_k(t) = \exp\left[-\frac{\Gamma(1-\alpha)|k|^{2\alpha}}{\Gamma(1+\alpha)2^{2\alpha}} t \right] n_k(0) \, . \tag{11}$$

To the equation (10) in usual space there corresponds the equation:

$$\frac{\partial n}{\partial t} = \frac{\Gamma(\alpha+1/2)}{\Gamma(\alpha)\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{n(v')dv'}{|v-v'|^{2\alpha+1}} \, . \tag{12}$$

Its solution is possible to present as:

$$n(v,t) = \int_{-\infty}^{\infty} G(v-v',t) n(v',0) dv' \quad . \tag{13}$$

The integral in equation (12) should be understood in sense of the main meaning. In this case this integral is definition of the fractional derivative [7]. Thus, the integro-differential equation (12) is equivalent to the differential equation with fractional derivative. The Green function in equation (13) has enough bulky expression. We shall not write out it. We shall use the fact that as in usual diffuse equations at large time ($t \to \infty$), the structure of Green function becomes smooth and it is possible to take out from integral:

$$n(v,t) \to N \cdot G(v,t) \qquad N = \int_{-\infty}^{\infty} n(v,0)dv \quad . \tag{14}$$

At the large time ($t \to \infty$), as it follow from expression (11) for Fourier-image of the density the contribution will be given only harmonics with $k \to 0$. This fact essentially simplifies Green function. Finally, at $t \to \infty$ and $v >> 1$ the expression for density of particles gets a simple form:

$$n(v,t) = N \frac{\Gamma(\alpha+1/2)}{\sqrt{\pi} \cdot \Gamma(\alpha)} \frac{t}{v^{2\alpha+1}} \quad . \tag{15}$$

The more significant result, which are follow from this expression is that the density of the particles with growth of velocity is decreasing not as exponent (as it take place at the conventional diffuse), but under power law.

4. DYNAMIC TUNNELING

The short-cat equation of the system (2) in nonrelativistic case ($p_\perp << 1$) and for $k_z = 0$ we can represent in form:

$$\frac{dP_\perp}{d\tau} = Re(iJ_s'(\mu)e^{i\theta_s}\varepsilon_0) \tag{16}$$

$$\frac{d\theta_s}{d\tau} = \frac{s\omega_H}{\gamma} - 1 - \mathrm{Re}(\frac{1}{\omega_H}\left(\frac{s^2}{\mu^2}\right) J_s(\mu)e^{i\theta_s}\varepsilon_0) \, .$$

The significant difference of this system from investigated in works [3,4] is the fact that the phase portrait is not mathematical pendulum portrait. This portrait has or only one special point that is "center" or has two such points and one point that is type “saddle”. In last case this phase portrait is topology equivalent to the phase portrait of Duffing oscillator ($\ddot{x}+\alpha\cdot x+\beta\cdot x^3 = f(t)$). The dynamic tunneling regime for Duffing oscillator is well investigated. We have studied system (16) and Duffing oscillator for behavior of the higher moments. It’s happened, that in this regime the higher moments, as well as in section 3, has unrestricted growing, i.e. for these regimes the super-diffuse is characteristic.

In figures 1-3 potential Duffing oscillator, its phase portrait and characteristic realization are submitted. From the realization it is visible, that in a regime of dynamic tunneling the particle quickly jumps from one potential hole in another. Thus, in this regime the particles do not wander in the stochastic sea.

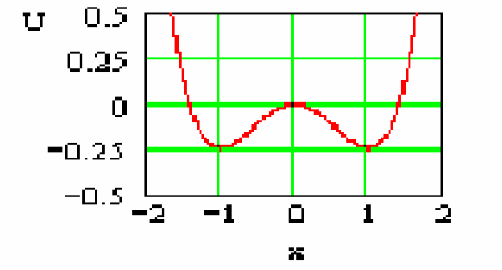

Fig.1. The potential of Duffing oscillator

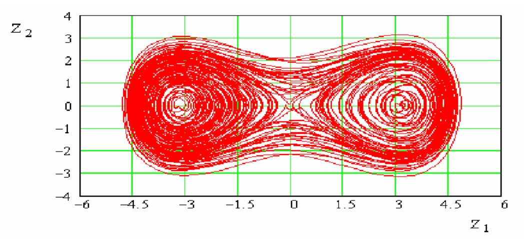

Fig.2. Phase portrait of Duffing oscillator

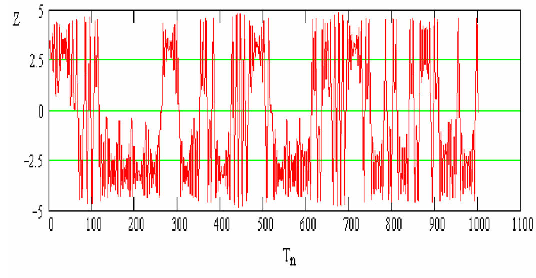

Fig.3. Localization of a particle. One can see quick jumps from left hole to right and back

REFERENCES

1. A.V. Buts, V.A. Buts. Dynamic of the charged particles in a field intense transversal electromagnetic wave // *JETPh*. 1996, v.110, N. 3(9), p.818-831.
2. V.A. Buts, V.V. Kuzmin. Dynamic of the particles in a field’s large intensity // *Modern radioelectronics-Uspekhi*. 2005, № 11, p. 5-20.
3. V.A. Buts, V.A. Balakirev, A.P. Tolstoluzhskiy, Yu.A. Turkin. Chaotization of the moving beams of the phased oscillators // *JETPh*. 1983, v.84, N 4, p. 1279-1289.
4. V.A. Buts, V.A. Balakirev, A.P. Tolstoluzhskiy, Yu.A. Turkin. Dynamic of the charged particles in fields of the two electromagnetic waves // *JETPh*. 1989, v. 95, N 4, p. 1231-1245.
5. K.B. Chukbar // *JETPh Letters*. 1993, v.58, p. 87.
6. A.J. Lichtenberg, M.A. Lieberman. Regular and Stochastic Motion // *Springer-Verlag*. New York: Heidelberg Berlin, 1983, p. 499.
7. S.G. Samko, A.A. Kilbas, O.I. Marichev. Integrals and Derivatives With Fractional Order and Some Their Application // *Science and Technik*. Minsk, 1987, p.687.

ДВЕ ОСОБЕННОСТИ ДИНАМИКИ ЗАРЯЖЕННЫХ ЧАСТИЦ ВО ВНЕШНЕМ МАГНИТНОМ ПОЛЕ И В ПОЛЕ ЭЛЕКТРОМАГНИТНОЙ ВОЛНЫ

В.А. Буц

Показано, что режимы супердиффузии в пространстве скоростей присущи движению заряженных частиц, которые движутся во внешнем магнитном поле и в полях электромагнитных волн. В этих же условиях могут реализоваться режимы динамического туннелирования.

ДВІ ОСОБЛИВОСТІ ДИНАМІКИ ЗАРЯДЖЕНИХ ЧАСТОК В ЗОВНІШНЬОМУ МАГНІТНОМУ ПОЛІ ТА В ПОЛІ ЕЛЕКТРОМАГНІТНОЇ ХВИЛІ

В.О. Буц

Показано, що режими супердифузії у просторі швидкостей притаманні руху заряджених часток, які рухаються в зовнішньому магнітному полі та в полях електромагнітних хвиль. В цих умовах можуть реалізуватися режими динамічного тунелювання.