EIGEN DIPOLAR ELECTROMAGNETIC WAVES OF COAXIAL NON-UNIFORM PLASMA-METALL WAVEGUIDE WITH EXTERNAL AZIMUTH MAGNETIC FIELD

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The purpose of this paper is to study the electrodynamic properties of eigen dipolar electromagnetic waves of coaxial metal waveguide filled by slightly axially non-uniform and strongly radially non-uniform cold dissipative plasma immersed in non-uniform azimuth external magnetic field. The influence of external azimuth magnetic field, geometric parameters of the waveguide structure and the plasma electron collisions on the dispersion properties, spatial attenuation and radial wave field structure of these waves for different radial plasma density profiles is studied.

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INTRODUCTION

At present time the intensive theoretical and experimental studies of plasma, produced and sustained by the eigen traveling along the waveguide structure electromagnetic waves and properties of these waves are carried out in the leading scientific laboratories of the world [1]. These studies are stipulated by the fact that such waveguide systems of different radial structure are widely used in the devices of plasma electronics [2] and also as the discharge chambers in plasma-technological processes [3]. The properties of eigen waves and plasma are simultaneously determined by different factors but the external magnetic field and the azimuth structure of the electromagnetic wave considered exert substantial influence on it [2, 3]. The eigen dipolar waves with azimuth wave number m = 1 are often used for various technological applications in plasma electronics and for sustaining discharges in cylindrical waveguides [4]. At the same time coaxial waveguide structures are widely used in different technological applications [5]. The properties of eigen dipolar waves in cylindrical waveguide structure are studied well but propagation of such waves in long strong radially non-uniform coaxial structures are studied insufficiently. These facts determine the urgency of the presented study.

1. BASIC EQUATIONS

The considered waveguide structure is composed of the central cylindrical metal conductor of radius R_1 that is immersed in the cylindrical plasma layer with outer radius R_2. The vacuum gap $(R_2 < r < R_3)$ separates the plasma layer from outer metal wall with radius R_3. The direct current J_z flows along the central conductor and produces the radially non-uniform azimuth magnetic field $H_0(r)$. Plasma is considered in the hydrodynamic approach as cold slightly dissipative medium with constant effective collision frequency $\tilde{\nu}$. It is supposed that plasma density n varies slightly along the plasma column at the distances of wavelength order [1, 3, 4, 6]. It is also considered that plasma density radial profile $n(r)$ along all plasma column has the form: $n(r) = n(r_{\max})\exp\left(-\delta(r - r_{\max})^2 / r_\delta^2\right)$. The non-uniformity parameter δ describes the plasma density shape and varies from $\delta = 0$ (radially uniform plasma) to $\delta = 1$ (completely radially non-uniform plasma). The parameter $r_{\max}$ is radial coordinate, where plasma density reaches the maximum, and the parameter r_δ characterizes the width of profile [7]. In this research it is supposed that $r_{\max}$ corresponds to the centre of plasma layer and $r_\delta = 0.1(R_2 - R_1)$. Under these assumptions the permittivity tensor of collisional plasma in azimuth magnetic field $\varepsilon_{i,j}$ was obtained in [8] with components $\varepsilon_{1,2,3}$ which depend on radial position r [2] and slightly depend on axial coordinate.

The dipolar wave propagation is governed by the system of Maxwell equations that in cylindrical coordinates (r, φ, z) possesses the solutions in the form:

$$A(\vec{r}) = A(r,z)\exp\left(i[\int_{z_0}^{z} k_3(z')dz' + m\varphi - \omega t] \right), \quad (1)$$

where A denotes the electric and magnetic wave field component; ω is given wave frequency; k_3 is axial wave number. Due to slight plasma density changing in axial direction at the distances of wavelength order we follow the authors of [6] and neglect all the terms proportional to $\Im^{-1}\left(\partial\Im/\partial z\right)$ and higher derivatives, where $\Im$ denotes all the quantities which slightly depend on z.

For the plasma layer $(R_1 < r < R_2)$ one can obtain the system of ordinary differential equations that describes the radial distribution of tangential wave field components and two algebraic equations which describe the radial wave field components [8]. For arbitrary parameters of plasma region and waveguide structure the solution of this system can be found with the help of special numerical methods.

In the vacuum region $(R_2 < r < R_3)$ the corresponding system of Maxwell equations can be solved analytically [8] and wave field components can be expressed in terms of linear combination of modified Bessel functions [2]. Values $C_{1,2,3,4}$ which are present in the expressions for wave field components can be obtained with the help of boundary conditions consisting in the continuity of tangential wave field components at plasma-vacuum interface. The values of plasma wave field components at plasma-vacuum interface ($r = R_2$), can be obtained by the direct numerical solution of the system of differential equations [8].

The analogue of the local dispersion equation that connects ω and k_3 can be obtained from the boundary conditions at $r = R_3$ and can be written in the following form:

$$\begin{cases} C_1 I_m(\kappa_v R_3) + C_2 K_m(\kappa_v R_3) = 0 \\ C_3 I_m^{'}(\kappa_v R_3) + C_4 K_m^{'}(\kappa_v R_3) = 0 \end{cases}, \quad (2)$$

where $\kappa_v^2 = k_3^2 - k^2$, $k = \omega / c$ and C_i are the values which are present in the expressions for vacuum wave field components and are connected with wave field components in plasma due to boundary conditions.

2. MAIN RESULTS

The dipolar wave $m = 1$ possesses all six components of electromagnetic wave field, so the solution of the problem is rather hard and bulky. First the influence of direct current value and waveguide geometric parameters on the phase properties of the wave for the case of collisionless plasma ($\nu = \tilde{\nu} / \omega = 0$) is studied. The dependence of the normalized parameter $\mu = \omega / \omega_p(r_{\max})$ ($\omega_p(r_{\max})$ is electron plasma frequency) that depends on the plasma density that slightly varies along the cylindrical plasma column on the normalized axial wavenumber $x = \mathrm{Re}(k_3) R_2$ for different normalized direct current values $j = eJ_z / (2mc^3)$ is shown on the Fig. 1. In the considered case the dispersion equation (2) possesses the number of solutions. Three solutions of the eq. (2) with the larger μ values for different direct current values are shown on the Fig. 1.

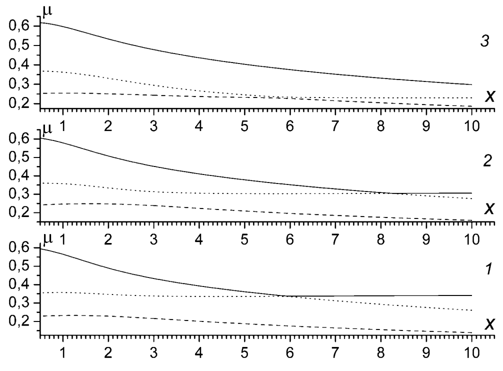

Fig. 1. The solutions of the equations (2) $\mu = \omega / \omega_p$ on the dimensionless wave number $x = \mathrm{Re}(k_3) R_2$ under the parameters values: $R_1 / R_2 = 0.1$; $R_2 \omega / c = 0.5$; $R_3 / R_2 = 2.0$. The numbers at the upper right corner of the graph correspond to such current values: 1 – $j = 1.5$; 2 – $j = 2.0$; 3 – $j = 4.0$

The presented results correspond to the eigen waves with $\omega < \omega_p$. It's necessary to mention that the presented solutions don't intersect with each other. These solutions come close to each other, but they are essentially different modes with different radial wave field structure.

Solutions which are represented on Fig. 1 correspond to the eigen modes which have different radial wave field structure especially in plasma region (Figs. 1,a,b) and different dependence of phase and group velocities on the wavenumber. The mode with the lower dimensionless frequency μ under the fixed axial wavenumber value possesses smaller scale length of spatial wave field oscillations in radial direction (see Figs. 1,a; 1,b).

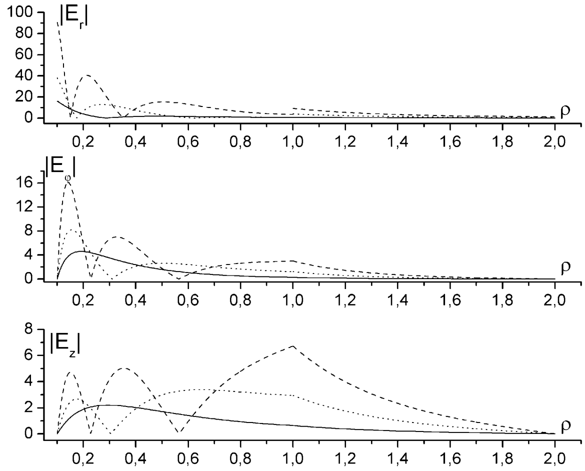

Fig. 1,a. The electric wave field components (normalized on the $H_\varphi(R_1)$) for the first tree solutions of the equations (2). The score parameters and numbering of the curves are the same as for Fig. 1, $\rho = r / R_2$

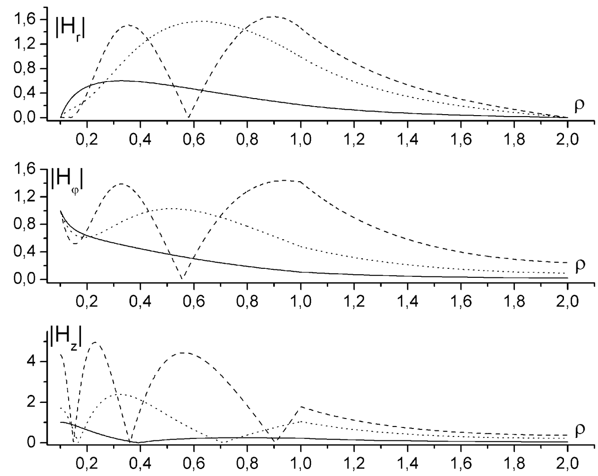

Fig. 1,b. The magnetic wave field components (normalized on the $H_\varphi(R_1)$) for the first tree solutions of the equations (2). The score parameters and numbering of the curves are the same as for Fig. 1, $\rho = r / R_2$

The radial distribution of the normalized wave field components for the first three solutions for $x = 2.0$ are presented on the Fig. 1,a (electric field components) and Fig. 1,b (magnetic field components). The presented wave field components are normalized on the $H_\varphi(R_1)$. The numerical study shows that the considered eigen dipolar wave $m = 1$ is neither pure surface one nor pure volume wave. This wave demonstrates complex radial structure that corresponds to the pseudo-surface wave according to the classification presented in [3]. This means that wave field is composed from the surface and volume radial mode. In our study we cannot separate this radial modes but the effect is analogous to those in [3].

It is obtained that each of three displayed solutions has different type of dependence of the frequency on the normalized direct current value j (see subplots 1-3 on Fig. 1). While the direct current j increases its value from $j = 1.5$ up to $j = 4.0$ the first solution (solid curve on Fig. 1) and the third solution (dashed curve) increases its phase velocity on the whole range of wavenumber x (this can be obtained from subplots 1-3). The second solution (dotted curve) has different type of dependence. In the range of small ($x \le 1.2$) and large ($x \ge 7.4$) wavenumber values the increase of j value from 1.5 up to 4.0 leads to the increase of μ for the fixed x value. In the range $1.2 < x < 7.4$ the increase of the current j leads to the decrease of wave phase velocities.

It is necessary to mention that the increase of direct current j leads to the essential changing of the solutions behavior. So, when j raises up to 4.0 the first solution (solid curve on Figs. 1; 1,a; 1,b) becomes well separated from the others.

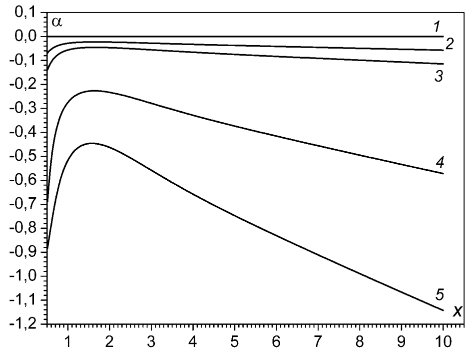

Fig. 2. The dependence of dimensionless attenuation coefficient α on the dimensionless wave number x for the first root (solid curve on Fig. 1). Numbers just near the curves correspond to the different ν values: 1 – $\nu = 0$; 2 - 0.005; 3 – $\nu = 0.01$; 4 – $\nu = 0.05$; 5 – $\nu = 0.1$. Other parameters are the same as for Fig. 1, except $j = 4.0$

The influence of geometric parameters of the waveguide structure on the dipolar wave dispersion and attenuation is studied as well. The width of vacuum gap that is characterized by the parameter $\eta = R_3 / R_2$ strongly influences mainly the wave dispersion in the range of small and moderate values ($\eta < 2.0$). When parameter η grows up to rather large values ($\eta > 2$) it has negligible influence on the dispersion. The numerical calculations show that the variation of parameter η slightly influences on the attenuation coefficient α.

The influence of effective electron collision frequency ν on the spatial attenuation coefficient $\alpha = \mathrm{Im}(k_3)R_1$ is studied for the first solution (solid curve on the Fig. 1) and is presented at the Fig. 2. It is obtained that the increase of the ν value leads to the increase of the absolute value of wave attenuation coefficient. It is necessary to mention that characteristic feature of this solution is negative value of spatial attenuation coefficient α. This fact means that this wave cannot be effectively used for gas discharge sustaining, but the waves of such type are widely used in plasma electronics [2].

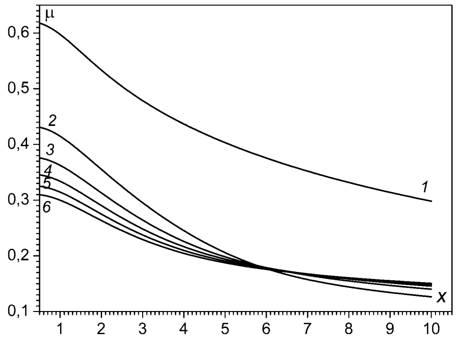

Fig. 3. The dependence of frequency μ on the wave number x for the first root (solid curve on fig. 1). Numbers of the curves correspond to the non-uniformity parameter δ value: 1 – $\delta = 0$; 2 – $\delta = 0.2$; 3 – $\delta = 0.4$; 4 –$\delta = 0.6$; 5 –$\delta = 0.8$; 6 –$\delta = 1$. Other parameters are the same as for Fig. 1, except $j = 4.0$ and $\nu = 0.001$

The influence of plasma density non-uniformity on the dispersion and attenuation properties at gradually increase of the non-uniform parameter δ from 0 up to 1 is studied. The results of numerical study of plasma density non-uniformity on the wave dispersion for some parameter δ values are presented on the Fig. 3. It is shown that the dispersion has different behavior in the case of uniform (curve 1) and non-uniform plasma (curves 2-6). For the non-uniform plasma the increase of the non-uniformity parameter δ leads to the decrease of the dipolar wave phase velocity in the region of short wave lengths (curves 2-6 in the range $x > 6$). In the region of long wave lengths (curves 2-6 in the range $x < 5$) the increase of parameter δ leads to the increase of the phase velocity.

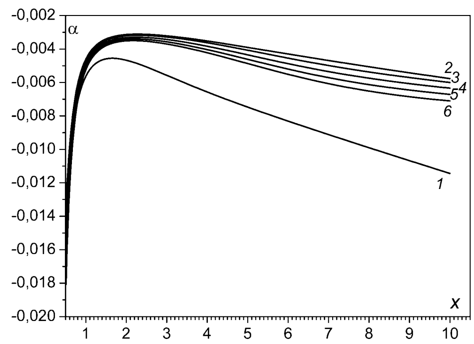

Fig. 4. The dependence of spatial attenuation coefficient α on the wave number x for the first root (solid curve on Fig. 1). Numbers near the curves and parameters values are the same as for Fig. 3

The results of studying the influence of radial plasma density non-uniformity on the wave attenuation are presented on the Fig. 4. Similarly to the dispersion of dipolar wave the spatial attenuation coefficient α has different behavior in the case of uniform (curve 1) and non-uniform plasma (curves 2-6). The absolute value of the coefficient α reaches the minimum at axial wave number $x \approx 2$.

In the region of long wave lengths ($x < 2$) it is observed the strong dependence of the increase of absolute value coefficient α on the wave length. In the region of short wave lengths ($x > 2$) $|\alpha|$ has smooth dependence on the wave length. It is obtained that the $|\alpha|$ takes on some minimum value at some $\delta \neq 0$ value. This can be explained by the fact that under given parameters the electric and magnetic field strength raises its maximum in the region where plasma density tends to zero. Further growth of the parameter δ value leads to the gradual concurrence of the radial positions of the maximum values of plasma density and wave field amplitude. This process causes the increase of spatial wave attenuation due to the Joule wave energy losses under plasma density non-uniformity parameter growth.

CONCLUSIONS

This paper presents the developed new method of calculation of eigen waves of coaxial waveguides partially filled by the dissipative non-uniform plasma immersed in azimuth external magnetic field. It is shown that the number of eigen dipolar modes can propagate in the considered coaxial waveguide structure. These modes differ by their phase and spatial attenuation properties and by their radial wave field structure. It is shown that the solution with the greatest μ value is backward wave with poor trend for gas discharge sustaining in long coaxial waveguide structures, but it has good prospect of usage in wave amplifiers. Other solutions possess the ranges of forward propagation that depends mainly on the external azimuth magnetic field value and demands on further study.

REFERENCES

1. M. Moisan, J. Hubert, J. Margot, Z. Zakrewski. The development and use of surface-wave sustained discharges for applications // *Advanced Technologies Based on Wave and Beam Generated Plasmas* / Ed. by H. Shluter, A. Shivarova. Amsterdam: "Kluwer Academic Publisher", 1999.

2. A.N. Kondratenko, V.M. Kuklin. *Osnovy Plasmennoy Electriniky*. M: "Energoatomizdat", 1988 (in Russian).

3. I. Zhelyazkov, V. Atanasov. Axial structure of low-pressure high-frequency discharges sustained by travelling electromagnetic surface waves *// Physics Reports.* 1995, v. 225, p. 79-201.

4. M. Djourelova, T. Petrova, I. Ghanashev, I. Zhelyazkov. Axial structure of a shielded plasma column sustained by a dipolar electromagnetic wave // *J. Phys. D: Appl. Phys.* 1993, v. 26, p. 1601-1610.

5. P.I. Markov, I.N. Onishchenko, G.V. Sotnikov. Uhf-generation in a coaxial slowing down structure filled with plasma // *Problems of Atomic Science and Technology. Series "Plasma Physics"*. 2002, v. 5(8), p. 86-88.

6. G.G. Lister, T.R. Robinson. Strongly damped surface waves in plasmas. I: the wkb approximations // *J. Phys. D: Appl. Phys.* 1991, v. 24, p. 1993-1999.

7. N.I. Karbushev, Yu.A. Kolosov, A.I. Polovkov, et al. Vluyanie neodnorodnosti plasmy na transportirovky mikrovolnovogo islucheniya cherez koaksialny plasmmenny volnovod // *Physica Plasmy*. 1992, v. 18, № 8, p. 1027-1038 (in Russian).

8. V.P. Olefir, A.E. Sporov. Symmetric and dipolar electromagnetic waves in coaxial structure filled by non-uniform dissipative plasma with azimuth magnetic field // *Problems of Atomic Science and Technology. Series "Plasma Physics" (71)*. 2011, № 1, p. 47-49.

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СОБСТВЕННЫЕ ДИПОЛЬНЫЕ ЭЛЕКТРОМАГНИТНЫЕ ВОЛНЫ КОАКСИАЛЬНЫХ ПЛАЗМЕННО-МЕТАЛЛИЧЕСКИХ ВОЛНОВОДОВ С НЕОДНОРОДНОЙ ПЛАЗМОЙ ВО ВНЕШНЕМ АЗИМУТАЛЬНОМ МАГНИТНОМ ПОЛЕ

Н.А. Азаренков, В.П. Олефир, А.Е. Споров

Исследованы электродинамические свойства собственных дипольных электромагнитных волн, распространяющихся в коаксиальном металлическом волноводе, заполненном слабо неоднородной в аксиальном и сильно неоднородной в радиальном направлениях холодной диссипативной плазмой, находящейся во внешнем неоднородном азимутальном магнитном поле. Изучено влияние величины внешнего азимутального магнитного поля, геометрических параметров волноводной структуры, частоты столкновений электронов на дисперсионные свойства, пространственное затухание и радиальную структуру поля волны для различных радиальных профилей плотности плазмы.

ВЛАСНІ ДИПОЛЬНІ ЕЛЕКТРОМАГНІТНІ ХВИЛІ КОАКСІАЛЬНИХ ПЛАЗМОВО-МЕТАЛЕВИХ ХВИЛЕВОДІВ З НЕОДНОРІДНОЮ ПЛАЗМОЮ В ЗОВНІШНЬОМУ АЗИМУТАЛЬНОМУ МАГНІТНОМУ ПОЛІ

М.О. Азарєнков, В.П. Олефір, О.Є. Споров

Досліджено електродинамічні властивості власних дипольних електромагнітних хвиль коаксіального металевого хвилеводу, заповненого слабко неоднорідною в аксіальному напрямку та сильно радіально неоднорідною холодною дисипативною плазмою, що знаходиться в зовнішньому неоднорідному азимутальному магнітному полі. Вивчено вплив величини зовнішнього азимутального магнітного поля, геометричних параметрів хвилеводної структури, частоти зіткнень електронів на дисперсійні властивості, просторове загасання і радіальну структуру поля хвилі для різних радіальних профілів густини плазми.