FORMATION OF THE CURRENT ATTACHMENTS IN PLASMA ACCELERATOR CHANNEL UNDER INFLUENCE OF THE LONGITUDINAL MAGNETIC FIELD

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Results of numerical simulations of axisymmetric plasma flows in accelerator channel with longitudinal magnetic field are presented. The investigations of two-dimensional flows are carried out within the framework of one-component MHD-model and two-component model taking into account the Hall effect. It is found that the current attachments are appeared in case of respective strong longitudinal magnetic field.

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1. INTRODUCTION

In accelerators, the plasma dynamics is investigated in various current transport regimes [1-2]. In the electron current transport regime, the lines of the ion plasma current lie on the surfaces of the impermeable electrodes (cathode and anode) and the electrode surfaces are nonequipotential. In contrast, in the ion current transport regime, the electrode surfaces are equipotential and should be permeable to the plasma. Exclusively the Hall effect governs the self-consistent plasma flow through the electrodes. Ignoring the Hall effect ($\mathbf{V}_e = \mathbf{V}_i$), we arrive at the regime of impermeable continuous equipotential electrodes. Under real conditions ($\mathbf{V}_e \neq \mathbf{V}_i$), this regime can be regarded as degenerate. Most experiments [3-5] and models [6-7] are based on ion current transport.

In one-fluid model, Hall effect is ignored and problem does not depend on the electrode polarity. This model provides a qualitative description of the above regimes.

The paper is aimed at discussing the qualitative features of rotating plasma flows in a longitudinal magnetic field in accelerator channels. Applying a longitudinal field $H_z >> H_r$ in addition to the traditional azimuthal magnetic field H_φ opens up new possibilities for controlling the dynamic processes.

The hierarchism of numerical models implies that, in the first step, the plasma dynamics should be investigated in a one-fluid MHD-model with allowance for the finite conductivity of the medium. Such a model is developed in [8]. Some aspects of the formulation of the problem and the results of first numerical experiments within the framework of two-component MHD-model were reported in [9]. The corresponding transport coefficients were obtained in explicit form in [10]. The numerical experiments were tested against the two-dimensional plasma flows considered earlier in analytical model [11].

2. ONE-COMPONENT MHD-MODEL

We will consider a two-dimensional axial-symmetrical plasma flow when the two electrodes profiles reproduced in Fig. 2 specify channel geometry. In the presence of a longitudinal field and the arising rotation whole three components of field and velocity participate in model. To be specific, we will investigate a plasma formed from atomic hydrogen when the inertia of electrons ($m_e << m_i = m$). The medium is assumed to be quasineutral $n_i = n_e = n$. Within framework of the one-component approximation ($\mathbf{V}_e = \mathbf{V}_i = \mathbf{V}$) the construction of the model is based on the traditional MHD-equations taking into account of conductivity: We have the following equations in the dimensionless form

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{V}) = 0 \; ; \qquad \rho \frac{d \mathbf{V}}{d t} + \nabla P = \mathbf{j} \times \mathbf{H} \qquad (1)$$

$$\rho \frac{d\varepsilon}{d t} + P \, \nabla \bullet \mathbf{V} = \nu \, \mathbf{j}^2 \; ; \qquad \frac{\partial \mathbf{H}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{H}) - \nabla \times (\nu \, \mathbf{j})$$

Here, $P = P_i + P_e = \beta \, \rho \, T$ is the total pressure; $\varepsilon = \beta \, T / (\gamma - 1)$ is the intrinsic energy per unit mass; $\mathbf{j} = \nabla \times \mathbf{H}$ is the electric current. We will restrict our attention to the case of a single-temperature mixture $T_i \approx T_e = T$. In accordance with Ohm’s law, the electrical field is given by the relation

$$\mathbf{E} = \nu \, \mathbf{j} - \mathbf{V} \times \mathbf{H} \qquad (2)$$

The initial quantities are related to the dimensionless parameters of the problem as follows: $\beta = 8\pi \, P_0 / H_0^2$ is the ratio between the gas-kinetic and magnetic pressure at entrance; $\nu = 1/\mathrm{Re}_m = c^2 / 4\pi \, L \, V_0 \, \sigma$ is magnetic viscosity, which is inversely proportional to Reynolds number with Spitzer conductivity $\mathrm{Re}_m = \sigma_0 \, T^{3/2}$. $H_0 = 2 J_p / c R_o$, J_p - discharge current, $P_0 = 2 \, k \, n_0 \, T_0$, $V_0 = H_0 / \sqrt{4\pi \, \rho_0}$ - characteristic Alfven velocity.

The equations and boundary conditions define flow dynamics. The conditions at the channel inlet $(z = 0)$ correspond to subsonic plasma inflow with $\rho(r) = f_1(r)$, $T(r) = f_2(r)$, where f_1 and f_2 are known functions. We will assume that the total electric current flowing in the system is maintained constant. This generates the boundary condition at the inlet $r \, H_\varphi = r_0 = const$. The inflow is carried out along the coordinate lines, for example. Plasma at entrance is no rotating $V_\varphi = 0$.

We specify longitudinal field at entrance $H_z \neq 0$. For any β value in the radial-equilibrium case, traditional conditions $\rho = 1$, $T = 1$ yield $H_z = H_z^0 = const$ at $z = 0$.

For $\beta << 1$, the plasma can be injected in an arbitrary manner, for instance, in accordance with the analytic model of [13], in which $\rho(z = 0, r) = r_0^2 / r^2$. In this case, it is possible to compare results with the analytic solution.

At the outlet $z = 1$ the boundary conditions correspond to a supersonic plasma flow in transonic case.

We suppose that the electrodes with given profiles are equipotential $E_\tau = 0$ and non-penetrable $V_n = 0$ ones.

It is necessary additional relation in the presence of longitudinal magnetic field. Equality $H_n = 0$ is typical and natural condition in plasmadynamics. This condition leads to conservation law of magnetic flux along channel.

3. TWO-COMPONENT MHD-MODEL

The two-component model and computation of axial-symmetrical plasma flow in the presence of a longitudinal magnetic field are based on the MHD-equations taking into account the Hall effect ($\mathbf{V}_e \neq \mathbf{V}_i$), electrical conductivity tensor and transport coefficients in magnetic field depending on the $x = \omega_e \tau_e$ [10]. Ohm's law and the electrical field can be represented in the form

$$\mathbf{E} = A_1 \nu\, \mathbf{j} - [\,\mathbf{V}\,,\ \mathbf{H}\,] + \mathbf{R}_1 + \mathbf{R}_2 \qquad (3)$$

A_1 are known functions of $x = \omega_e \tau_e$.

Dimensionless parameters of model: β, ν, $\omega_e \tau_e$ and $\xi = c\ m_i / e L \sqrt{4\pi\ \rho_0}$ is the local parameter characterizing the Hall effect in the two-fluid model.

In [9] two-component model of two-dimensional flow corresponds to the ion current transport regime. In this case on anode equipotential ($E_\tau = 0$) surface the density $\rho(z)$ and azimuthal velocity $V_\varphi(z)$ were chosen in accordance with analytical model to compare the numerical and analytical solutions. However, this formulation of problem did not give possibility to investigate the plasma dynamics for different parameters, including the longitudinal magnetic field.

The present investigation is aimed at discussing the numerical model in the ion current transport regime with the self-consistent plasma flow through the electrodes. In the absence of longitudinal magnetic field the model with the self consistent plasma flow was used earlier [6] to investigate the processes with the traditional azimuthal component of field H_φ. In the presence of the longitudinal magnetic field $(H_\varphi << H_z << H_r)$ in the case of the self-consistent plasma flow it is assumed that on the equipotential ($E_\tau = 0, E_\varphi = 0$) electrodes the jumps and breaks of thermodynamic values are absent.

4. MAIN RESULTS

In the present computations the initial dimensional parameters correspond to the experiments within the framework of QSPA program. For example, if $n_0 = 3.6 \cdot 10^{20} m^{-3}$, $T_0 = 2.3 \cdot 10^4 K$, $J_p = 300\, kA$, $L = 0.6\, m$, dimensionless parameters are $\beta = 0.005$, $\sigma_0 = 812.8$. In addition, as the longitudinal magnetic field value we will take, for example, $H_z^0 = 0.1$. This sufficient small value H_z^0 makes it possible to produce the transonic flow in accordance with the analytical model. The channel geometry (Fig. 2) and density at inlet correspond also the analytical investigation.

The steady-state flows are calculated by the relaxation method for the initial time-dependent equations. As result of calculations, at a mid-channel the flow velocity passes through the local velocity of the fast magnetosonic wave. We can observe the peculiarities of the vector magnetic field distribution. The value H_z increases as a function of r. It has maxim at the surface of the external electrode in the neighborhood of the narrowest section of the accelerator channel. The azimuthal velocity $V_\varphi(z, r)$ increases along the radial and axis directions. In other words, a small longitudinal magnetic field leads to the increasing plasma rotation, which have maxim in the neighborhood of the external electrode closer to the channel exit. Nevertheless, at the outlet the kinetic energy part $K_\varepsilon = \frac{\varepsilon_\varphi}{\varepsilon_z} 100\,\%$ associating with the rotation is less the kinetic energy of flow along the axis. This value is equal to $K_\varepsilon = 7\,\%$ in calculation in one-component model and $K_\varepsilon = 12\,\%$ in two-component model.

The longitudinal magnetic field determines behavior of density $\rho(z, r)$ in neighborhood of the external electrode. In the presence of H_z the level lines of the function $r\, H_\varphi$ (plasma current) change the inclination in the neighborhood of the external electrode. Simultaneously, at this place the density increases due to plasma rotation. In Fig. 1 we have illustrated this effect. Here, we can see the density distribution along the external electrode $\rho(z, r = r_0)$ for different values H_z^0. Continuous curves 1 and 2 correspond to calculations within the framework of one-component MHD-model in cases $H_z^0 = 0$ and $H_z^0 = 0.1$ respectively. Thus, the density enlarges near the external electrode. Due to this circumstance there is possibility to overcome or weaken the current crisis in plasma accelerator channel.

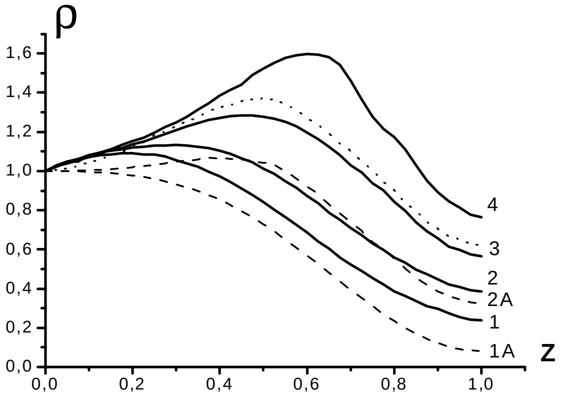

Fig. 1. Density distribution along external electrode: lines 1 (1A) – calculation (theory) in case $H_z^0 = 0$; lines 2 (2A) – calculation (theory) in case $H_z^0 = 0.1$; line 3 – calculation at $H_z^0 = 0.15$; line 4 - $H_z^0 = 0.2$

Moreover, we can compare the calculation of steady-state plasma flow with the analytical solution (dotted lines in Fig.1). These solutions may be different because the analytical model was constructed in the smooth channel approximation for ideal magnetohydrodynamic equations of cold plasma ($\nu = 0$, $\beta = 0$). We have not observed the principal qualitative distinctions of two solutions.

Further investigations based on the full MHD-model made it possible to reveal the following regular features. The increase of the longitudinal magnetic field leads to the growth of the density in the neighborhood of the external electrode. The curves 3 and 4 in Fig. 1 were calculated for $H_z^0 = 0.15$ and $H_z^0 = 0.2$, respectively. Starting with the some critical value of longitudinal field, we have observed the qualitative changing of flow. The value of critical field is a function of initial parameters. In case of $H_z^0 > 0.2$ the current layer is formed in the moving plasma in the neighborhood of the external electrode. In Fig. 2 we have plotted the current layer in case of $H_z^0 = 0.25$. The dotted line in this figure corresponds to the value $H_\varphi = 0$. Such closing of plasma current in plasma on the external electrode points to the possible formation of current attachments in case of sufficient strong longitudinal field H_z^0.

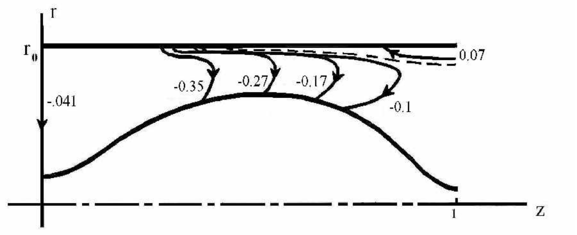

Fig.2. Plasma current (level lines rH_φ) at $H_z^0 = 0.25$

The numerical experiments carried out within the framework of the two-component model confirmed the main peculiarities of plasmadynamic processes, found before by means of the one-component model. Dash dotted line in fig.1 corresponds to the two-component model in case $H_z^0 = 0.15$. Also we observe excellent accordance the two-component model with analytical model. In case of two-component model it is possible to detect that azimuthal velocity have greater values in contrast one-component model. Normal component of plasma velocity on anode is very small value during the inflow through the external electrode. The increase of the longitudinal magnetic field also leads to the formation of current attachments as well as current layers in moving plasma. In case of two-component model we observe the enlargement of angle between electrode and current layer.

5. CONCLUSIONS

We observed that the weak longitudinal field can generate transonic flows on different conditions at the inlet. In this case at channel outlet the rotation energy part is much less kinetic energy of plasma flow along the axis.

A longitudinal magnetic field having effect along the channel leads to the rotational plasma motion, gradually intensifying it. As a result the density increases near the external electrode. This circumstance makes it possible to weaken the current crisis phenomenon in plasma accelerator channel. At the same time, the formation of the current layers and attachments to the external electrode is observed in the moving plasma in case of respective strong longitudinal magnetic field.

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ФОРМИРОВАНИЕ ПРИВЯЗОК ТОКА В КАНАЛЕ ПЛАЗМЕННОГО УСКОРИТЕЛЯ ПОД ВЛИЯНИЕМ ПРОДОЛЬНОГО МАГНИТНОГО ПОЛЯ

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Представлены результаты численного моделирования осесимметричных течений плазмы в канале ускорителя при наличии продольного магнитного поля. Исследования двумерных течений проведены на основе одножидкостной МГД-модели и двухжидкостной модели с учетом эффекта Холла. Наблюдается возникновение привязок тока к электродам при сравнительно больших значениях продольного магнитного поля.

ФОРМУВАННЯ ПРИВ'ЯЗОК ТОКУ В КАНАЛІ ПЛАЗМОВОГО ПРИСКОРЮВАЧА ПІД ВПЛИВОМ ПОЗДОВЖНЬОГО МАГНІТНОГО ПОЛЯ

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Представлено результати чисельного моделювання вісесиметричних течій плазми в каналі прискорювача при наявності поздовжнього магнітного поля. Дослідження двовимірних течій проведені на основі однорідинної МГД-моделі і дворідинної моделі з урахуванням ефекту Холла. Спостерігається виникнення прив'язок струму до електродів при порівняно великих значеннях поздовжнього магнітного поля.