ELECTRON MOTION
IN THE FIELD OF INTENSIVE PLANE LIGHT WAVE

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In the present work the equation of electron motion in the field of intensive plane light wave is approximately integrated. In the integration the force of bremsstrahlung produced by electron radiation was taken into account. The probability of periodic motion of a radiating electron is shown.

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1. The method of obtaining a short-wave radiation by scattering the intensive electromagnetic wave on electrons moving towards the wave front is widely discussed in the scientific literature [1-5]. Usually, in the theoretical study of the dynamics of an electron in the wave field one makes the assumptions, which are not acceptable to relativistic electron velocities and high electromagnetic field strengths. In modern lasers the electric field strength reaches high values (10^{12} v/m and more), and for increasing the frequency of scattered electromagnetic radiation one uses the electrons moving with a velocity close to the velocity of light.

In work [6] was noted, that the process of laser–electron interaction could result in radiation compression of the phase volume of the electron beam. This phenomenon is supposed to be used in electron storage rings to increase the density of electronic beam. The laser–electron interaction is now under theoretical consideration, as an electron motion in the undulator with a certain fictitious magnetic field [5, 7, 8].

Much earlier the solution of the equation of electron motion in the field of plane light wave, without taking into account the bremsstrahlung of electron radiation, was obtained in [9]. However, in this work the assumption is made that in the initial moment the electron is at rest.

In our work the equation of electron motion in the field of plane light wave with allowance for bremsstrahlung forces produced by electron radiation was approximately integrated. Two possibilities of electron interaction with a plane light wave are considered: the electron goes towards a wave front distribution, and the electron goes in the direction of wave front distribution.

The main results of the present work are obtained with the help of the integration method stated in [9]. The found motion integral, connecting the longitudinal and transversal components of electron velocities without radiation, has allowed reducing the equations of motion, with taking into account the bremsstrahlung forces, to the simple integrable equations.

2. The equation of electron motion in the electromagnetic field with allowance for bremsstrahlung forces looks like [10]

$$\frac{d}{dt} m\mathbf{v} = \mathbf{F}_L + \mathbf{F}_R , \qquad (1)$$

where F_L is the Lorentz force,

$$\mathbf{F}_L = eE + e\left[\frac{\mathbf{v}}{c}\mathbf{H}\right],$$

where $\mathbf{F}_R$-is the bremsstrahlung force,

$$\mathbf{F}_R = \frac{2e^3}{3m_0^2c^3}\left(1-\beta^2\right)^{-1/2} \times$$
$$\times\left\{\left(\frac{\partial}{\partial t}+(\mathbf{v}\nabla)\right)\mathbf{E}+\frac{1}{c}\left(\mathbf{v}\left(\frac{\partial}{\partial t}+(\mathbf{v}\nabla)\right)\right)\mathbf{H}\right\}$$
$$+\frac{2e^4}{3m_0^2c^4}\left\{[\mathbf{EH}]+\frac{1}{c}[\mathbf{H}[\mathbf{Hv}]]+\frac{1}{c}\mathbf{E}(\mathbf{vE})\right\}-$$
$$-\frac{2e^4}{3m_0^2c^5\left(1-\beta^2\right)}\mathbf{v}\left\{(\mathbf{E}+\frac{1}{c}[\mathbf{vH}])^2-\frac{1}{c^2}(\mathbf{Ev})\right\},$$

$m = m_0\Big/\left(1-\beta^2\right)^{\frac{1}{2}}$, $\beta = \frac{v}{c}$, m_0 - rest mass of an electron, c–light velocity, **v**–vector of electron velocity, e–electron charge, t–time, **E**, **H** - vector of electrical and magnetic fields, respectively.

Let us suppose that normal to the wave front is parallel to the coordinate axis x, and has the same or opposite direction. The electromagnetic field of a light wave will be set as

$$E_z = E_a \cos\left[2\pi\nu\left(t-\frac{\mathbf{nr}}{c}\right)+\delta\right], \qquad (2)$$

where ν - frequency, **n** - normal to the wave front, **r** -a radius-vector, δ - initial significance of a phase. The vector of magnetic field **H**

$$\mathbf{H} = [\mathbf{nE}]. \qquad (3)$$

Let us integrate Eq. (1) by the method of [9], considering $\mathbf{F}_R = 0$, and generalizing this method for the case when the initial electron velocity is not equal to zero.

Substituting the value of **H** from Eq. (3) in Eq. (1) and multiplying scalarly the equation obtained by a unit vector **n**, we obtain

$$\frac{d}{dt}\left(m\mathrm{v}_n\right) = \frac{e}{c}\left(\mathbf{Ev}\right), \tag{4}$$

where $\mathrm{v}_n = \mathbf{vn}$.

The magnitude $e\mathbf{Ev}$ represents the work of electrical force per time unit. Therefore:

$$\frac{d}{dt}\left(c^2 m\right) = e\mathbf{Ev}. \tag{5}$$

Multiplying Eq. (4) by c, and deducting from Eq. (5) the equation obtained we have:

$$\frac{d}{dt}\left(c^2 m\right) - \frac{d}{dt}\left(cm\mathrm{v}_n\right) = 0. \tag{6}$$

Integrating Eq. (6), we obtain the integral of motion:

$$\left(1 \mp \beta_x(t)\right) \Big/ \left(1 - \beta^2(t)\right)^{1/2} = \left(1 \mp \beta_x(0)\right) \Big/ \left(1 - \beta^2(0)\right)^{1/2} \tag{7}$$

where $\beta_x = |\mathrm{v}_x|/c$, v_x is the velocity projection onto the axes x, "0" in brackets designates a value at the initial time instant. The upper sign in Eq. (7) relates to the case, when the direction of wave front motion and the velocities v_x of an electron coincide, lower - when these directions are opposite.

Let us enter a new parameter, with which the integration of Eq. (1) with taking into account the forces $\mathbf{F}_R$ will be carried out. For this purpose we designate

$$t - \frac{\mathbf{rn}}{c} = S. \tag{8}$$

Let us take the derivative from S by t. Then we obtain:

$$\frac{dS}{dt} = 1 \mp \beta_x. \tag{9}$$

If we express the magnitude $\left(1 \mp \beta_x\right)$ from Eq. (7) and substitute it in Eq. (9), then we obtain the variable S.

$$\frac{dS}{dt} = \left\{\left(1 \mp \beta_x(0)\right) \Big/ \left(1 - \beta^2(0)\right)^{1/2}\right\}\sqrt{1 - \beta^2} \tag{10}$$

$$S = \left\{\left(1 \mp \beta_x(0)\right) \Big/ \left(1 - \beta^2(0)\right)^{1/2}\right\}\int_0^t \sqrt{1 - \beta^2}\, d\xi. \tag{11}$$

Using (7), as well as (8) and (10), we proceed in Eq. (1) and formula (2) from time "t" to variable "S".

$$\begin{aligned} \frac{d^2\mathbf{r}}{dS^2} = {} & \frac{e}{\mu_0}\mathbf{E} + \frac{e}{\mu_0 c}\mathbf{n}\left(\mathbf{E}\frac{d\mathbf{r}}{dS}\right) + \\ & + \frac{2}{3}\frac{1}{\mu_0} r_e^2 E_a^2 \cos^2(2\pi\nu S + \delta)\left[\pm\, \mathbf{i} - \frac{\left(1 \mp \beta_x(0)\right)^2}{\left(1 - \beta^2(0)\right)}\frac{d\mathbf{r}}{dS}\Big/ c\right] + \\ & + \frac{2}{3}\frac{1}{\mu_0}\frac{2\pi r r_e e E_a \sin(2\pi\nu S + \delta)}{\lambda}\frac{\left(1 \mp \beta_x(0)\right)}{\sqrt{1 - \beta^2(0)}} \times \\ & \times \left[-\mathbf{k} \pm \mathbf{i}\frac{dz}{dS}\Big/ c\right], \end{aligned} \tag{12}$$

where $r_e = e^2/m_0 c^2$ - radius of electron, $\lambda = c/\nu$ - light wavelength, $\mathbf{k}$ and $\mathbf{i}$ - unit vectors of the axes z - and axes x, respectively.

$$\mu_0 = m_0\left(1 \mp \beta_x(0)\right) \Big/ \left(1 - \beta^2(0)\right)^{1/2}. \tag{13}$$

The equation (12) can be integrated as follows

$$\begin{aligned} y' = {} & y'(0)\exp\left\{-\frac{A}{2}\left[S + \right.\right. \\ & \left.\left. + \frac{1}{2(2\pi\nu)}\left[\sin 2(2\pi\nu S + \delta) - \sin 2\delta\right]\right]\right\} \end{aligned} \tag{14}$$

$$\begin{aligned} y = {} & y'(0)\int_0^S \exp\left\{-\frac{A}{2}\left[\xi + \right.\right. \\ & \left.\left. + \frac{1}{2(2\pi\nu)}\left[\sin 2(2\pi\nu\xi + \delta) - \sin 2\delta\right]\right]\right\} d\xi + y(0) \end{aligned} \tag{15}$$

$$\begin{aligned} z' = {} & z'(0)\exp\left\{-\frac{A}{2}\left[S + \right.\right. \\ & \left.\left. + \frac{1}{2(2\pi\nu)}\sin 2(2\pi\nu S + \delta) - \sin 2\delta\right]\right\} + \\ & + \int_0^S f_z(\xi)\exp\left\{-\frac{A}{2}\left[S + \frac{1}{2(2\pi\nu)} \times \right.\right. \\ & \left.\left. \times \left[\sin 2(2\pi\nu S + \delta) - \sin 2(2\pi\nu\xi + \delta)\right]\right]\right\} d\xi \end{aligned} \tag{16}$$

$$\begin{aligned} z = {} & \int_0^S\left\{\int_0^\eta f_z(\xi)\exp\left\{-\frac{A}{2}\left[(\eta - \xi) + \frac{1}{2(2\pi\nu)} \times \right.\right.\right. \\ & \left.\left.\left. \times \left[\sin 2(2\pi\nu\eta + \delta) - \sin 2(2\pi\nu\xi + \delta)\right]\right]\right\} d\xi\right\} d\eta + \\ & + z'(0)\int_0^S \exp\left\{-\frac{A}{2}\left[\xi + \frac{1}{2(2\pi\nu)} \times \right.\right. \\ & \left.\left. \times \left[\sin 2(2\pi\nu\xi + \delta) - \sin 2\delta\right]\right]\right\} d\xi + z(0) \end{aligned} \tag{17}$$

$$\begin{aligned} x' = {} & \int_0^S f_x(\xi)\exp\left\{-\frac{A}{2}\left[(S - \xi) + \frac{1}{2(2\pi\nu)} \times \right.\right. \\ & \left.\left. \times \left[\sin 2(2\pi\nu S + \delta) - \sin 2(2\pi\nu\xi + \delta)\right]\right]\right\} d\xi + \\ & x'(0)\exp\left\{-\frac{A}{2}\left[S + \frac{1}{2(2\pi\nu)} \times \right.\right. \\ & \left.\left. \times \left[\sin 2(2\pi\nu S + \delta) - \sin 2\delta\right]\right]\right\} \end{aligned} \tag{18}$$

$$\begin{aligned} x = {} & \int_0^S\left\{\int_0^\eta f_x(\xi)\exp\left\{-\frac{A}{2}\left[(\eta - \xi) + \frac{1}{2(2\pi\nu)} \times \right.\right.\right. \\ & \left.\left.\left. \times \left[\sin 2(2\pi\nu\eta + \delta) - \sin 2(2\pi\nu\xi + \delta)\right]\right]\right\} d\xi\right\} d\eta + \\ & + x'(0)\int_0^S \exp\left\{-\frac{A}{2}\left[\xi + \frac{1}{2(2\pi\nu)} \times \right.\right. \\ & \left.\left. \times \left[\sin 2(2\pi\nu\xi + \delta) - \sin 2\delta\right]\right]\right\} d\xi + x(0), \end{aligned} \tag{19}$$

where $z' = dz/dS$, $x' = dx/dS$, $y' = dz/dS$,

$$A = \frac{2}{3}r_e^2\left(1 \mp \beta_x(0)\right)E_a^2 \Big/ \left\{cm_0\left(1 - \beta^2(0)\right)^{1/2}\right\},$$

$$f_z = \frac{e}{\mu_0} E_a \cos(2\pi \nu S + \delta) -$$
$$- \frac{2}{3} \frac{2\pi r_e e E_a \sin(2\pi \nu S + \delta)}{m_0 \lambda},$$

$$f_x = \pm \frac{e}{\mu_0 c} E_a \cos(2\pi \nu S + \delta) z' \pm$$
$$\pm \frac{2}{3} \frac{2\pi r_e e E_a \sin(2\pi \nu S + \delta) z'}{m_0 c \lambda} \pm$$
$$\pm \frac{2}{3} \frac{1}{\mu_0} r_e^2 E_a^2 \cos^2(2\pi \nu S + \delta).$$

The equation (19) should be used to calculate S with the given x. After calculation "S" should be substituted in the formulas (14-18) for calculations of values y', z', x', y, z. To calculate **r** and **v** as functions of time t it is possible to take the formula (10) for calculation of t, as a function of S.

By definition of $\beta(t)$ we have:

$$\beta^2 = c^{-2}(dS/dt)^2 \left[x'^2 + z'^2 + y'^2\right]. \quad (20)$$

Substituting the value of (dS/dt) from (10) into (20), we obtain:

$$\beta^2 = \frac{\left(c^{-2}B^2\left[z'^2 + z'^2 + y'^2\right]\right)}{\left(1 + c^{-2}B^2\left[x'^2 + z'^2 + y'^2\right]\right)}, \quad (21)$$

where $B = (1 \mp \beta_x(0)) / (1 - \beta^2(0))^{1/2}$.

Substituting the values of x', z', y' from the formulas (14), (16), (18), as functions of S into (21) and integrating Eq. (10), we obtain the relation between S and t.

$$t = B^{-1} \int_0^S \sqrt{1 + c^{-2}B^2\left(x'^2 + z'^2 + y'^2\right)}\, d\xi. \quad (22)$$

3. Using the formulas obtained, we can show that for certain entry conditions there is a periodic motion of a radiating electron in the field of plane light wave. The periodic motion of a radiating electron is possible due to the fact that under some conditions the electric field of light wave can make the work equal to the energy, which an electron expends for radiation. This result differs fundamentally from conclusions of [5], where the electron motion in the electromagnetic field of light wave is reduced to motion in an undulator with a static magnetic field. In the static magnetic field there cannot be periodic motion of an electron losing the energy for radiation. The conditions of periodic motion and the periodic trajectory can be approximately calculated neglecting the bremsstrahlung. For this purpose in the formulas (14-19) it is necessary to put $r_e \equiv 0$.

The solution of Eq. (1) when $\mathbf{F}_R = 0$ looks like:

$$y' = y'(0); y = y'(0)S + y(0); \quad (23)$$

$$z' = \frac{eE_a}{(2\pi \nu)\mu_0} \sin(2\pi \nu S + \delta) + C_1; \quad (24)$$

$$z = -\frac{eE_a}{(2\pi \nu)^2 \mu_0} \cos(2\pi \nu S + \delta) + C_1 S + C_2; \quad (25)$$

$$x' = -\frac{e^2 E_a^2}{(4\pi \nu)^2 \mu_0^2 c} \cos 2(2\pi \nu S + \delta) + \frac{eE_a C_1}{(2\pi \nu)\mu_0 c} \sin(2\pi \nu S + \delta) + a_1; \quad (26)$$

$$x = -\frac{e^2 E_a^2}{(4\pi \nu)^3 \mu_0^2 c} \sin 2(2\pi \nu S + \delta) - \frac{eE_a C_1}{(2\pi \nu)^2 \mu_0 c} \cos(2\pi \nu S + \delta) + a_1 S + a_2, \quad (27)$$

where $C_1 = z'(0) - \frac{eE_a}{\mu_0} \frac{1}{2\pi \nu} \sin\delta$;

$$C_2 = z(0) + \frac{eE_a}{\mu_0} \frac{1}{(2\pi \nu)^2} \cos\delta;$$

$$a_1 = x'(0) + \frac{e^2 E_a^2}{\mu_0^2 c (4\pi \nu)^2} \cos 2\delta - \frac{eE_a}{\mu_0 c (2\pi \nu)} \left[z'(0) - \frac{eE_a}{\mu_0 (2\pi \nu)} \sin\delta\right] \sin\delta;$$

$$a_2 = x(0) + \frac{e^2 E_a^2}{\mu_0^2 c (4\pi \nu)^3} \sin 2\delta + \frac{eE_a}{\mu_0 c (2\pi \nu)^2} \left[z'(0) - \frac{eE_a}{\mu_0 (2\pi \nu)} \sin\delta\right] \cos\delta.$$

From the given formulas it is seen, that the magnitudes z', x' are the periodic functions with the period $\Delta S = \nu^{-1}$. If to put

$$z'(0) = \frac{eE_a}{\mu_0 (2\pi \nu)} \sin\delta, \quad (28)$$

than z also will be the periodic function with the same period. Eq. (28) is one of conditions of a periodic solution. The second condition of periodicity follows from (23): $y'(0) = 0$.

From (27) follows that by changing S to ΔS the change of x will be equal to $\Delta x = a_1 \Delta S$. From (8) it follows that:

$$\Delta t = \Delta S[1 \pm a_1/c]. \quad (29)$$

Having expressed Δt from (22) and having equated this value to the value, given in (29), we shall obtain the third condition of periodicity connecting the parameters of the wave F_a, ν, δ and the initial conditions of the periodic trajectory $\beta_x(0)$ and $\beta(0)$

$$B^{-1}\int_{0}^{\nu^{-1}} sqrt(L\cos^2 2(2\pi\nu S+\delta)+ \\ + M\cos 2(2\pi\nu S+\delta)+N)d\xi = \\ = \nu^{-1}[1\pm a_1/c], \qquad (30)$$

$$L=2^{-4}c^{-4}B^2p^4;\; M=-\frac{1}{2}p^2c^{-2}B^2[a_1/c+1];$$

$$N=\left(a_1^2+\frac{p^2}{2}\right)c^{-2}B^2+1;\; p=\frac{eE_a}{2\pi\nu\,\mu_0}.$$

Under realization of periodicity conditions the period on x is determined by the formula

$$\Delta x=\left[x'(0)+2^{-2}c^{-1}p^2\cos 2\delta\right]\nu^{-1}. \qquad (31)$$

The magnitudes $x'(0), p, \delta$, included in (31), are calculated with the help of (28), (30).

From the given formulas it follows, that the decrease of a beam phase volume most greatly happens during motion of an electron with $\beta(0)\approx 1$ towards the wave front.

The formulas (14-22) enable to calculate trajectories of electrons and, thus, to investigate the process of beam phase volume decrease. Besides, as is known from [9], the knowledge of the electron trajectory enables one to calculate the spectral angular characteristics of radiation for periodic motion of an electron along the axes x. We guess, that the spectrum of Compton scattering thus can be calculated in the limits of classic electrodynamics.

4. The boundaries of applicability of the obtained formulas are determined by the boundaries of applicability of Eq. (1), established in [10]:

a) Restriction by the wavelength:

$$\lambda/\left(1-\beta^2\right)^{1/2} >> r_e. \qquad (32)$$

b) Restriction by the electric field strength:

$$e^3E_a/m_0^2c^4\left(1-\beta^2\right)^{1/2} << 1. \qquad (33)$$

In the ultra relativistic case the bremsstrahlung force can become more than the Lorentz force [10]. In this case the formulas of the given paper describing electron dynamics taking into account the bremsstrahlung forces are not suitable for analysis of problems of dynamics and radiation of an electron in the light wave field. It is connected with the fact that the integral of motion in Eq. (7), which was used for integration of Eq. (1), was obtained at equality of the bremsstrahlung force to zero. Because of that the formulas (12-19) are usable, if they satisfy the condition:

$$\frac{F_R}{F_L}\approx\frac{e^3F_a}{m_0^2c^4\left(1-\beta^2\right)}=\varepsilon << 1. \qquad (34)$$

Using Eq. (34) it is possible to be convinced, that, for example at $\varepsilon=10^{-2}$, $E_a=10^{12}$ V/m the energy of an electron can be of about 63 TeV. These estimates allow one to make the conclusion that the formulas obtained in the paper are usable in the range of parameters of laser and electron beams, achievable now and, apparently, in the rather long-term future.

In the summary it should be noted that entering into Eq. (1) the fluctuation forces with a quantum character of radiation, as well as forces conditioned by a space heterogeneity of the laser beam, will allow on this way to investigate a problem of «laser cooling» of electron beam.

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