RESTORING OF FAST NEUTRON ENERGETIC SPECTRA ON THE BASIS OF AMPLITUDE DISTRIBUTION OF SIGNALS MEASURED BY SINGLE CRYSTAL NEUTRON SPECTROMETER

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The technique of restoring fast neutron spectra on the basis of amplitude spectra registered by a scintillation detector has been developed. The typical instability of restoring, that is boundary ejection and periodic perturbations, has been found out. The quality criteria of restoring, such as values of empirical risk and conditional characteristics of a matrix in the set of linear equations have been proposed. The main reasons of solution instabilities and methods of their overcoming have been found.

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1. INTRODUCTION

A wide application of nuclear installations generating penetrating radiation in science, engineering and medicine requires a control on quantitative characteristics of such radiation. The integrated type means of measurement (dosimeters, radiometers etc.), in fact, are not sufficiently informative. The fullest information on neutron and gamma fields can be obtained with the help of a special equipment– spectrometers of appropriate radiation [1,2].

The authors have developed a model of a single crystal spectrometer of fast neutrons operating on the basis of a method of neutron registration considering "recoil protons" [3]. Such spectrometers are described in papers [1,2] and are characterized by simplicity of the experimental equipment, high counting efficiency, satisfactory energy resolution and universality of use. They consist of a scintillator (stilben, liquid or solid organic scintillator etc.), a photoelectronic multiplier and electronic blocks of selection, blocks for shaping and operating signals, registered from the detector.

The spectrometers operate satisfactorily in the neutron energy range 0,2-20 MeV. The initial neutron spectra are restored by the special mathematical processing of the measured amplitude distributions of signals. The restoring is carried out with the help of complicated and rather bulky software of the spectrometer [4].

At present both various approaches to solving the problem of restoring, and appropriate algorithms and programs for applying this or that method are described in [5-8]. The characteristic feature of restoring is a well-known problem of solution instability [5-7]. Its appears in deviation of a spectra restored from the initial one, moreover, the character of deviations can be in a wide range, according to a class of selected approximating functions, algorithms of restoring and experimental data obtained [5,6,8].

Therefore, to apply the published programs [5] for restoring the certain spectra of fast neutrons it was necessary to conduct a number of "numerical" experiments imitating experimental factors, which affect the solution, and to reveal an effect of parameters used in computing procedures.

The results allowed working out a technique of restoring fast neutrons on the basis of measured distributions. This technique permitted to optimize parameters of computing procedures, to develop criteria of optimum calculation variants and to conduct a priori evaluation of solution quality.

The main mathematical premises of an algorithm of restoring spectral dependences, the technique of making a stable solution and examples of typical instabilities obtained in the process of restoring smooth neutron spectra are indicated below.

2. MATHEMATICAL FUNDAMENTALS OF RESTORING ALGORITHM

The problem of determining a fast neutron spectrum on the basis of measured amplitude distributions is one of the mathematical problems on interpreting results of indirect experiments [5]. The solutions of such problems are based on a theory of restoring dependences selected from a limited volume of data. These dependences can be determined with the same mathematical scheme – minimization of average risk on empirical data [5,6].

In general the problem under consideration is described by the Fredholm integral equation of kind I [5]:

$$f(x) = \int_a^b R(x,t)g(t)dt\,, \tag{1}$$

where g(t) is the required dependence – neutron spectrum, R(x,t) is the core of integral equation, f(x) is the amplitude spectrum of signals registered by the detector. In this case, the function of two variables R(x,t) is formed of the functions of a detector response to monoenergetic neutrons.

The solution of Eq. (1) belongs to a class of reverse problems of incorrect type characterized by instability of a solution, which depends on both the components of

Eq. (1), and the algorithm of the solution with its program realization.

Two programs were used to restore neutron spectra. In these programs the approximation of spectral dependence g(t) was presented either as a linear combination of Chebyshev polynomials or as a normal system of cubic spline-functions (programs POLIL and SPLIL, respectively) [5].

The results of "numerical" experiments have shown, that the best approximation of the solution g(t) can be achieved with the help of the program SPLIL. Therefore, the further presentation of research results will be connected with this program.

In a spline-solution algorithm the section [a,b] is divided into (N+1) parts by equidistant points (a_1...,a_N) − knots of conjugation. The number N is determined in the process of algorithm fulfilment. On each section [a_j,a_{j+1}] (j=0...,N; a_0=a; a_{N+1}=b) the approximate solution is represented by a cubic polynomial $P_j(t)$. The coefficients of these polynomials are agreed in such a way that the solution of Eq. (1) was continuous and had the first and the second continuous derivatives along the whole section [a,b].

The solution (1) is found with a normal system of cubic splines:

$$g(t) = \sum_{j=1}^{N+4} \alpha_j^* P_j(t), \tag{2}$$

where α_j^* is the coefficients of Eq. (2), are determined from the condition of minimizing the empirical risk functional:

$$I_e(\alpha) = \frac{1}{l}\sum_{i=1}^{l}\left(y_i - \int_a^b R(x_i,t) \sum_{j=1}^{N+4} \alpha_j P_j(t)dt \right)^2 / \sigma_i^2, \tag{3}$$

where σ_i is the dispersion of measurements $y_i=f(x_i)+\varepsilon_i$, $M\varepsilon_i=0$, $D\varepsilon_i<\infty$, i=1,...,l.

The minimum value of the empirical risk functional (3) is achieved in the solution of a normal set of linear algebraic equations:

$$S^T S \;\alpha = S^T y \;, \tag{4}$$

where $\alpha=(\alpha_1,...,\alpha_{N+4})^T$ is the vector of cubic spline coefficients, $y=(y_1,...,y_l)$ is the vector of experimental values of dependence observed, S is the matrix of a size lx(N+4) of image values of fundamental splines in data points x_i (i=1,...,l), and $\alpha^*=(S^TS)^{-1}S^Ty$.

The value of empirical risk magnitude (3) determines the quality of the constructed approximation of solution of Eq. (1). The evaluation of approximation quality is determined by the expression:

$$\mathrm{crit}(N) = \frac{I_e(\alpha^*)}{1-\sqrt{\dfrac{(N+4)(\ln l - \ln(N+4)+1) - \ln\eta}{l}}}, \tag{5}$$

where $1-\eta$ is the probability, when the evaluation is true.

The value of N, when evaluation (5) of the spline quality is of the least value, is the optimum number of cubic spline conjugations; and the spline itself is taken for the approximate solution of the integral equation (1).

3. TECHNIQUE OF CALCULATINS AND RESULTS

The technique of restoring neutron spectra includes a number of special programs, developed by the authors. These programs imitate neutron spectra as Gaussian distributions, and a response function of the detector − as linear associations $F(E_e)$=aE+b, where the constats a<0 and b>0. $F(E_n)$=0, where E_n is the neutron energy (corresponds in (1) to the variable t), $E_e$is the amplitude of a signal from the detector in the scale of equivalent electron energies [3] (corresponds in (1) to the variable x).

For the a priori evaluation of restoring quality the following dependences are analyzed: a) crit(N) is the criterion of solution quality (1) defined by Eq. (5) and b) cond(N) is the degree of conditioning the matrix of linear equation system (4). To analyze the parameter of the cond(N) in the used program SPLIL, a subprogram for solution of the linear equation system GELS [5] was replaced by the subprograms DECOMP and SOLVE [9].

Besides, the upper El and lower Eh thresholds of cutting off the amplitude spectrum corresponding to the real conditions of measurements were introduced as an element of a restoring technique.

Practically, all the typical peculiarities of solutions are seen in the process of restoring three types of imitative neutron spectra (Gaussian distributions): a "broad" spectra with a dispersion σ1=3 MeV, an "average" − with σ2=1 MeV and a "narrow" − with σ 3=0.25 MeV. The positions of maximum of these Gaussian distributions (E_m=7 MeV) coincided, and the definition regions corresponded to the values E_m±4σ, respectively.

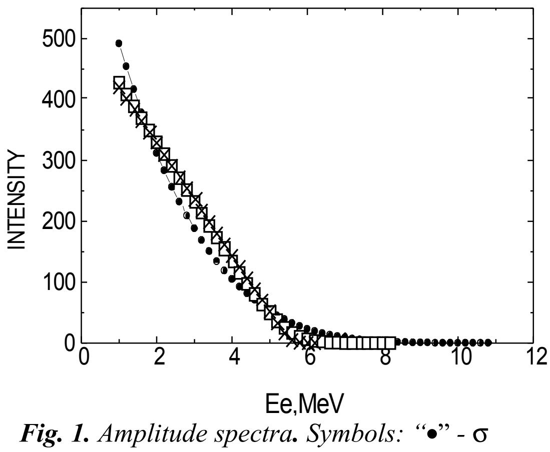

***Fig. 1.** Amplitude spectra. Symbols: "•" - σ1=3 MeV, " " - σ2=1 MeV, "+" - σ3=0.25 MeV.*

Fig. 1 shows the "registered" amplitude spectra, corresponding to the Gaussian distributions, which served as those for restoring initial neutron spectra. The results of such restoring are presented in Fig. 2 as symbols, and initial imitative neutron spectra − as continuous curves.

It is seen, that restoring the "average" spectra with σ 2=1 MeV (see Fig. 2) is the most qualitative; the region of its definition is exceeded completely with a shaping range (E_n=1-14 MeV) of the core R(x, t) in integral equation (1). In the case if the range of spectra definition is wider than the range of shaping R(x, t), there is a typical solution instability – boundary ejection (curve with σ1=3 MeV in Fig. 2).

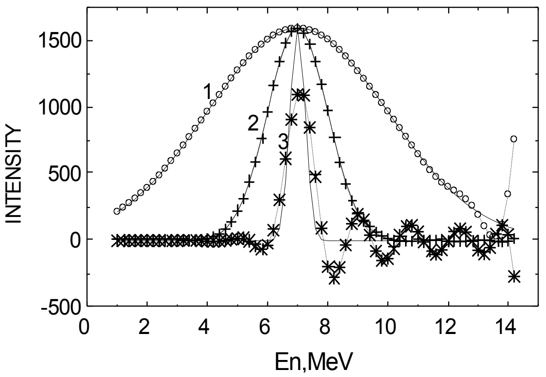

Fig. 2. Neutron spectra. Symbols: "o" - σ1=3 MeV, "+" - σ2=1 MeV, "" - σ3=0.25 MeV.*

Another limiting case takes place in the process of restoring the "narrow" spectra (curve with σ 3=0,25 MeV in Fig. 2); besides a noticeable deviation of the dependence restored from the initial one, there is a typical instability of the solution – periodic (wave–type) perturbation outside the region of initial spectra definition, and a boundary ejection too.

The degree of restored spectrum adequacy for three cases under consideration, on the whole, correlates with a behaviour of parameters crit(N) and cond(N) the analysis of which can give a priori qualitative information on the solution stability.

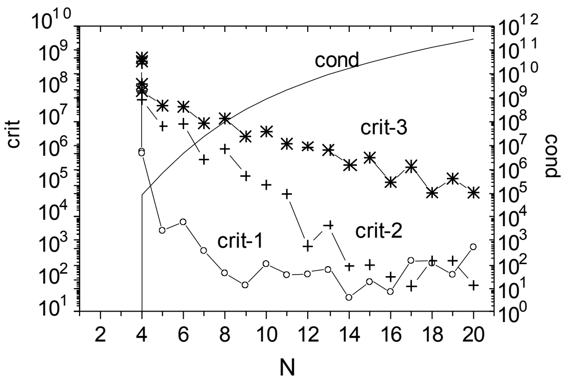

Fig. 3. Parameters of restoring the neutron spectra. Symbols: "o" - σ1=3 MeV, "+" - σ2=1 MeV, "" - σ 3=0.25 MeV*

Fig. 3 presents the dependences crit(N) and cond(N) expressing the process of determining the solutions of Eq. (1) shown by the appropriate curves in Fig. 2. The approximation to a stable solution is accompanied by the fast changing in the parameter crit(N) before reaching a minimum and by rather smooth changing in the field of the minimum (curves crit–1 and crit–2 in Fig. 3). The unstable solution is characterized by the slow change of crit(N), and its minimum is usually on the boundary of changing matrix grade (n_{max}) of the set of linear Eq. (4) (curve crit–3 in Fig. 3). In this case n_{max}=20.

The research of solution instabilities (1) show that the main factor of their origin when restoring the "broad" neutron spectra (σ1=3 MeV), is the disagreement of a definition range of the "measured" peak spectra and the sub-integral core R(x,t). The availability of such a difference can lead to full loss of adequacy of the solution. The result of such restoring is represented in Fig. 4, when the definition range of the core R(x,t) by the variable "x" is 1…14 MeV, and that of the "measured" amplitude spectra is 3…14 MeV.

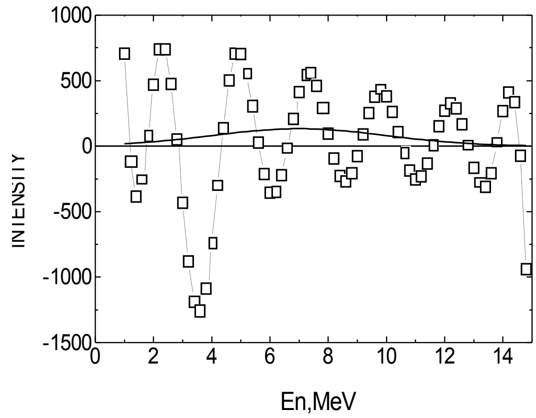

Fig. 4. Inadequate restoring of neutron spectra with σ1=3 MeV.

For the "narrow" spectra (curve with σ3=0,25 MeV in Fig. 3) it is possible to reach the suppression of periodic instability to the right of the peak by introduction of the upper threshold Eh, the value of which is determined by the maximum value of the registered signal amplitude (see Fig. 1). In this case, the quality of restoring (the approximation to the initial spectra) is improved, and perturbations to the left of the peak decrease (see Fig. 5).

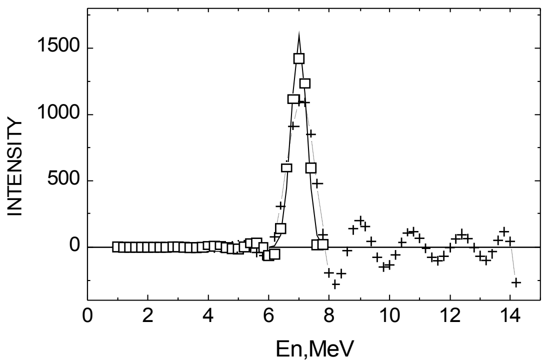

Fig. 5. Results of restoring the neutron spectra with σ3 = 025 MeV. Symbols: "+" – Eh "switch off" (corresponds to curve 3 in Fig. 2); " " - Eh=8 MeV.

Simultaneous introduction of upper and lower thresholds can lead to unexpected effects. The results of restoring the "narrow" spectra σ3=0,25 MeV for two values of upper and lower thresholds are given in Fig. 5. In the process of restoring the neutron spectra with $El=E_m-4\sigma$=6 MeV and $Eh=E_m+4\sigma$=8 MeV (Fig. 5) the obtained solution does not correspond to the spectra form we are looking for (continuous curve in Fig. 6), though the position of a peak maximum is given correctly.

The increase in the width of the Eh-El window results in a satisfactory approximation of the solution to the initial dependence, however, outside the region of dependence definition there are periodic and boundary instabilities (see Fig. 6).

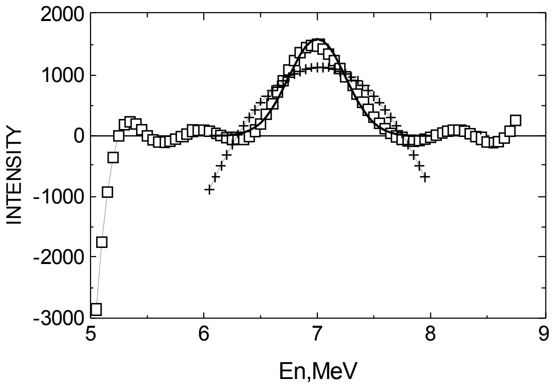

Fig. 6*. Results of restoring the neutron spectra with σ3 = 0.25 MeV. Symbols: " " - El=5 MeV, Eh=9 MeV; "+" - El=6 MeV, Eh=8 MeV.*

While restoring the real spectra of a complicated form the satisfactory solution within the framework of the above mentioned technique can be achieved in case if the spectrum has not any narrow peaks. In Fig. 7, one can see an illustration of the results of restoring then imitative spectrum of the Pu-Be-source simulated by a set of Gaussian distributions. The defined solution (symbols in Fig. 7), in main, corresponds to the initial spectrum (continuous curve). However, small periodical perturbations can arise in the right part of the restored spectrum. The methods of avoiding these perturbations are described above.

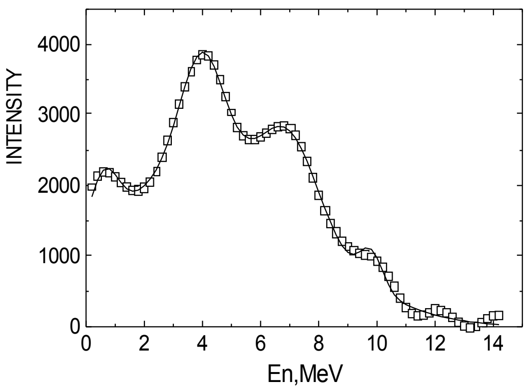

Fig. *7. Neutron spectra of the Pu-Be-source.*

CONCLUSIONS

The studies on quality of restoring the neutron spectra from amplitude distributions based on response functions of detecting neutron equipment have been done within the framework of the most general mathematical approach to the numerical solution of the I kind Fredholm equation.

A number of special programs and approaches has been developed. For the first time they served as a base for simulation of calculations that permitted to find numerical characteristics of computing parameters, which depend on both the form of energetic neutron spectra, and response functions. This circumstance has allowed developing a criterion for determining the stable solutions of the integral equation describing the real experimental data.

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