

ELECTROMAGNETIC FIELD EXCITATION BY AN ELECTRON BUNCHES IN A PARTIALLY FILLED CYLINDRICAL DIELECTRIC RESONATOR

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A nonlinear self-consistent theory of wakefield excitation by an electron bunches in cylindrical resonator with longitudinally nonuniform filling by dielectric is constructed. A bunch-excited field are presented in the form of superposition solenoidal and potential fields. The formulated nonlinear theory allows carrying out numerical analysis of resonator excitation by electron bunches for the best interpretation of the results received in experiment.

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INTRODUCTION

Charged particles acceleration by wake field, excited by an electron bunches in a dielectric waveguide structures, is one of the advanced acceleration methods, which is currently investigated in many research centers. Intensive development of this research are due to current progress in the development of artificial dielectrics (with a greater level of amplitude of electric field breakdown, in comparison with metallic accelerating structures), and an advance in pulsed high current electronics.

Recently renewed interest (due to using in the wakefield acceleration methods) to electrodynamics structures that are partially filled with a dielectric in the longitudinal direction [1]. In the previous paper [2] was examined the radiation, arisen in a longitudinally infinite waveguide during the charged particle transition of a boundary between two media, and was obtained the formula for the total energy of the radiation and its spectral distribution. Under the experimental conditions waveguide structure used is limited longitudinally in the direction of bunch propagation, so during the charged particles bunches passage through it there are effects associated with the transition radiation, with the formation and dynamics of the group front of excited electromagnetic field [3]. The simplest formulation to account for these effects is resonator statement of the problem.

The report presents the results of the development of self-consistent theory of electromagnetic field excitation by electron bunches in a dielectric resonator. The theory takes into account both multifrequency character of an excited electromagnetic field, and its feedback on the dynamics of particles. The results of numerical simulation of resonator excitation by the train of electron bunches are presented. Longitudinal profiles of an excited axial electric field and dynamics of the particles are investigated.

STATEMENT OF THE PROBLEM. EQUATIONS FOR EXCITED FIELD

Let's consider a cylindrical metallic resonator with inner radius b , partially filled with isotropic material with dielectric constant ε . We suppose that the end

walls of the resonator are closed by metal grids, transparent for charged particles and nontransparent for an excited electromagnetic field. The resonator geometry under investigation is shown on Fig. 1.

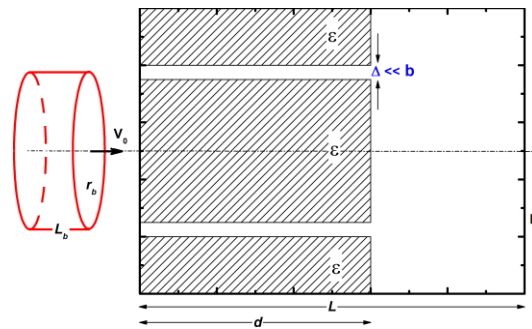


Fig. 1. General view of the dielectric structure, excited by a sequence of electron bunches. The electron bunches (red color) with an initial velocity v_0 are injected along an axis of the metal cylindrical resonator of radius b and length L with a dielectric insertion of length d . Bunches are modelled by hollow cylinders and extend from left to right. Thickness of the vacuum channel Δ is small in comparison with resonator radius

An electron bunches will be described in terms of macroparticles, therefore the charge and current density will be written as:

$$\rho = \sum_{p \in V_R} q_p \delta(\mathbf{r} - \mathbf{r}_p(t)), \quad \mathbf{j} = \sum_{p \in V_R} q_p \mathbf{v}_p(t) \delta(\mathbf{r} - \mathbf{r}_p(t)), \quad (1)$$

where q_p is a charge of a macroparticle, \mathbf{r}_p and \mathbf{v}_p are its time-dependent coordinates and velocity. Summation in the (1) is carried out over the particles, which are being in the volume of the resonator V_R .

We introduce the solenoidal \mathbf{E}^t , \mathbf{H}^t and the potential \mathbf{E}^l electric and magnetic fields satisfied to equations

$$\text{div} \varepsilon \mathbf{E}^t = 0, \text{div} \mu \mathbf{H}^t = 0, \text{rot} \mathbf{E}^t = 0,$$

and which are defined from the Maxwell's and the Poisson equations:

$$\text{rot} \mathbf{H}^t = \frac{\varepsilon(z)}{c} \frac{\partial \mathbf{E}^t}{\partial t} + \frac{4\pi}{c} \mathbf{j}, \quad \text{rot} \mathbf{E}^t = -\frac{\mu(z)}{c} \frac{\partial \mathbf{H}^t}{\partial t}, \quad (2)$$

$$\text{div} \varepsilon \mathbf{E}^l = 4\pi \rho.$$

The solenoidal \mathbf{E}^t and potential $\mathbf{E}^l = -\nabla\Phi$ electric fields are mutually orthogonal and satisfy the boundary conditions, making their tangential components vanish on the metal walls of the resonator. Poisson equation should be complemented by boundary conditions:

$$\begin{aligned} \Phi(z=0) &= \Phi(z=L) = \Phi(r=b) = 0, \\ \Phi(z=d-0) &= \Phi(z=d+0), \quad \varepsilon \frac{\partial\Phi}{\partial z} \Big|_{z=d-0} = \frac{\partial\Phi}{\partial z} \Big|_{z=d+0}. \end{aligned} \quad (3)$$

By using the expansion by eigen functions method, the Poisson equation with boundary conditions (3) can be solved. Finally, we obtain the potential in the form:

$$\Phi = \sum_{n,s} \frac{4J_0(\kappa_n r)Z_s(z)J_0(\kappa_n r_b)}{(\mu_s^2 + \kappa_n^2)b^2 J_1^2(\kappa_n b) \|Z_s\|^2} \sum_{p \in V_R} q_p Z_s(z_p). \quad (4)$$

In the above (and below in the paper) symbols n and s enumerate, respectively, radial, and axial indexes; J_0 and J_1 are, respectively, Bessel functions of zero and first order; $\kappa_n = \lambda_n^{(0)}/b$ are the radial eigenvalues, where $\lambda_n^{(0)}$ are zeros of the function J_0 . $Z_s(z)$ are the axial eigenfunctions, and $\|Z_s\|^2$ are their norms:

$$\begin{aligned} Z_s(z) &= \sin \mu_s z, \quad (0 \leq z < d) \\ Z_s(z) &= \varepsilon \frac{\sin \mu_s (z-L) \cos \mu_s d}{\cos \mu_s (L-d)}, \quad (d \leq z \leq L) \end{aligned} \quad (5)$$

$$\|Z_s\|^2 = \int_0^L dz \varepsilon Z_s^2(z).$$

Axial eigenvalues μ_s satisfies the equation

$$tg \mu d + \varepsilon tg \mu (L-d) = 0. \quad (6)$$

The solenoidal part of the electromagnetic field can be determined by expanding the required fields into solenoidal fields of the empty dielectric resonator [4]. Let's write down the fields \mathbf{E}^t and \mathbf{H}^t in the form:

$$\mathbf{E}^t = \sum_s A_s(t) \mathbf{E}_s(\mathbf{r}), \quad \mathbf{H}^t = -i \sum_s B_s(t) \mathbf{H}_s(\mathbf{r}). \quad (7)$$

The functions \mathbf{E}_s and \mathbf{H}_s , which describe the spatial structure of solenoidal fields, satisfy the Maxwell sources-free equations. By using the orthonormality conditions of eigen modes

$$\int_{V_R} dV \varepsilon \mathbf{E}_s \mathbf{E}_{s'}^* = \int_{V_R} dV \mu \mathbf{H}_s \mathbf{H}_{s'}^* = 4\pi N_s \delta_{ss'}, \quad (8)$$

one can obtain the differential equations for calculation the expansion coefficients $A_s(t)$ and $B_s(t)$

$$\frac{dA_s}{dt} = -\omega_s B_s - \frac{1}{N_s} \int_{V_R} dV \mathbf{j} \mathbf{E}_s^*(\mathbf{r}), \quad \frac{dB_s}{dt} = \omega_s A_s. \quad (9)$$

Eigenfrequencies ω_s are determined from the equation

$$k_1 tg k_1 d + \varepsilon k_2 tg k_2 (L-d) = 0, \quad (10)$$

$k_1^2 = \varepsilon \omega_s^2 / c^2 - (\lambda_n^{(0)}/b)^2$, $k_1^2 = \omega_s^2 / c^2 - (\lambda_n^{(0)}/b)^2$ are the axial wave numbers, respectively, in dielectric and vacuum. The self-consistent dynamics of bunch particles is described by relativistic equations of motion in the electromagnetic fields excited by bunches:

$$\frac{d\mathbf{p}_p}{dt} = q_p \left(\mathbf{E} + \frac{1}{m_p c \gamma_p} \mathbf{p}_p \times \mathbf{B} \right), \quad \frac{d\mathbf{r}_p}{dt} = \frac{\mathbf{p}_p}{m_p \gamma_p}, \quad (11)$$

where $\gamma_p^2 = 1 + (\mathbf{p}_p / m_p c)^2$.

NUMERICAL ANALYSIS

The obtained analytical expressions for components of the excited electromagnetic field together with motion equations of particles of bunches allow to carry out the numerical analysis of excitation of the resonator, taking into account the reverse influence of a field on the exciting bunches. For numerical calculations we used the following parameters of structure and a bunch: resonator radius $b = 4.25$ cm, it's length $L = 50.036$ cm, length of a dielectric insertion $d = 30.318$ cm, insertion it is made of teflon with dielectric permittivity $\varepsilon = 2.045$, the initial electrons velocity of a bunch is equal to $\beta_0 = 0.89$, a bunch charge are 0.32 nC, bunch radius $r_b = 5$ mm, bunch rms-length $\sigma_z = 8.7$ mm, full length of a bunch $L_b = 4\sigma_z$, bunches repetition rate $f_0 = 2.805$ GHz.

The choice of parameters of the considered resonator isn't arbitrary. Length of the resonator was defined as follows. Necessary condition of resonance excitation is coincidence of one of eigen frequencies of the resonator with a repetition frequency of bunches. I.e. in the equation (10) one of possible eigen frequencies is already set. Fixing the length of a dielectric insertion, from the dispersion equation (10) we find values of length of the resonator. Special case is the choice of the resonator at which length of dielectric area is multiple to wavelength of any resonance eigenmode. In the resonator used for the current numerical analysis the length of dielectric area was equal to three wavelengths in dielectric of E_{01} mode of oscillations resonant with bunches. The length of vacuum part was selected equal to half wavelength of E_{01} mode of oscillations of the vacuum resonator with the same frequency f_0 . In case of such choice of lengths of vacuum and dielectric areas the radial component of an electric field of a resonant mode of oscillations equals to zero at $z=0$, $z=d$ and $z=L$, i.e. for this mode dielectric and vacuum parts are being uncoupled resonators (for remaining eigenmodes it's not so). The velocity of bunches, injected into the resonator, is set for the following reasons. The phase velocity of eigenmode of oscillations in the dielectric which frequency is equal to the repetition frequency of bunches, shall equal the initial velocity of electrons of a bunch. I.e. the main mechanism of excitation of the resonator is Cherenkov radiation.

Fig. 2 shows a frequency spectrum of longitudinal electric field excited by a single electron bunch, and by train of bunches. The spectrum is calculated from field realization beginning from the time when the last bunch

left the resonator. The length of this sampling was equal to ten times of the free flight of a bunch through the resonator.

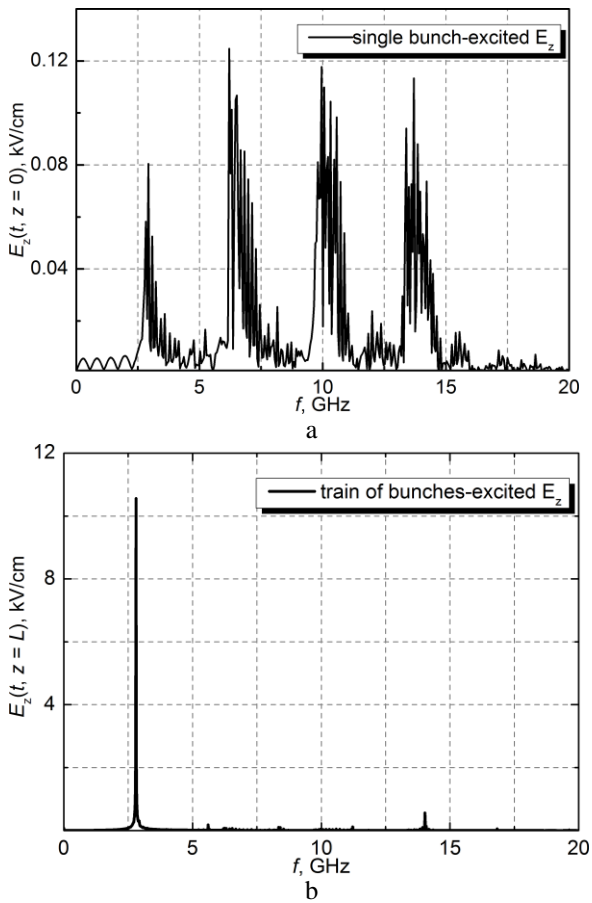


Fig. 2. The spectrum of a longitudinal electric field. The electric probe is located on an axis of the structure at the end of the resonator. The resonator was excited by a single bunch (a), and by a train of bunches (b)

In Fig. 3 the axial profiles of longitudinal electric field excited in the resonator and also the phase planes "energy-longitudinal coordinate" of bunch particles for the time when the last bunch of sequence of 300 bunches completely entered the resonator are provided.

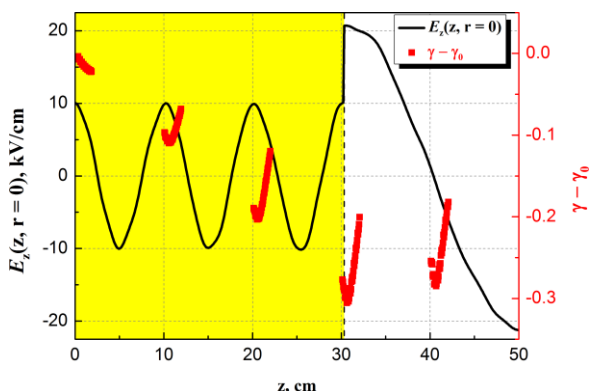


Fig. 3. Axial distribution of longitudinal electric field after injection of the 300th bunch, and also the phase planes of particles of bunches. Bunches moves from left to right, the plane of injection corresponds to $z = 0$

The vacuum part of the resonator in the conducted experimental studies is a diagnostic part – in it the output of electron bunches is carried out (after passing

through dielectric part), and also in it diagnostic electric probes are located. From distribution of the excited electric field depicted in Fig. 3 it is visible that adding of vacuum part, with length equal to the multiple half vacuum wavelength of eigenmode of oscillations (a particular case a mode, resonant with bunches), to dielectric part, with a length, the multiple wavelength of an oscillation mode in dielectric (a particular case a mode, resonant with bunches), doesn't make change to excitation of the resonator at a frequency equal to repetition frequency of bunches. As a result – existence of such vacuum part isn't a negative factor for the resonator concept of wakefield accelerating section, and also allows realizing measurements of parameters of the excited electromagnetic field without diagnostics directly in dielectric. Introduction of probes into a dielectric insertion can lead to unwanted change of values of eigen frequencies of the resonator, and also topography of components of the excited electromagnetic field. In process of increase in number of the injected bunches the field amplitude in the resonator increases, and in the longitudinal distribution becomes more monotonic with the characteristic spatial period of a resonant mode of oscillations, both in dielectric part of the resonator, and in vacuum (see Fig. 3).

In Fig. 4 dynamics of change of energy of particles of bunches in the resonator for four sequential times, since the moment of full entrance 300 (last) bunches of the regular sequence is provided.

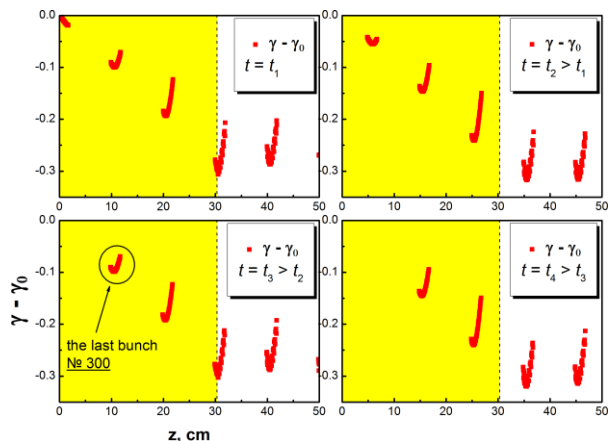


Fig. 4. Time dependence of changing of energy of bunch particles. The time $t = t_1$ corresponds to entrance of the last bunch of sequence. Injection of bunches wasn't carried out at the next time

The excited electromagnetic field changes in time so that all bunches passing through dielectric part of the resonator remain in the braking field phases, giving energy to the field. At that an energy of bunches at the output of vacuum part of the resonator practically doesn't differ from their energy at an input in it (see Fig. 4).

The important task of development and implementation of accelerating structures is the accuracy of maintenance of a resonance between resonant eigenfrequency of the resonator and repetition frequency of bunches. Therefore the dependence of amplitude of a wakefield on the accuracy of maintenance of a resonance was studied by us numerically (Fig. 5).

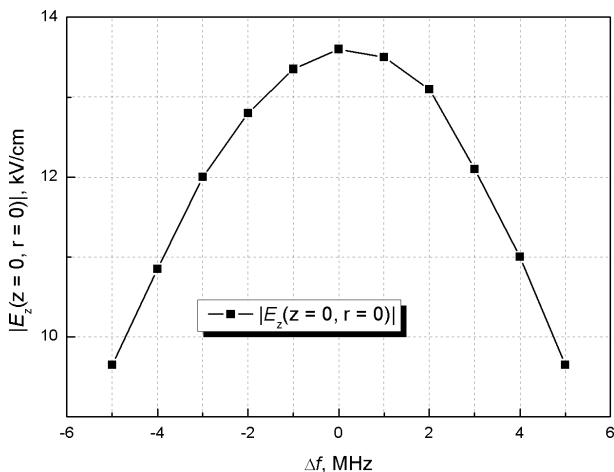


Fig. 5. Dependence of amplitude of a longitudinal electric field E_z on an axis of structure at $z=0$ as a function of detuning of repetition frequency of bunches from an exact resonance. The structure was excited by sequence of 300 bunches

From Fig. 5 one can see that for the most effective excitation of the resonator it is necessary to support precisely a resonance between eigen frequency of the resonator and frequency of injection of electron bunches. Deviation of repetition frequency on ± 3 GHz leads to change of amplitude of a wakefield on $\approx 5\%$.

CONCLUSIONS

Self-consistent theory of wakefield excitation by an electron bunches in cylindrical resonator with longitudinally nonuniform filling by dielectric is constructed.

Presented theory allows to analyze the self-consistent dynamics of excitation of the dielectric resonator by charged bunches. By results of numerical simulation are obtained and analyzed: amplitude-frequency distributions of the excited electromagnetic field (both a single electron bunch, and their regular sequence); longitudinal distribution of amplitude of axial and radial components of the excited electric field; dependence of amplitude of a longitudinal electric field on detuning of repetition frequency of bunches on an exact resonance. The obtained results can be used when carrying out the appropriate experimental studies, in particular, in case of a choice of parameters, both accelerating structure, and the injected electron bunches for the purpose of excitation of the necessary operation mode with the greatest amplitude of the accelerating field.

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ВОЗБУЖДЕНИЕ ЭЛЕКТРОМАГНИТНОГО ПОЛЯ ЭЛЕКТРОННЫМИ СГУСТКАМИ В ЧАСТИЧНО ЗАПОЛНЕННОМ ДИЭЛЕКТРИЧЕСКОМ РЕЗОНАТОРЕ

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Построена нелинейная самосогласованная теория возбуждения кильватерного поля электронными сгустками в цилиндрическом резонаторе с продольно-неоднородным диэлектрическим заполнением. Возбуждаемое поле представлено в виде суперпозиции потенциального и соленоидального полей. Разработанная теория позволяет провести численный анализ возбуждения резонатора электронным сгустком для наилучшей интерпретации полученных экспериментальных результатов.

ЗБУДЖЕННЯ ЕЛЕКТРОМАГНІТНОГО ПОЛЯ ЕЛЕКТРОННИМИ ЗГУСТКАМИ В ДІЕЛЕКТРИЧНОМУ РЕЗОНАТОРІ, ЩО Є ЧАСТКОВО ЗАПОВНЕНИМ

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Побудовано нелінійну самоузгоджену теорію збудження кильватерного поля електронними згустками в циліндричному резонаторі з поздовжньо-неоднорідним діелектричним заповненням. Поле, що збуджується, представлено у вигляді суперпозиції потенційного та соленоїдального полів. Розроблена теорія дозволяє провести числовий аналіз збудження резонатора електронним згустком для найліпшої інтерпретації отриманих експериментальних результатів.