FW PROPAGATION AND ABSORPTION IN REACTOR PLASMAS IN THE ICR FREQUENCY RANGE AT THE QUASIPERPENDICULAR PROPAGATION

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On the base of the numerical model, taking into account the strong transverse and longitudinal dispersion of plasma, the absorption and dispersion of the Fast Mode of fast magnetosonic wave (FW) near ICR harmonics $\omega = n\omega_{ci}$ at the quasiperpendicular propagation regime were investigated. It was shown that in the region of small longitudinal refractive index values $N_{\parallel}$ there appears the additional absorption that provides the value of optical thickness τ being constant in the entire region of additional absorption. At first and second harmonics the additional absorption can be interesting for FW plasma heating in the reactor-size devices. For higher harmonics it can be important for HF diagnostic purposes and for investigation of astrophysical plasmas.

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INTRODUCTION

Relativistic effects, associated with fast electrons mass increasing, have an appreciable influence on the absorption and dispersion of plasma waves in the ECR frequency range, even at relatively low electron temperatures ($T_e \approx 1\,\mathrm{keV}$) [1]. It is convenient to divide these effects into two groups in accordance with the method of their theoretical description.

The first method involves the use of the relativistic generalization of the Vlasov kinetic equation to describe the motion of electrons and the relativistic Maxwellian distribution of background electrons in this equation to derive the plasma dielectric tensor [2]. The evaluation of this tensor leads to the use of a fully relativistic plasma dispersion functions (PDFs). These PDFs describe the longitudinal (with respect to magnetic field) movement of electrons, i.e., the longitudinal spatial plasma dispersion, in part, the relativistic effects associated with the longitudinal Doppler broadening of the cyclotron zones and shift of those zones into the "red" side relatively to the corresponding resonances [3]. These longitudinal effects are particularly strong at the regime of propagation almost perpendicular to the magnetic field (at the quasi-perpendicular propagation regime), that corresponds to the direction of the most effective synchrotron radiation of fast plasma electrons.

The second method involves the use of Maxwell's equations (or the dispersion equation for the case of homogeneous plasma) in addition to the plasma dielectric tensor. In accordance with the Lorentz transform these equations describes relativistic effects associated with the transverse motion of the plasma particles, i.e. the effects of transverse spatial dispersion. These effects are particularly strong in the regimes with high wave harmonics, since the higher harmonic number, the faster electron moves in order to maintain the conditions of cyclotron resonance.

Influence of relativistic effects on the motion of more inert ions on the properties of plasma waves in the ICR frequency range seems much more exotic phenomenon, since the main relativistic parameter $\mu = c^2/V_T^2 = c^2 m_0/T$ (m_0 – the rest mass of plasma particles) is very high for the ions. Indeed, $\mu_i/\mu_e \sim 2\cdot 10^3$ and, therefore, for ions relativistic effects should be on three orders smaller than for electrons. This argument is usually used to neglect the ions emission in comparison with the electrons emission when the total plasma synchrotron radiation is estimated.

However even for ions, relativistic effects may appear for FW at the fundamental ion cyclotron resonance in the regime of oblique propagation in plasma with temperature, typical for tokamak JET [4]. Accurate description of the FW absorption and dispersion in this case requires a more accurate taking into account the effects of strong transverse spatial dispersion, namely FLR_4-approximation (approximation in the finite Larmor radius up to the 4th order). Analysis in the framework of this approach showed that the absorption of FW at this cyclotron resonance has two-peak structure, if it is represented as a function of the angle of propagation. An additional peak at small angles of propagation is well described in the FLR_4-approximation, while commonly used FLR_2-approximation in the region of this peak gives unphysical effects such as, for example, the negative dissipation of energy. Effect of the appearance of the additional peak in the absorption of the fundamental ion cyclotron resonance is determined by the anti-Hermitian part of the dispersion equation for FW or, more precisely, by the structure of the elements of the plasma dielectric tensor. But the structure of these elements is similar for any of harmonic number, $n > 0$. Therefore, this effect can occur at higher cyclotron resonances in the corresponding $FLR_{2(n+1)}$-approximation as well. However calculations, accounting only the effects of strong transverse spatial dispersion, for higher harmonics in the quasi-perpendicular propagation regime may not be accurate enough because they also require accurate taking into account the longitudinal spatial dispersion.

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The main purpose of this work is to perform accurate calculations, taking into account both strong transverse and strong longitudinal spatial dispersions, for FW absorption and dispersion near the fundamental and higher harmonics in the quasi-perpendicular propagation regime.

1. DISPERSION EQUATION FOR FW

Dispersion equation for FW excited with the frequency ω and longitudinal refractive index $N_{\parallel} = k_{\parallel} c/\omega$ in the ICR frequency range can be presented in the form

$$(\varepsilon_{11} - N_{\parallel}^2)N_{\perp}^2 - (\varepsilon_{11} - N_{\parallel}^2)(\varepsilon_{22} - N_{\parallel}^2) - \varepsilon_{12}^2 = 0\,, \qquad (1)$$

where ε_{ik} are components of a plasma dielectric tensor in FLR_{∞} – approximation; $N_{\perp} = k_{\perp} c/\omega$ is the transverse refractive index. Neglecting by the electron mass or connection with slow mode (SW) of fast magneto-sonic wave can be justified in the same manner as in the case of resonance $\omega = \omega_{ci}$ in [4]. Equation (1) is a transcendental equation in a complex region for a square of the transverse refractive index, $N_{\perp}^2$, which can be solved numerically. However, in the case of quasi-transverse FW propagation it is also possible the analytical approach based on the perturbation method in the small parameters $\lambda_i = (k_{\perp}\rho_i)^2$ and $N_{\parallel}$ that allows one to reduce the equation (1) to a polynomial of $n+1$ degree:

$$P_{n+1}(N_{\perp}^2) = 0, \qquad (2)$$

where n is the number of the resonance harmonic. In more detail, polynomial (2) can be represented as

$$-\frac{\mu_i}{2^n (n+1)!}\left(\frac{N_A}{W}\right)^2 Z_{|n|+2+3/2}^{-n}\left(\frac{N_{\perp}\rho_i\omega}{c}\right)^{2(n+1)} +$$
$$N_{\perp}^2\left[1 + \Delta_e^{(0)} + \Delta_i^{(1)} - 2(\varepsilon_1^{(1)} - \varepsilon_2^{(1)}) + \left(\varepsilon_1^{(0)} - \varepsilon_2^{(0)} - N_{\parallel}^2\right)2\lambda_i/n\right] =$$
$$= \left(N_F^{(n-1)}\right)^2\left(1 + \Delta_e^{(0)} + \Delta_i^{(1)} - 2(\varepsilon_1^{(1)} - \varepsilon_2^{(1)})\right), \qquad (3)$$

where $Z_{|n|+k+3/2}^{-n} = Z_{|n|+k+3/2}(a_i, z_{-n}^i, \mu_i)$ is the exact relativistic plasma dispersion function for ions [3], $z_{-n}^i = (\omega - n\omega_{ci})/(\sqrt{2}k_{\parallel}V_{Ti})$, $W = \omega/\omega_{ci}$, $N_A = \omega_{pi}/\omega_{ci}$, $a_i = \mu_i N_{\parallel}^2/2$, $\lambda_i = (k_{\perp}\rho_i)^2$, ρ_i is the ion Larmor radius, $\varepsilon_1 = \varepsilon_{11}$, $\varepsilon_2 = -i\varepsilon_{12}$, values of Δ_e and Δ_i follows from the definition $\varepsilon_{22} = \varepsilon_{11} - (\Delta_e + \Delta_i)N_{\perp}^2$, superscripts in the terms Δ_e, Δ_i ε_1, ε_2 denotes the order of this term in the Larmor expansion. At last $\left(N_F^{(n-1)}\right)^2$ is a solution of the equation

$$N_{\perp}^2\left(1 + \Delta_e + \Delta_i^{(1)} - 2(\varepsilon_1^{(1)} - \varepsilon_2^{(1)})\right) = \frac{\left(\varepsilon_1^{(0)} - \varepsilon_2^{(0)} - N_{\parallel}^2\right)\left(\varepsilon_1^{(n-1)} + \varepsilon_2^{(n-1)} - N_{\parallel}^2\right)}{\varepsilon_1^{(n-1)} - N_{\parallel}^2}\,. \qquad (4)$$

The equation (4) is the classical expression, which was used in the lowest nonzero ("cold") approximation in the parameter λ_i for FW absorption and dispersion (see, for example, [5]).

The roots of the equation (3) describe FW properties and properties of ion Berstein waves (with less level of accuracy) near resonances $\omega = n\omega_{ci}$.

2. CORRECTIONS TO FW ABSORPTION AND DISPERSION FOR QUASI-PERPENDICULAR PROPAGATION AT THE RESONANCES ω = nω$_{ci}$ (n=1, 2, 3)

When n=1, 2, 3 equation (3) is bi-quadratic, bi-cubic and bi-4th degree, respectively, and consequently it can be solved analytically. The dependencies of $Re(N_{\perp}^2)$ and $Im(N_{\perp}^2)$ versus $N_{\parallel}$ are plotted in Figs. 1-3, obtained from dispersion relation (3) for n=1, 2, 3 in the deuterium plasma. The following plasma parameters have been considered: $B_0 = 5\,T$, $n_e = 2\cdot 10^{14} cm^{-3}$, $T_i = T_e = 40\,keV$, which correspond to the ITER-tokamak [6]. In those figures the solutions of the equations (3) and (4) are presented for comparison. Eq. (4) is the dispersion relation corresponding to the lowest significant order approximation in the parameter λ_i.

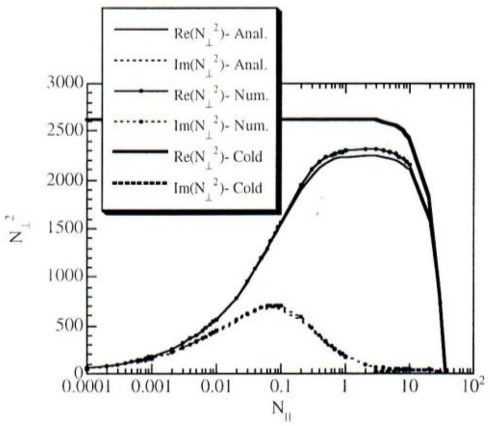

Fig. 1. The dependencies of $Re(N_{\perp}^2)$ *and* $Im(N_{\perp}^2)$ *versus* $N_{\parallel}$ *for n=1*

As it follows from these pictures, the solutions of equations (3) and (4) coincide when $N_{\parallel} > 20$ (for n=1), $N_{\parallel} > 1$ (for n=2) and $N_{\parallel} > 0.1$ (for n=3). When the value of $N_{\parallel}$ decreases in the regions $N_{\parallel} < 20$ (for n=1), $N_{\parallel} < 1$ (for n=2) and $N_{\parallel} < 0.1$ (for n=3) the imaginary part of the square of perpendicular index, $Im(N_{\perp}^2)$, corresponding to the solution of equation (3), begins first gradually and stronger and stronger to be higher in comparison with the cold value of $Re(N_{\perp}^2)$, obtained from equation (4). These curves achieve maximums at the values $N_{\parallel} = 0.2$ (for n=1), $N_{\parallel} = 0.02$ (for n=2) and $N_{\parallel} = 0.002$ (for n=3), respectively. $Im(N_{\perp}^2)$ tends to 0 for lower values of $N_{\parallel}$.

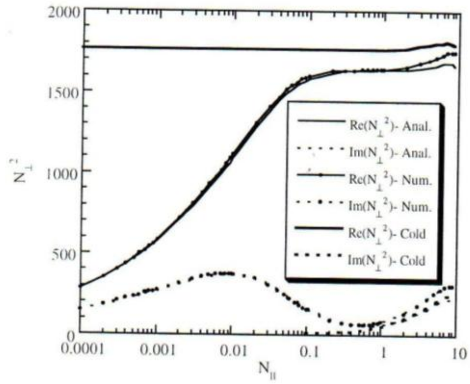

Fig. 2. The dependencies of $\mathrm{Re}(N_\perp^2)$ *and* $\mathrm{Im}(N_\perp^2)$ *versus* $N_{\parallel}$ *for n=2*

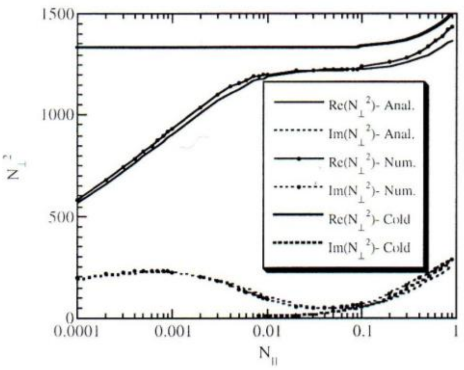

Fig. 3. The dependencies of $\mathrm{Re}(N_\perp^2)$ *and* $\mathrm{Im}(N_\perp^2)$ *versus* $N_{\parallel}$ *for n=3*

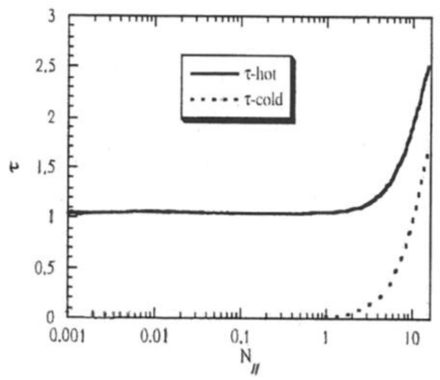

Fig. 4. The optical thickness versus $N_{\parallel}$ *for n=1*

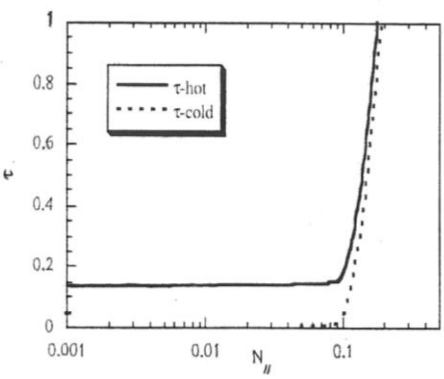

Fig. 5. The optical thickness versus $N_{\parallel}$ *for n=2*

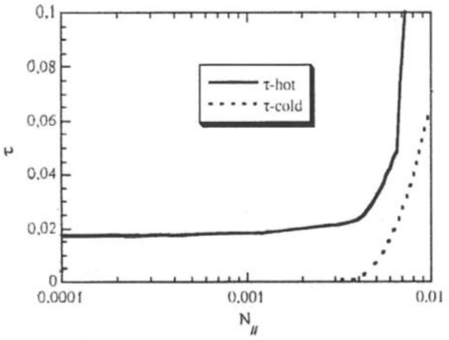

Fig. 6. The optical thickness versus $N_{\parallel}$ *for n=3*

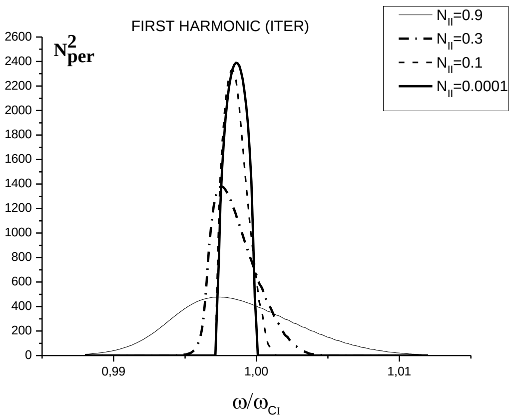

Fig. 7. The Doppler broadening for some values $N_{\parallel}$

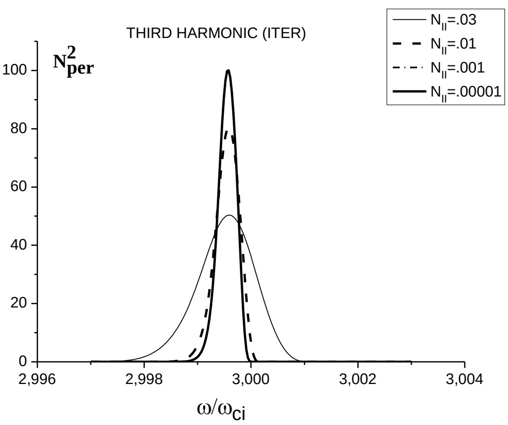

Fig. 8. The Doppler broadening for some values $N_{\parallel}$

The real parts of the squared perpendicular index, $\mathrm{Re}(N_\perp^2)$, obtained from equation (3) begin to differ from the “cold” ones near the gap of values of $N_{\parallel}$ where $\mathrm{Im}(N_\perp^2)$ present maximums. When $N_{\parallel}$ tends to 0 $\mathrm{Re}(N_\perp^2)$ sharply decreas and tend to 0 with $\mathrm{Im}(N_\perp^2)$.

To estimate the enhancement of absorption, it is useful to calculate the optical thickness $\tau = \int \mathrm{Im}(k_\perp)dR$ assuming that magnetic field changes as 1/R. In Figs. 4-6 the dependencies of τ on $N_{\parallel}$ are presented for n=1 (see Fig. 4), for n=2 (see Fig. 5) and for n=3 (see Fig. 6) for ITER tokamak ($R_0 = 8$ and plasma parameters correspond to Figs. 1-3).

It can be seen in Figs. 4-6 that when $N_{||} < 5$ for n=1, $N_{||} < 0.7$ for n=2 and $N_{||} < 0.07$ for n=3, the optical thickness is given by additional absorption and takes the values τ=1.1, τ=0.14, τ=0.02, respectively. Thus, if the longitudinal spatial dispersion is exactly taken into account, i.e., instead non-relativistic PDF the exact relativistic PDFs are used then optical thickness retains these values at a constant level for all the values $N_{||}$ in the entire region of localization of additional absorption. In Figs. 7, 8 it is also given the Doppler broadening for some $N_{||}$-values in the cases of absorption at the fundamental and third harmonics in order to clarify the retaining these constant values of the optical thickness. When $N_{||} > 20$ for n=1, $N_{||} > 1$ for n=2, and $N_{||} > 0.1$ for n=3 additional absorption is very small and optical thickness will be given by the solutions of equation (4). The calculations show that values of $N_{||}$ defining the boundary for the apparition of additional absorption can be obtained from the equation $2\sqrt{a_i}\lambda_i^n\mu = 1$. It can be also seen in Figs. 2, 4, 6 that, due to the existence of this additional absorption, FW may totally be absorbed in less than a single pass for n=1, in 7 passes for n=2 and in 50 ones for n=3, for the corresponding values of $N_{||}$. This means that this effect can be interesting for FW heating plasmas confined in a reactor size-device at first and second harmonics. For higher harmonics, the additional absorption is rather small but this effect can be still important for HF diagnosis and for astrophysical plasmas.

CONCLUSIONS

1. Taking into account the longitudinal relativistic effects additionally to the transverse relativistic effects is very important at the quasi-perpendicular FW propagation since in the region of small $N_{||}$ there appears additional absorption that provides the value τ being constant in the entire region of additional absorption.

2. The additional absorption due to the strong longitudinal and transverse dispersion at first and second harmonics can be interesting for FW plasma heating in the reactor-size devices. For higher harmonics, it can be still important for HF diagnostic purposes and for astrophysical plasmas as well.

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РАСПРОСТРАНЕНИЕ И ПОГЛОЩЕНИЕ БВ В РЕАКТОРНОЙ ПЛАЗМЕ В ИЦР-ДИАПАЗОНЕ ЧАСТОТ ПРИ КВАЗИПЕРПЕНДИКУЛЯРНОМ РАСПРОСТРАНЕНИИ

С.С. Павлов

Дисперсия и поглощение быстрой моды быстрой магнитозвуковой волны (БВ) в области ионных циклотронных гармоник $\omega = n\omega_{ci}$ исследованы на основе численной модели, учитывающей сильную продольную и поперечную дисперсии плазмы. Показано, что в области малых продольных замедлений появляется дополнительное поглощение, которое обеспечивает постоянное значение оптической толщи во всей области проявления дополнительного поглощения. На первой и второй гармониках такое поглощение может быть интересно для нагрева реакторной плазмы. Для более высоких гармоник оно может быть важным для ВЧ-диагностики, а также для исследования астрофизической плазмы.

ПОШИРЕННЯ І ПОГЛИНАННЯ ШХ У РЕАКТОРНОЇ ПЛАЗМІ В ІЦР-ДІАПАЗОНІ ЧАСТОТ ПРИ КВАЗІПЕРПЕНДІКУЛЯРНОМУ РОЗПОВСЮДЖЕННІ

С.С. Павлов

Дисперсія і поглинання швидкої моди швидкої магнітозвукової хвилі (ШХ) в області іонних циклотронних гармонік $\omega = n\omega_{ci}$ досліджені на основі чисельної моделі, що враховує сильну поздовжню і поперечну дисперсії плазми. Показано, що в області малих поздовжніх уповільнень з'являється додаткове поглинання, яке забезпечує постійне значення оптичної товщі у всій області прояви додаткового поглинання. На першій і другій гармоніках таке поглинання може бути цікавим для нагріву реакторної плазми. Для більш високих гармонік воно може бути важливим для ВЧ-діагностики, а також для дослідження астрофізичної плазми.