PECULIARITIES OF A WAKE POTENTIAL FORMATION DUE TO A POLARIZATION INTERACTION OF CHANNELING ELECTRON WITH IONIC CRYSTAL SOLID STATE PLASMA

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Under the influence of electric and magnetic fields generated by a nonrelativistic electron moving in the regime of a plane channeling, the induced time dependent ion electric dipole moments of a crystal medium are calculated. Accounting the causality principle for this dipole system a general scalar potential which can be named a “wake” one is found. It is shown that this potential in some crystallographic directions (for example, in ionic crystals) may lead to an essential reverse influence on the regime of the channeling particle motion.

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INTRODUCTION

Obviously, the motion of any particle in arbitrary medium is accompanied by the mutual influence of medium on a particle and a particle, in its turn, on medium. This mutual influence in quite a complicated way depends on medium characteristics, kind and parameters of particles. For example, at the channeling of relativistic electron (positron) its reverse effect on crystal medium is negligible [1]. Just at the motion of a heavy ion, on one hand, a polarization of crystal medium occurs and on the other hand, a delayed in time and space screening of a potential of an ion itself appears [2]. It leads to an origination of so-called “wake potential”. Note, that for the first time a qualitative theory of a wake interaction for a charged particle moving in plasma has been worked out in [3]. Similar wake effects generate just in other cases. For example, a wake potential which is possible at swift ions motion in solids [4] is used for electron beams acceleration [5]. It appears in carbon nanotubes [6], etc.

This paper deals with the investigation of the channeling weak relativistic electron influence on electron and nucleus subsystems of a crystal grid of an ionic crystal which to some extent can be treated as solid plasma. It should be emphasized that for the first time the idea of the possibility of wake potential generation at the channeling of relativistic electrons in ionic crystals was stated in [7]. The case when a particle effect on medium is not too strong is treated. Then the problem can be solved by means of the successive approaches (in the given paper a zero approach is developed). In the opposite case when a particle effect on medium may be compared with the effect of medium on a particle (it is possible at the channeling of charged particles along charged planes of some ionic crystals and also at the motion along high indices directions in many ionic and ionic covalent crystals) the problem, evidently, should be solved in a self-consistent way by means of a variation method.

Thus, supposing that crystal medium influence on a moving electron is given (electron in a crystal is in the states $\psi_n(x)$ of a planar channeling with the possibilities w_n), let’s come proceed to the consideration of a reverse influence of the electron on a crystal medium, i.e. let’s calculate the induced electric dipole crystal moments.

1. CALCULATION OF ELECTRIC DIPOLE MOMENTS OF CRYSTAL IONS

The expression for an electric charge density moving with $\vec{v} = v\vec{e}_z$ velocity of a relativistic electron (in $\psi_n(x)$ state) has a form

$$\rho_n(\vec{r},t) = -e\psi_n^2(x)\,\delta(y)\,\delta(z-vt). \qquad (1)$$

In this case electric and magnetic field densities using (1) and in the framework of the relativistic approach are determined by the following expressions (see, for example, [8]):

$$\vec{E}_n(\vec{r},t) = -\frac{e}{\gamma^2}\int_{-\infty}^{\infty}\psi_n^2(\xi)\,\frac{\vec{R}(\vec{r},t,\xi)}{R^{*3}(\vec{r},t,\xi)}\,d\xi, \qquad (2a)$$

$$\vec{H}_n(\vec{r},t) = \frac{1}{c}\left[\vec{v},\vec{E}_n(\vec{r},t)\right], \qquad (2b)$$

where γ is a Lorentz-factor, $\vec{R}(\vec{r},t,\xi) = (x-\xi, y, z-vt)$ – radius-vector from e charge to (x,y,z) field observation point, $R^{*2}(\vec{r},t,\xi) = \left[(x-\xi)^2 + y^2\right]/\gamma^2 + (z-vt)^2$. The induced electric dipole moment $\vec{p}_n(\vec{r}_{\vec{n}},t)$ of a separate crystal ion in the point with a radius-vector, $\vec{r}_{\vec{n}} = (n_x a_x, n_y a_y, n_z a_z)$ (here $a_{x,y,z}$ are the crystal grid periods along x,y,z axes; $n_{x,y,z} = 0,\pm1,\pm2,\ldots$), is calculated by means of the following equation of the oscillation type:

$$\ddot{\vec{p}}_n(\vec{r}_{\vec{n}},t) + g\dot{\vec{p}}_n(\vec{r}_{\vec{n}},t) + \omega_0^2\vec{p}_n(\vec{r}_{\vec{n}},t) = -\frac{e^2}{m}\int d\vec{r}\,\kappa\left(\left|\vec{r}-\vec{r}_{\vec{n}}\right|\right)\left\{\vec{E}_n(\vec{r},t) + \frac{1}{c}\left[\vec{v},\vec{H}_n(\vec{r},t)\right]\right\}, \qquad (3)$$

where m – an electron rest mass; $\kappa(r)$ – the density of an electron distribution in an ion; g – a fading constant bound to electron and nucleus oscillations; ω_0 – the density of dipole proper oscillations. To solve the equation (3) one expands $\vec{E}_n(\vec{r},t)$ and $\vec{H}_n(\vec{r},t)$ fields as it was done just in [3] into the following Fourier integrals:

$$\left\{\begin{matrix}\vec{E}_n(\vec{r},t)\\ \vec{H}_n(\vec{r},t)\end{matrix}\right\}=\int\frac{d\vec{k}d\omega}{(2\pi)^4}\exp\left[i(\vec{k}\vec{r}-\omega t)\right]\left\{\begin{matrix}\vec{E}_n(\vec{k},\omega)\\ \vec{H}_n(\vec{k},\omega)\end{matrix}\right\}, \quad (4)$$

where Fourier-components of the expressions (2a) and (2b) by means of [8] are represented in the form:

$$\vec{E}_n(\vec{k},\omega)=4\pi ief_n(k_x)\frac{\left(\vec{k}-\frac{\omega}{c^2}\vec{v}\right)}{k^2-\frac{\omega^2}{c^2}}\delta(\omega-k_z v), \quad (5a)$$

$$\vec{H}_n(\vec{k},\omega)=\frac{4\pi ie}{c}f_n(k_x)\frac{\left[\vec{v},\vec{k}\right]}{k^2-\frac{\omega^2}{c^2}}\delta(\omega-k_z v). \quad (5b)$$

Here $f_n(k_x)=\int_{-\infty}^{\infty}\psi_n^2(\xi)\exp(-ik_x\xi)d\xi$. Substituting (4) into (3), the right side of the equation (3) is transformed into the form

$$-\frac{e^2}{m(2\pi)^4}\int d\vec{k}d\omega\kappa(k)\exp\left[i(\vec{k}\vec{r}_{\vec{n}}-\omega t)\right]\times$$
$$\times\left\{\vec{E}_n(\vec{k},\omega)+\frac{1}{c}\left[\vec{v},\vec{H}_n(\vec{k},\omega)\right]\right\}, \quad (6)$$

where $\kappa(k)=\int\kappa(r)\exp(i\vec{k}\vec{r})d\vec{r}$ – a special Fourier-component of $\kappa(r)$ function. Substituting (5a) and (5b) into the expression (6), from the expression (3) we find a Fourier-component of $\vec{p}_n(\vec{r}_{\vec{n}},t)$ electric dipole moment:

$$\vec{p}_n(\vec{k},\omega)=\frac{4\pi ie^3}{m\gamma^2}\frac{\vec{k}f_n(k_x)\kappa(k)\delta(\omega-k_z v)}{(\omega^2-\omega_0^2+i\omega g)\left(k^2-\frac{\omega^2}{c^2}\right)}. \quad (7)$$

At last, substituting the expression (7) into a Fourier integral $\vec{p}_n(\vec{r}_{\vec{n}},t)=\int\frac{d\vec{k}d\omega}{(2\pi)^4}\exp\left[i(\vec{k}\vec{r}_{\vec{n}}-\omega t)\right]\vec{p}_n(\vec{k},\omega)$, we come after long calculations to the expression for the value $\vec{p}_n(\vec{r}_{\vec{n}},t)$:

$$\vec{p}_n(\vec{r}_{\vec{n}},t)=C\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\psi_n^2(\xi)d\xi dk_z}{\left[(k_z^2-\omega_0^2/v^2)^2+k_z^2g^2/v^2\right]}F_{\xi}^{\vec{n}}(k_z)\times$$
$$\times f_{\lambda_{n_z}}^{k_z}\left[\vec{e}_x(x_{n_x}-\xi)-\vec{e}_y y_{n_y}\right]+\vec{e}_z G_{\xi}^{\vec{n}}(k_z)g_{\lambda_{n_z}}^{k_z}, \quad (8)$$

where $f_{\lambda_{n_z}}^{k_z}=\frac{k_z g}{v}\sin(k_z\lambda_{n_z})-\left(\frac{\omega_0^2}{v^2}-k_z^2\right)\cos(k_z\lambda_{n_z})$,

$$g_{\lambda_{n_z}}^{k_z}=\frac{k_z g}{v}\cos(k_z\lambda_{n_z})+\left(\frac{\omega_0^2}{v^2}-k_z^2\right)\sin(k_z\lambda_{n_z}),$$

$$F_{\xi}^{\vec{n}}(k_z)=\frac{\frac{|k_z|}{\gamma}K_1\left(\frac{|k_z|r_{\xi}^{\vec{n}}}{\gamma}\right)-\sigma_z K_1(\sigma_z r_{\xi}^{\vec{n}})}{r_{\xi}^{\vec{n}}\beta_z^4}-\frac{K_0(\sigma_z r_{\xi}^{\vec{n}})}{2\beta_z^2},$$

$$G_{\xi}^{\vec{n}}(k_z)=\frac{K_0\left(\frac{|k_z|r_{\xi}^{\vec{n}}}{\gamma}\right)-K_0(\sigma_z r_{\xi}^{\vec{n}})}{\beta_z^4}-\frac{r_{\xi}^{\vec{n}}K_1(\sigma_z r_{\xi}^{\vec{n}})}{2\beta_z^2\sigma_z},$$

$x_{n_x}=n_x a_x$, $y_{n_y}=n_y a_y$, $z_{n_z}=n_z a_z$,

$\sigma_z=\sqrt{k_z^2+4/b_0^2}$, $r_{\xi}^{\vec{n}}=\sqrt{(x_{n_x}-\xi)^2+y_{n_y}^2}$,

$\beta_z^2=k_z^2\beta^2+4/b_0^2$, $\beta=v/c$, $\lambda_{n_z}=z_{n_z}-vt$,

$C=-16\alpha e^3/\pi^2 mv^2 b_0^2\gamma^2$, $K_{0,1}(x)$ – MacDonald functions. Note that calculating the expression (8) one uses $\kappa(k)=32\alpha/(4+k^2b_0^2)^2$ function arising for electron distribution density $\kappa(r)$, chosen in an exponential form, namely $\kappa(r)=(2\alpha/\pi b_0^3)\exp(-2r/b_0)$, where α is a degree of crystal atoms ionicity. Similar situation, for example, takes place for negatively charged hydrogen ions H^- in LiH, ionic crystal for which $b_0=16a_0/11$, where a_0 – a Bohr radius (see, for example, [9]). Analogous situations are possible just in other ionic crystals.

2. CALCULATION AND ANALYSIS OF A WAKE POTENTIAL

A wake potential, i.e. electrostatic potential, generated by the system of electric dipoles induced, in its turn, by a channeling particle, can be calculated by means of the following expression:

$$\Phi(\vec{r},t)=\sum_{n,\vec{n}}\left[\vec{p}_n(\vec{r}_{\vec{n}},t)\cdot(\vec{r}-\vec{r}_{\vec{n}})/|\vec{r}-\vec{r}_{\vec{n}}|^3\right]w_n. \quad (9)$$

In particular, if we are restricted by the only crystallographic plane ($n_x=0$) and by the only one axis ($n_y=0$) on this plane, the total scalar potential accounting the expressions (8) and (9) is determined by the formula

$$\phi(\rho,\lambda_z)=\frac{4C}{a_z}\sum_n w_n\int_0^\infty\int_0^\infty\frac{\psi_n^2(\xi)\,G(k_z,\xi)\,d\xi dk_z}{\left[\left(k_z^2-\omega_0^2/v^2\right)^2+k_z^2g^2/v^2\right]}\times$$

$$\times\int_{-\infty}^{vt}\frac{(z-\zeta)\left[\left(\frac{\omega_0^2}{v^2}-k_z^2\right)\sin(k_z\lambda_\zeta)+\frac{k_z g}{v}\cos(k_z\lambda_\zeta)\right]}{\left[\rho^2+(z-\zeta)^2\right]^{3/2}}d\zeta=$$

$$=\frac{4C}{a_z}\int_0^\infty\int_0^\infty\frac{\psi_n^2(\xi)\,G(k_z,\xi)\,D(k_z,\lambda_z,\rho)\,d\xi dk}{\left[\left(k_z^2-\omega_0^2/v^2\right)^2+k_z^2g^2/v^2\right]},\qquad(10)$$

where

$$D(k_z,\lambda_z,\rho)=\left[\left(\frac{\omega_0^2}{v^2}-k_z^2\right)\sin(k_z\lambda_z)+\frac{k_z g}{v}\cos(k_z\lambda_z)\right]\times$$

$$\times\left[\frac{1}{\rho}\,{}_1F_2\left(1;\frac{1}{2},\frac{1}{2};\frac{k_z^2\rho^2}{4}\right)-\frac{\pi k_z I_0(k_z\rho)}{2}-\int_0^{\lambda_z}\frac{\nu\cos(k_z\nu)\,d\nu}{\left(\rho^2+\nu^2\right)^{3/2}}\right]-$$

$$-\left[\left(\frac{\omega_0^2}{v^2}-k_z^2\right)\sin(k_z\lambda_z)+\frac{k_z g}{v}\cos(k_z\lambda_z)\right]\times$$

$$\times\left\{k_zK_0(k_z\rho)-\int_0^{\lambda_z}\frac{\nu\sin(k_z\nu)\,d\nu}{\left(\rho^2+\nu^2\right)^{3/2}}\right\},$$

${}_1F_2\left(1;\frac{1}{2},\frac{1}{2};x\right)$ – a hypergeometric function; $I_0(x)$ – a modified Bessel function; $\lambda_\zeta=\zeta-vt$, $\lambda_z=z-vt$, $\rho=\sqrt{x^2+y^2}$. In the formula (10) the causality principle is accounted and the transition from the summarizing by n_z to integration $a_z^{-1}\int\ldots d\zeta$, has been realized, where a_z is a distance between ions along z axis.

In Fig. 1 the potential (10) is illustrated for the example of nonrelativistic ($\gamma\approx 2$) channeling in ionic LiH crystal when in H^-(111) planes with the potential $U(x)=-U_0/\mathrm{ch}^2(x/b)$ ($b\approx 0,22A$, $U_0\approx 0,26\,\mathrm{eV}$ [10]) pit the only one state with a wave function $\psi_0(x)=2^{-s}\left(2/b\,\mathrm{B}(s,s)\right)^{1/2}\mathrm{ch}^{-s}(x/b)$ is realized, where $s=\left(2m\gamma U_0b^2/\hbar^2+1/4\right)^{1/2}-1/2$ (g and ω_0 were chosen on the basis of heuristic conformations: $g\sim v/b_0$, $\omega_0\sim U_0/\hbar$).

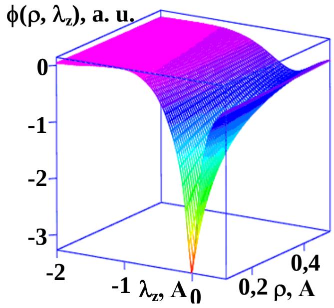

Fig. 1. The wake potential $\phi(\rho,\lambda_z)$ behavior, arising fo non-relativistic electron channeling along H^- planes of ionic LiH crystal (in arbitrary units) depending on the distance ρ and λ_z

As it is clear from Fig. 1, the wake potential has the reverse effect on the channeling particle. Let's analyze this phenomenon.

3. POTENTIAL WAKE REVERSE EFFECT ON THE CHANNELING ELECTRON

The channeling electron interaction energy in a wake potential (10), averaged by thermal oscillations can be written down in the form

$$V(\rho)=-e\exp\left(-\frac{\rho^2}{2u^2}\right)\int_0^\infty\iota\exp\left(-\frac{\iota^2}{2}\right)I_0\left(\frac{\rho\iota}{u}\right)\phi(u\iota)\,d\iota,\quad(11)$$

where u is an amplitude of crystal ions thermal oscillations, $\phi(\rho)=\phi(\rho,\lambda_z=0)$. Fig. 2 shows the function $U_{H^-}(\rho)=V(\rho)_{H^-}/V(0)_{H^-}$ graph for the above mentioned example (for H^- $u\approx 0,26A$ ions [10]).

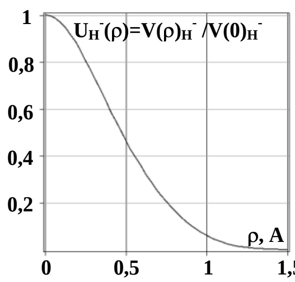

Fig. 2. Interaction energy $U_{H^-}(\rho)$ of the channeling electron in a wake potential as a function of ρ distance from z axis

Numerical evaluations of dependence (11) show that the wake potential can have a significant impact on thecharacter of a channeling particle motion (especially in anomalous cases).

CONCLUSIONS

Thus, in the paper it was shown that the channeling of non-relativistic electrons in ionic crystals generates a wake potential in the result of polarization of these crystals. Besides, it is shown that a wake potential affects a channeling particle. First, it is interesting from the general scientific point of view since a certain succession with analogous effects in many media is traced, and second, it may be used in practice (for example, at the investigation of the defects in crystals as it is supposed in [11]).

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ОСОБЕННОСТИ ФОРМИРОВАНИЯ КИЛЬВАТЕРНОГО ПОТЕНЦИАЛА ЗА СЧЕТ ПОЛЯРИЗАЦИОННОГО ВЗАИМОДЕЙСТВИЯ КАНАЛИРУЕМОГО ЭЛЕКТРОНА С ТВЕРДОТЕЛЬНОЙ ПЛАЗМОЙ ИОННОГО КРИСТАЛЛА

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Под воздействием электрического и магнитного полей, создаваемых движущимся в режиме плоскостного каналирования релятивистским электроном, рассчитываются зависящие от времени наведенные электрические дипольные моменты ионов кристаллической среды. С учетом принципа причинности для этой системы диполей находится суммарный скалярный потенциал, который можно назвать "кильватерным потенциалом". Показывается, что этот потенциал в некоторых кристаллографических направлениях (например, в ионных кристаллах) может приводить к существенному обратному влиянию на режим движения каналируемой частицы.

ОСОБЛИВОСТІ ФОРМУВАННЯ КІЛЬВАТЕРНОГО ПОТЕНЦІАЛУ ЗА РАХУНОК ПОЛЯРИЗАЦІЙНОЇ ВЗАЄМОДІЇ КАНАЛЬОВАНОГО ЕЛЕКТРОНА З ТВЕРДОТІЛЬНОЮ ПЛАЗМОЮ ІОННОГО КРИСТАЛА

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Під дією електричного та магнітного полів, створених рухомим у режимі площинного каналювання релятивістським електроном, розраховуються залежні від часу наведені електричні дипольні моменти іонів кристалічного середовища. На підставі врахування принципу причинності для цієї системи диполів знаходиться сумарний скалярний потенціал, який можна назвати "кільватерним потенціалом". Показується, що цей потенціал у деяких кристалографічних напрямках (наприклад, в іонних кристалах) може приводити до суттєвого зворотного впливу на режим руху канальованої частинки.