INFLUENCE OF THE FINITE Q-FACTOR ON EXCITATION OF WAKEFIELD OSCILLATIONS IN DIELECTRIC CAVITY BY A SEQUENCE OF RELATIVISTIC ELECTRON BUNCHES IN THE PRESENCE OF DETUNING BETWEEN THE RESONANT FREQUENCY AND THE BUNCH REPETITION FREQUENCY

V.A. Balakirev, I.N. Onishchenko, A.P. Tolstoluzhsky

National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine

Excitation of Cerenkov wakefield oscillations in dielectric cavity with finite value of *Q*-factor in the presence of detuning between the bunch repetition frequency and the resonant frequency of the excited oscillations is investigated. It is shown that the picture of wake oscillations excitation is determined by the ratio between the value of the Q -factor and frequency detuning parameter $\pi\omega/\delta\omega$. For the high Q -factor dielectric cavity $Q >> \pi\omega/\delta\omega$ picture of wake oscillations excitation is the same as in the cavity without ohmic losses. If the opposite condition is satisfied, the process of wakefield excitation occurs almost as well as in the case of resonant excitation. Linear growth of the wakefield amplitude at initial stage reaches its saturation level.

PACS: 41.75.Jv, 41.75.Lx, 41.75.Ht, 96.50.Pw

INTRODUCTION

In paper [1] process of excitation of wakefield oscillations in planar dielectric cavity with a finite *Q*-factor by a sequence of relativistic electron bunches in conditions of exact resonance between the sequence bunches and one of the cavity modes is investigated. In this paper it is shown that the accounting of the *Q*-factor leads to a restriction of the resonance oscillations amplitude. Especially strong influence of *Q*-factor is shown in the case of a long sequence (the order of several hundred or even thousands of low-current bunches). In the excitation process of resonant oscillation involving only a finite number of bunches (in order $N = Q/\pi$) located in the leading front of the chain. The remaining bunches contribution to the amplitude of the resonant oscillation is not given, i.e. oscillation amplitude saturates. As is known [2], by introducing a detuning between the frequency of the excited oscillations and the repetition frequency of bunches in the dielectric cavity self-acceleration mode can be realized. In this case, bunches adjacent to the leading front will transfer its energy to excite oscillations, and bunches near the back front will gain energy. The first part of bunches with numbers $\omega/2\delta\omega \geq m \geq 1$, where $\delta\omega$ is the frequency detuning, will lose their energy. In turn bunches with numbers $\omega/\delta\omega \geq m \geq \omega/2\delta\omega$ will be accelerated. Then the process according to the value of detuning and the total number of bunches in the sequence can be repeated periodically. The above considerations are valid for the dielectric cavity without ohmic losses ($Q = \infty$). Meanwhile the accounting of *Q*-factor of the dielectric cavity, as in the resonance case may significantly change picture of wakefield excitation in the dielectric cavity in the presence of the frequency detuning and even disrupt auto-acceleration regime when *Q*-factor of dielectric cavity is low. As the estimates show, *Q*-factor will not influence on the process of auto-acceleration of bunches sequence when the requirement for *Q*-factor values $Q > \pi\omega/\delta\omega$ is satisfied. If this condition on the *Q*-factor of the dielectric cavity is not satisfied, then the process of auto-acceleration is disrupted, as in this case, the amplitude will saturate at low level and cannot be formed beating oscillations.

In this paper detailed picture of the wakefield excitation of oscillations in the dielectric cavity with finite *Q*-factor by sequence of relativistic electron bunches in the presence of detuning between the repetition frequency of bunches and the resonance frequency of oscillation is presented.

1. STATEMENT OF PROBLEM. BASIC EXPRESSIONS

Let's consider the problem in the following statement. The dielectric cavity is formed by a perfectly conducting metal cylindrical cavity whose volume is completely filled with a uniform dielectric permittivity ε. Dielectric losses determined by *Q*-factor, where $Q = \varepsilon_1/\varepsilon_2$, $\varepsilon_1, \varepsilon_2$ is the real and imaginary part of permittivity, respectively. On the left end of the cavity in its volume injected periodic sequence of electron bunches with arbitrary longitudinal and transverse profiles of the electron density. The repetition frequency of bunches ω_b is different from the frequency of oscillations ω_{mn} in the value of detuning $\delta\omega_{mn} = \omega_{mn} - \omega_b$.

Let's determine wakefield in approximation of given motion of a single bunch in the dielectric cavity with *Q*-factor. The problem will be solved in a standard way. Firstly we find wakefield of elementary charge, having the form of a thin ring with charge dQ_b. Elementary charge density of an infinitely thin ring has the form

$$d\rho_b = \frac{dQ(r_0,t_0)}{v_0}\frac{\delta(r-r_0)}{2\pi r_o}\delta(t-t_0-\frac{z}{v_0}), \quad (1)$$

$$dQ(r_0,t_0) = \frac{Q_b}{v_0\sigma_b t_b}R(r_0/r_b)T(t_0/t_b)2\pi r_0 dr_0 dt_0, \quad (2)$$

where Q_b is full charge of bunch, t_0 is time of entry of elementary charge, r_0 is radius ring, v_0 is bunch velocity, t_b, r_b are characteristic duration and transverse bunch size, $R(r_0/r_b)$ is function described transversal profile of bunch density, $\sigma_b = \pi r_b^2$ is characteristic square of bunch transverse section, and $T(t_0/t_b)$ is longitudinal profile. These functions are satisfied of the normalization condition $\int_0^1 R(x)dx = 1, \quad \int_0^1 T(\tau)d\tau = 1$.

Elementary charge is within the volume of the cavity in the time interval $t_0 + L/v_0 \geq t \geq t_0$, L/v_0 is the passing time of elementary charge through cavity, L is cavity length. After determine field of the elementary ring bunch (1) $E_{ZG}(r_0, t_0, r, z, t-t_0)$ we determine the wakefield of entire bunch by integrate on initial radial coordinate r_0, and time of entry t_0:

$$E_{zw} = \int_0^{r_b} 2\pi r_0 dr_0 \int_0^{t_b} dt_0 E_{ZG}(r_0, t_0, r, z, t-t_0). \qquad (3)$$

After realization the algorithm described above, we obtain the following expression for the wake field excited by a single bunch in the dielectric cavity

$$E_{zw} = \frac{8Q_b c^2(\beta_0^2\varepsilon - 1)}{v_0 \varepsilon^2 a^2 L t_b} \sum_{n=1}^{\infty} \Pi_n \frac{J_0(\lambda_n r/a)}{J_1^2(\lambda_n r/a)} \sum_{m=0}^{\infty} \alpha_m S_m(t) \cos(k_m z), \qquad (4)$$

where $\beta_0 = v_0/c$, a is cavity radius,

$$\Pi_n = \int_0^1 R(x) J_0(\lambda_n \frac{r_b}{a} x) x dx, \qquad (5)$$

$$S_m(t) = \int_0^1 T(t_0/t_b) Z_{mn}(t-t_0) dt_0, \qquad (6)$$

$$Z_{mn}(t-t_0) = \frac{1}{\omega_{mn}^2 - \omega_m^2} \{\theta(t - t_0 - L/v_0)$$
$$\left[\omega_m \sin\omega_m(t-t_0) - (-1)^m \omega_{mn} \sin\omega_{mn} e^{-\gamma_{mn}(t-t_0-L/v_0)}(t - t_0 - L/v_0)\right] -$$
$$-\theta(t-t_0)\left[\omega_m \sin\omega_m(t-t_0) - e^{-\gamma_{mn}(t-t_0)}\omega_{mn}\sin\omega_{mn}(t-t_0)\right]\},$$

$\omega_m = k_m v_0,\ k_m = \pi m/L, \quad \omega_{mn} = \frac{c}{\sqrt{\varepsilon}}\left(k_m^2 + \frac{\lambda_n^2}{a^2}\right)^{1/2}$ are frequencies of fundamental oscillations of dielectric cavity, $\gamma_{mn} = \omega_{mn}/2Q$ is damping decrement oscillation with indexes m, n,

$\theta(t) = \begin{cases} 1, & t > 0, \\ 0, & t \leq 0 \end{cases}$ is Heaviside unit function,

$\alpha = 1/2$ for $m = 0$ and $\alpha = 1$ for $m \geq 1$.

In the case of the chain of the N bunches wakefield is the sum of wakefields as bunches which are inside at the current time in the cavity volume and bunches passed through the cavity. We choose a particular form of profile bunches. We assume that the transverse density profile has a Gaussian shape

$$R(r_0/r_b) = (1/r_b^2) e^{-\frac{r^2}{2r_b^2}},$$

and longitudinal profile has rectangular form

$$T(t_0/t_b) = \begin{cases} 1, & 1 \geq t_0/t_b \geq 0 \\ 0, & t_0/t_b < 0,\ t_0/t_b > 1 \end{cases}.$$

For such model of the bunch profile, the expression for wakefield of a single bunch has the form

$$E_{zw}(t,r,z) = E_{1w}(t_b \geq t \geq 0, r, z) + E_{2w}(L/v_0 \geq t > t_b, r, z) +$$
$$+ E_{3w}(L/v_0 + t_b \geq t > L/v_0, r, z) + E_{4w}(t > L/v_0 + t_b, r, z).$$

First term

$$E_{1w} = E_0 \sum_{n=1}^{\infty} F_n(r) \sum_{m=0}^{\infty} \frac{\alpha_m \cos(k_m z)}{\left(\omega_{mn}^2 - \omega_m^2\right) t_b^2} \left[\cos\omega_m t - e^{\gamma_{mn} t}\cos\omega_{mn} t\right],$$

where $F_n(r) = \frac{J_0(\lambda_n r/a)}{J_1^2(\lambda_n r/a)} e^{-\frac{1}{2}\left(\frac{\lambda_n r_b}{a}\right)^2}$, $E_0 = \frac{8Q_b c t_b(\beta_0^2\varepsilon - 1)}{\beta_0 \varepsilon^2 a^2 L}$, describes the field, excited by a bunch at transition the entrance of the cavity $t_b \geq t \geq 0$.

Second term

$$E_{2w} = E_0 \sum_{n=1}^{\infty} F_n(r) \sum_{m=0}^{\infty} \frac{\alpha_m \cos(k_m z)}{\left(\omega_{mn}^2 - \omega_m^2\right) t_b^2} \begin{bmatrix} \cos\omega_m(t - t_b) - \cos\omega_m t) - \\ -e^{-\gamma_{mn}(t-t_b)}\cos\omega_{mn}(t - t_b) + \\ +e^{-\gamma_{mn} t}\cos\omega_{mn} t \end{bmatrix}$$

describes the wakefield excited by a bunch at its propagation inside the volume of the dielectric cavity $L/v_0 \geq t > t_b$. This time interval corresponds to the regime excitation which is fully equivalent to a semi-infinite dielectric waveguides [2, 3].

The third term accounts wakefield excited in the process of bunch exiting from cavity $L/v_0 + t_b \geq t > L/v_0$

$$E_{3w} = E_0 \sum_{n=1}^{\infty} F_n(r) \sum_{m=0}^{\infty} \frac{\alpha_m \cos(k_m z)}{\left(\omega_{mn}^2 - \omega_m^2\right) t_b^2} \times$$
$$\times \begin{cases} \cos\omega_m(t - t_b) + e^{-\gamma_{mn} t}\cos\omega_{mn} t) - e^{-\gamma_{mn}(t-t_b)}\cos\omega_{mn}(t - t_b) + \\ +(-1)^m\left[1 + e^{-\gamma_{mn}(t-t_b)}\cos\omega_{mn}(t - t_b)\right] \end{cases}.$$

And finally, the fourth term describes the field in the cavity after the bunch left cavity $t > L/v_0 + t_b$

$$E_{4w} = E_0 \sum_{n=1}^{\infty} F_n(r) \sum_{m=0}^{\infty} \frac{\alpha_m \cos(k_m z)}{\left(\omega_{mn}^2 - \omega_m^2\right) t_b^2} \begin{cases} e^{-\gamma_{mn} t}\cos\omega_{mn} t - \\ -e^{-\gamma_{mn}(t-t_b)}\cos\omega_{mn}(t - t_b) - \end{cases}$$
$$-(-1)^m \begin{bmatrix} e^{-\gamma_{mn}(t-L/v_0)}\cos\omega_{mn}(t - L/v_0) + \\ +e^{-\gamma_{mn}(t-t_b-L/v_0)}\cos\omega_{mn}(t - L/v_0 - t_b) \end{bmatrix} \Bigg\}.$$

2. WAKEFIELD EXCITATION IN DIELECTRIC CAVITY WITH FINITE Q-FACTOR BY SINGLE ELECTRON BUNCH

At the beginning we numerically consider the dynamics of wakefield excitation in dielectric cavity with finite Q-factor by a single bunch with density profile indicated in the previous section. Numerical calculations were performed for the following parameters of the dielectric cavity and the electron bunch: length of the cavity is $L = 63.85$ cm, its radius is $a = 3.92$ cm, dielectric permittivity is $\varepsilon = 2.1$, value of characteristic radius of bunch is $r_b/a = 0.25$, bunch duration is $t_b/T_1 = 5/32, \quad T_1 = 1/f_1$ is frequency oscillation which is in Cerenkov resonance with the bunch. Synchronous with respect to bunch is the oscillation with indexes $n=1, m=12$. For a given frequency the cavity length contains six wavelengths.

Particle energy of the electron bunch is 4.5 MeV, charge of bunch is $Q_b = 0.32\ nC$.

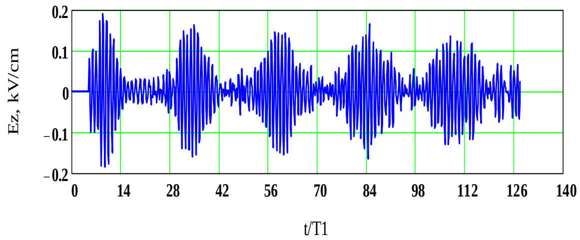

Fig. 1,a. The dependence of the field amplitude at the cavity output end in the case of single bunch, $Q=\infty$

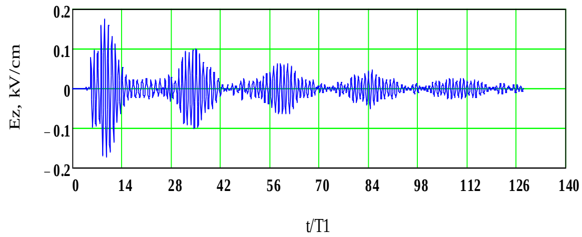

Fig. 1,b. The dependence of the field amplitude at the cavity output end in the case of single bunch, $Q=200$

In Fig. 1,a dependence on the wake electric field on time at the output end of the dielectric cavity with a large observation time $\tau/2\pi=126$ without ohmic losses is presented. It is shown dispersive spreading of the circulating pulse and its monochromatization. Note that increasing the duration of the bunch while keeping the total charge leads to reduction amplitude of the wakefield. This result is understandable, since the increasing in the duration of bunch degrades coherence radiation wakefield by bunch.

The time of flight through the cavity of the bunch is $\tau_{transit}/2\pi=6$. Group front passes through the cavity length over time $\tau_g/2\pi=12.6$. After bunch left cavity Cerenkov radiation pulse propagates with the group velocity. Multiple reflections and dispersive spreading of the wave packet leads to the decomposition of the field at the output of the cavity in the pulse sequence.

In Fig. 1,b the dependence of wake electric field from time to time at the output end dielectric cavity at *Q*-factor value $Q=200, (\nu/\omega=2.5\cdot10^{-3})$ is presented. Clearly that circulation of pulse in the volume of dielectric cavity is accompanied by attenuation of its amplitude.

3. WAKEFIELD EXCITATION IN THE DIELECTRIC CAVITY WITH OHMIC LOSSES BY SEQUENCE OF ELECTRON BUNCHES

In this section the picture excitation of the wakefield oscillations in dielectric cavity with the finite *Q*-factor by sequence of relativistic electron bunches in the presence of detuning between the repetition frequency bunches and the resonance frequency of oscillations is presented. Let's consider effect of influence of the finite values *Q*-factor on wakefield excitation in the dielectric cavity by resonance sequence bunches when the repetition frequency of bunches coincides with frequency of synchronous eigen oscillation of cavity. In Figs. 2a, b the dependences of the Cherenkov wakefield field at the exit of the cavity are presented in the case of the sequence of 100 bunches. The cavity length is $L=6\lambda=63.85$ cm, λ is wavelength, its radius is equal to $a=3.922\ cm$, the dielectric constant is $\varepsilon=2.1$. Parameters of each bunch are the same as in the previous section. Fig. 2,a corresponds to the $Q=\infty$ and Fig. 2,b corresponds to the $Q=200$.

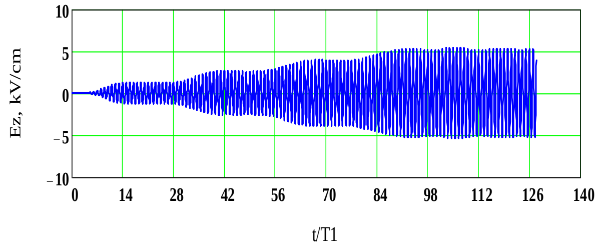

Fig. 2,a. Dependence of the field amplitude at the output of the cavity in the case of resonance sequence of bunches, $N=100$, $Q=\infty$

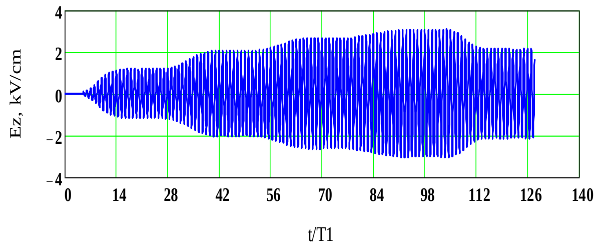

Fig. 2,b. Dependence of the field amplitude at the output of the cavity in the case of resonant sequence of bunches, $N=100$, $Q=200$

In this and in the other case in beginning non-monotonic growth of the amplitude of the wake field takes place. Nonmonotonic growth of amplitude is due to circulation of increasing signal through a feedback circuit, the nature of which is caused by the reflection of the signal from the perfectly conducting dielectric cavity ends. With decreasing *Q*-factor the maximum value of the amplitude wakefield also decreases. Moreover, in the second case after the last bunch passing some attenuation field is observed.

Let us now consider the case of wakefield excitation in dielectric cavity with different values Q-factor in the presence of detuning. The cavity length is $L=3\lambda=31.925$ cm, dielectric permeability is $\varepsilon=2.045$, the number of bunches in the chain is 400, the value of detuning is $\delta\omega/\omega=2.5\cdot10^{-3}$.

For this value of the detuning in the case of the lossless cavity phase shift of wakefield after the last bunch is 2π. Figs. 3,a-3,d shows series of curves for different values *Q*-factor lying within $2\cdot10^4 \ge Q \ge 1.25\cdot10^2$. For high *Q*-factor dielectric cavity $Q=2.0\cdot10^4$ (see Fig. 3,a) picture of the wakefield excitation is practically identical to the case of cavity without ohmic losses [2].

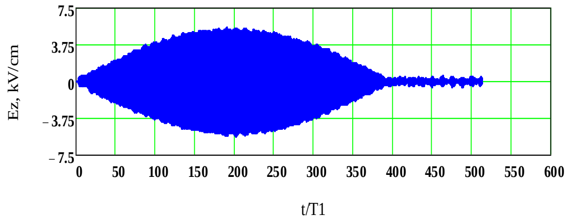

Fig. 3,a. Dependence of the field amplitude at the output of the cavity in the case of nonresonant sequence of bunches, $N = 400$, $Q = 2000$, value of detuning is

$\delta\omega/\omega = 2.5\cdot10^{-3}$

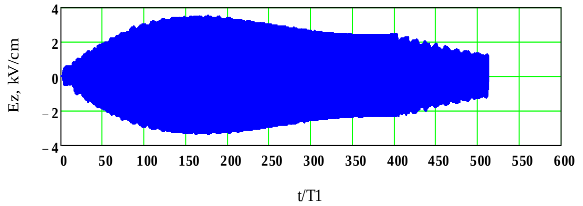

Fig. 3,b. Dependence of the field amplitude at the output of the cavity in the case of nonresonant sequence of bunches, $N = 400$, $Q = 1250$, value of detuning is

$\delta\omega/\omega = 2.5\cdot10^{-3}$

This case is intermediate, since the value of the detuning parameter $\pi\omega/\delta\omega = 1.2\cdot10^{3}$ is close to the value of Q-factor ($Q = 1.25\cdot10^{3}$). The picture of wakefield excitation has undergone a qualitative change. After the non-monotonic growth the wakefield amplitude relatively quickly reaches its maximum value. And the maximum shifted towards the region leading front of bunches sequence. The amplitude of the field decreases slowly until the last bunch. After last bunch field non-monotonically decreases with higher rate, compared with the region of bunches.

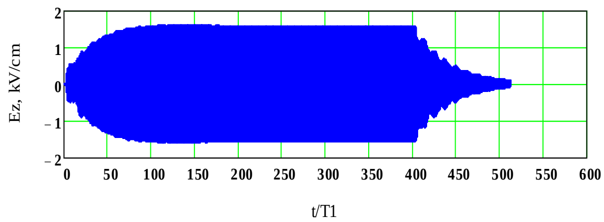

Fig. 3,c. Dependence of the field amplitude at the output of the cavity in the case of nonresonant sequence of bunches, $N = 400$, $Q = 125$, value of detuning is

$\delta\omega/\omega = 2.5\cdot10^{-3}$

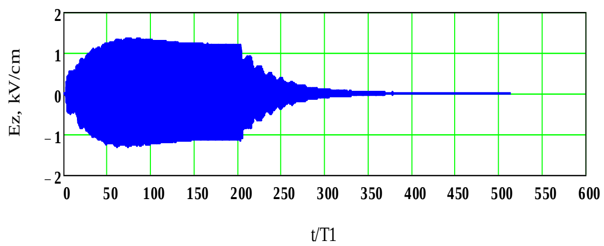

Fig. 3,d. Dependence of the field amplitude at the output of the cavity in the case of nonresonant sequence of bunches, $N = 200$, $Q = 125$, value of detuning is

$\delta\omega/\omega = 5\cdot10^{-3}$

In Fig. 3,b dependence of the wakefield amplitude on time for value of Q-factor $Q = 1.25\cdot10^{3}$ is presented. The picture of wakefield excitation practically does not differ from the case of resonant excitation [1]. After the non-monotonic growth amplitude reaches stationary constant value. Of 400 bunches at such value of Q-factor in process of the wakefield excitation participates only the first 100 bunches. With a constant value of amplitude decrease to almost 3.8 times compared with the case of high Q-factor cavity (see Fig. 3,a). After the last bunch wakefield amplitude non-monotonically decreases to zero (see Figs. 3,c; 3,d).

Thus, in this paper excitation of Cerenkov wakefield oscillations in dielectric cavity with finite value of Q-factor in the presence of detuning between the repetition frequency of bunches in the chain and the frequency of the excited oscillations is investigated. It is shown that the picture of wake oscillations excitation is determined by the ratio between the value of the Q-factor and frequency detuning parameter $\pi\omega/\delta\omega$. For the high Q-factor dielectric cavity $Q >> \pi\omega/\delta\omega$ picture of wake oscillations excitation is the same as in the cavity without ohmic losses. If the opposite condition is satisfied, the process of wakefield excitation occurs almost as well as in the case of resonant excitation. Finally, when both parameters are close in value $Q \sim \pi\omega/\delta\omega$, the intermediate case is realized (see Fig. 3,b).

ACKNOWLEDGEMENTS

This work was supported by the US Department of Energy/NNSA through the Global Initiatives for Proliferation Prevention (GIPP) Program in Partnership with the Science and Technology Center in Ukraine (Project ANL-T2-247-UA and STCU Agreement P522).

REFERENCES

1. I.N. Onishchenko, G.V. Sotnikov, T.C. Marshal. Amplitudes and Spectra of Wake Fields in a Planar Dielectric Cavity with Finite Q-Factor // *12th Advanced Accelerator Concepts Workshop, Lake Geneva, Wisconsin (USA), 10-15 July 2006*: *AIP Conf. Proceedings* / Editors: Manoel Conde and Catherine Eyberger, v. 877, p. 866-872.

2. V.A. Balakirev, I.N. Onishchenko, A.P. Tolstoluzhsky. Wakefield Excitation and Electron Acceleration at Detuning Bunch Repetition Frequency and Frequency of Eigen Principal Mode of Wakefield // *Problems of Atomic Science and Technology. Ser. "Plasma Electronics and New Acceleration Methods"*. 2013, № 6, p. 80-83.

3. V.A. Balakirev, I.N. Onishchenko, D.Y. Sidorenko, G.V. Sotnikov. Excitation of wakefield by relativistic electron bunch in a semi-infinite dielectric waveguide // *Zh. Eksp. Teor. Fiz.* 2001, v. 120, № 1, p. 41-51 (in Russian).

Article received 26.11.2014

ВЛИЯНИЕ КОНЕЧНОГО ЗНАЧЕНИЯ ДОБРОТНОСТИ НА ВОЗБУЖДЕНИЕ КИЛЬВАТЕРНЫХ КОЛЕБАНИЙ В ДИЭЛЕКТРИЧЕСКОМ РЕЗОНАТОРЕ ПОСЛЕДОВАТЕЛЬНОСТЬЮ РЕЛЯТИВИСТСКИХ ЭЛЕКТРОННЫХ СГУСТКОВ ПРИ НАЛИЧИИ РАССТРОЙКИ МЕЖДУ РЕЗОНАНСНОЙ ЧАСТОТОЙ И ЧАСТОТОЙ СЛЕДОВАНИЯ СГУСТКОВ

В.А. Балакирев, И.Н. Онищенко, А.П. Толстолужский

Изучен процесс возбуждения кильватерных колебаний в диэлектрическом резонаторе с конечным значением добротности Q при наличии расстройки между частотой следования сгустков и частотой резонансного колебания. Показано, что картина возбуждения кильватерных колебаний определяется соотношением между значением добротности Q и параметром частотной расстройки $\pi\omega/\delta\omega$. В случае высокой добротности диэлектрического резонатора $Q >> \pi\omega/\delta\omega$ картина возбуждения кильватерных колебаний такая же, как и в резонаторе без омических потерь. В диэлектрическом резонаторе формируется биение кильватерного поля. Если выполнено противоположное условие, картина возбуждения кильватерных колебаний практически не отличается от случая резонансного возбуждения. После начального линейного роста амплитуды имеет место ее насыщение на постоянном уровне.

ВПЛИВ СКІНЧЕННОЇ ВЕЛИЧИНИ ДОБРОТНОСТІ НА ЗБУДЖЕННЯ КІЛЬВАТЕРНИХ КОЛИВАНЬ У ДІЕЛЕКТРИЧНОМУ РЕЗОНАТОРІ ПОСЛІДОВНІСТЮ РЕЛЯТИВІСТСЬКИХ ЕЛЕКТРОННИХ ЗГУСТКІВ ЗА НАЯВНОСТІ РОЗСТРОЙКИ МІЖ РЕЗОНАНСНОЮ ЧАСТОТОЮ І ЧАСТОТОЮ СЛІДУВАННЯ ЗГУСТКІВ

В.А. Балакірєв, І.М. Оніщенко, О.П. Толстолужський

Вивчено процес збудження кільватерних коливань у діелектричному резонаторі із скінченним значенням добротності Q при наявності розстройки між частотою проходження згустків і частотою резонансного коливання. Показано, що картина збудження кільватерних коливань визначається співвідношенням між значенням добротності Q й параметром частотної розстройки $\pi\omega/\delta\omega$. У випадку високої добротності діелектричного резонатора $Q >> \pi\omega/\delta\omega$ картина збудження кільватерних коливань така ж, як і в резонаторі без омічних втрат. У діелектричному резонаторі формується биття кільватерного поля. Якщо виконана протилежна умова, картина збудження кільватерних коливань практично не відрізняється від випадку резонансного збудження. Після початкового лінійного росту амплітуди має місце її насичення на постійному рівні.