

# THE INFLUENCE OF THE ANISOTROPIC TEMPERATURE OF THE ELECTRON GAS ON ENERGY LOSSES OF A CHARGED PARTICLE IN A PLASMA

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Analytical expressions of real and imaginary parts of the dielectric susceptibility of the electron gas with an anisotropic velocity distribution are obtained within the quantum-field method. Energy losses of the projectile charged particle in an electron gas are calculated in the cases of an isotropic velocity distribution of electron gas and of the longitudinal motion in an medium with anisotropic temperature. The numerical results are confirmed by analytical expressions in the low temperatures approximation.

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## INTRODUCTION

Experimental studies in nuclear physics and elementary particle physics is closely related to the use of beams of charged particles. Almost always it's very important that beams involved in such experiments was monochromatic and well collimated, i.e. they have a small velocity spread or, in other words, the low temperature. There are a lot of methods to cool the beams. The method which allows produce beams of heavy charged particles with a record small momentum spread is electron cooling. Thus, for example, a cooled by electrons beam of antiprotons with momentum spread  $\Delta p/p \sim 10 \dots 5$  is planned to use in the experiment of researching of the strong interaction in collisions of protons and antiprotons in HESR (High Energy Storage Ring, Germany, FAIR Collaboration) [1].

A beam of heavy particles is mixed with the cold electron gas at a straight section of the storage ring. Due to Coulomb collisions the beam loses some of its heat energy. Originally, the theory of pair Coulomb collisions was used to describe the electron cooling [2-4]. Later, this theory was supplemented by a dielectric model which takes into account the collective effects at large impact parameters. Total energy losses were calculated by cross linking of results from both approaches.

In the papers [5, 6] quantum field method was proposed to describe energy losses of a massive charged particle in an electron gas. The quantum field method is more general and includes both the theory of binary collisions and plasma method as special cases, and does not use the phenomenological parameters.

The quantum field method allows to take into account the influence of the temperature of the electron gas on energy losses of a charged particle. The electrons can have an isotropic or an anisotropic velocity distribution. The last one is very important because during of electron cooling the projectile particle slows in the electron gas with anisotropic temperature which is, matter-of-course, a consequence of acceleration of electron gas (the Liouville's theorem).

An anisotropic velocity distribution of the electrons is one of the main reasons of, so-called, fast electron

cooling [4]. The effect of enhancing of energy transfer from the stored beam of massive charged particles to the electron gas is observed.

In this paper, the analytical expressions of the dielectric susceptibility of the electron gas with isotropic and anisotropic velocity distribution is obtained. Calculations are made for the energy loss of heavy charged particles in the approximation of low temperature electron gas. Energy losses in the electron gas with the temperature are numerically calculated. The dependence of energy losses of the massive charged particle on its velocity is presented. Numerical results were compared with approximate expressions.

## 1. ISOTROPIC VELOCITY DISTRIBUTION

### 1.1. POLARIZATION OPERATOR

In the frame of the quantum-field theory energy losses of the projectile particle in the electron gas is determined by equation (system of units:  $m_e = \hbar = e^2 = 1$ ) [5]

$$\frac{dE}{dt} = \frac{1}{(2\pi)^3} \int d^3q (\varepsilon_{\vec{p}_i} - \varepsilon_{(\vec{p}_i - \hbar\vec{q})}) W_{\vec{q}}, \quad (1)$$

where

$$W_{\vec{q}} = \frac{2V_q^2}{1 - \exp(-\beta\omega)} \text{Im} \frac{\Pi(\vec{q}, \omega)}{1 - V_q \Pi(\vec{q}, \omega)}, \quad (2)$$

$$\varepsilon_{\vec{p}_i} = \frac{p_i^2}{2M_i} \quad (3)$$

is the energy of projectile particle,

$$\hbar\omega = \varepsilon_{\vec{p}_i} - \varepsilon_{\vec{p}_i - \hbar\vec{q}} = \hbar \frac{\vec{p}_i \cdot \vec{q}}{M_i} - \frac{\hbar^2 q^2}{2M_i} \quad (4)$$

is the energy of the transition from a state with momentum  $\vec{p}_i$  to state with momentum  $\vec{p}_i - \hbar\vec{q}$ ,

$$V_q = \frac{4\pi}{q^2} \quad (5)$$

is Fourier component of the interaction potential,

$$\beta = \frac{1}{T} \quad (6)$$

is an inverse temperature (thermal energy,  $k_B = 1$ ),  $M_i$  is a mass of projectile particle;  $\vec{p}_i$  is a momentum of projectile particle;  $q$  is a wave vector;  $\Pi(\vec{q}, \omega)$  is a polarization operator.

The explicit form of the polarization operator  $\Pi(\vec{q}, \omega)$  in the cases of a sufficiently tenuous plasma or low-temperature high-density gas can be graphical represented in the form of a loop (Fig. 1)



Fig. 1. The Feynman diagram of the polarization operator in the one-loop approximation

Each solid line corresponds to the zero-particle Green's function.

In the one-loop approximation the polarization operator can be written as

$$\Pi(\vec{q}, \omega) = \frac{1}{(2\pi)^3} \int d^3p \frac{n_{\vec{p}+\frac{\hbar\vec{q}}{2}} - n_{\vec{p}-\frac{\hbar\vec{q}}{2}}}{\vec{p}\vec{q} - \omega - iv}, \quad (7)$$

where  $n_{\vec{p}}$  is a velocity distribution function.

We shall use the Maxwell distribution

$$n_p = \exp(-\varepsilon_p \beta). \quad (8)$$

Dielectric susceptibility  $\kappa(\vec{q}, \omega)$  is related with the polarization operator:

$$\kappa(\vec{q}, \omega) = -\frac{4\pi e}{q^2} \Pi(\vec{q}, \omega). \quad (9)$$

From equation (1) it follows that for the determination of the energy loss we need to know the real and imaginary part of the polarization operator (dielectric susceptibility).

$$-\frac{dE}{dt} = \frac{e^2 \omega_p^2}{V_i} \frac{1}{\pi^2} \int d^3g \frac{w}{g^2 \left(1 - \exp\left(-\frac{2\delta_0 w}{\tau}\right)\right)} \times \frac{\text{Im } \kappa(\vec{g}, w)}{\left(1 + \text{Re } \kappa(\vec{g}, w)\right)^2 + \left(\text{Im } \kappa(\vec{g}, w)\right)^2}, \quad (10)$$

where  $g = qv_e/\omega_p$ ,  $w = \omega/\omega_p$ ,  $\tau = T_e/m_e v_e^2$  are dimensionless wave vector, frequency, temperature, respectively.

To find the explicit form of the susceptibility one uses the well known solution for the permittivity in the one-dimensional case [7] and generalizes it to the 3-dimensional form. As a result:

$$\text{Im } \kappa(g, w) = \sqrt{\frac{\pi}{2\tau}} \frac{1}{2\delta_0} \frac{1}{g^3} \sum_{j=1}^2 (-1)^j \exp(-\xi_j^2), \quad (11)$$

$$\begin{aligned} \text{Re } \kappa(g, w) &= \quad (12) \\ &= \sqrt{\frac{\pi}{2\tau}} \frac{1}{2\delta_0} \frac{1}{g^3} \sum_{j=1}^2 (-1)^{j+1} \exp(-\xi_j^2) \text{erfi}(\xi_j), \end{aligned}$$

where

$$\xi_j = \frac{1}{g} \frac{1}{\sqrt{2\tau}} (w + (-1)^{j+1} \delta_0 g^2), \quad (13)$$

$$\delta_0 = \frac{\hbar \omega_p}{2m_e v_e^2}. \quad (14)$$

## 1.2. LOW TEMPERATURE CASE

Let consider the case when the velocity of the projectile particle is much higher than the average thermal velocity of the electrons,  $V_i \gg v_e$ . One uses the expansion for large values of the argument of function looks like

$$e^{-x^2} \text{erfi}(x) = \frac{2}{\sqrt{\pi}} \frac{1}{x} \left( \frac{1}{2} + \frac{1}{4x^2} + \frac{3}{8x^4} + \frac{15}{16x^6} \right) \quad (15)$$

$|x| \gg 1.$

Polarization operator in the approximation of low temperatures has the form

$$\begin{aligned} \text{Re } \Pi(q, \omega) &= \\ &= \frac{\omega_p^2}{4\pi e} \frac{q^2}{\omega^2} \frac{1}{\left(1 - \frac{q^2 \hbar q^2}{\omega^2 2m_e^2}\right)} \left[ 1 + v_e^2 \frac{q^2}{\omega^2} \frac{\left(3 + \frac{q^2 \hbar q^2}{\omega^2 4m_e^2}\right)}{\left(1 - \frac{q^2 \hbar q^2}{\omega^2 2m_e^2}\right)^2} + \right. \\ &\quad \left. + 3v_e^4 \frac{q^4}{\omega^4} \frac{\left(5 + 10 \frac{q^2 \hbar q^2}{\omega^2 2m_e^2} + \frac{q^4 \hbar^4 q^4}{\omega^4 16m_e^4}\right)}{\left(1 - \frac{q^2 \hbar q^2}{\omega^2 2m_e^2}\right)^4} \right]. \end{aligned} \quad (16)$$

The well-known in the classical approach expression for the plasma permittivity is obtained by neglecting quantum corrections.

$$\varepsilon(q, \omega) = 1 - \frac{\omega_p^2}{\omega^2} \left[ 1 + 3v_e^2 \frac{q^2}{\omega^2} + 15v_e^4 \frac{q^4}{\omega^4} \right]. \quad (17)$$

The ratio of permittivity and susceptibility is

$$\varepsilon(q, \omega) = 1 - \kappa(q, \omega). \quad (18)$$

At high speed of the projectile particles the imaginary part rapidly tends to zero. It can be neglected.

Substituting equation (18) in equation (10) one can obtain an expression for energy losses in the electron gas in the approximation of low temperatures in the second order on  $v_e/V_i$ :

$$-\frac{dE}{dt} = \frac{e^2 \omega_p^2}{V_i} \left( L_C - \frac{3}{2} \frac{T_e}{m_e V_i^2} \right), \quad (19)$$

where the Coulomb logarithm is  $L_C = \frac{m_e M V_i^2}{(m_e + M) \hbar \omega_p}$ .

## 1.3. NUMERICAL CALCULATION

Compare the expression (19) with the results of the numerical calculation of equation (10).

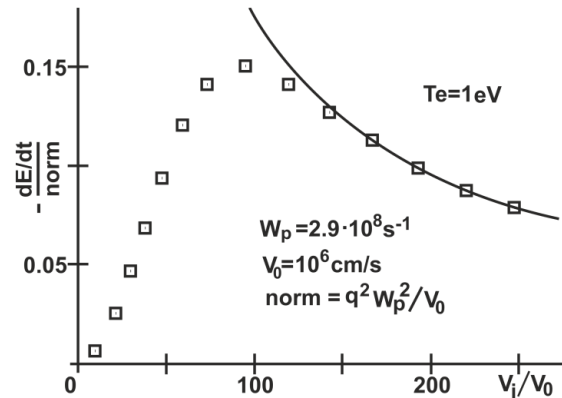


Fig. 2. The dependence of energy losses of heavy charged particles, the normalized by  $e^2 \omega_p^2 / V_0$ , in the electron gas with the average thermal energy  $T_e = 1\text{eV}$  on the speed. Speed is measured in units of  $V_0 = 10^6 \text{cm/s}$ . The solid line corresponds Eq. (19)

Results of numerical calculation of energy losses are presented on Fig. 2. The solid line corresponds to the low temperature approximation. Fig. 2 shows that results of numerical calculations are well described by an analytical expression (19) for sufficiently high velocities of the projectile particle.

Results of numerical calculation of energy losses of heavy charged particles in the electron gas are presented for different electron temperatures.

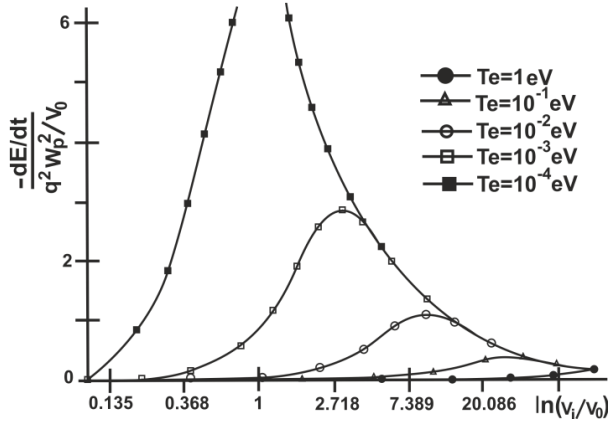


Fig. 3. Energy losses of heavy charged particles, the normalized by  $e^2 \omega_p^2 / V_0$ , in the electron gas for different electron isotropic temperatures on the speed

Curves correspond results with next temperature of electron gas: a) filled circles is  $T_e = 1 \text{ eV}$ ; b) open triangles is  $T_e = 10^{-1} \text{ eV}$ ; c) open circles is  $T_e = 10^{-2} \text{ eV}$ ; d) open squares is  $T_e = 10^{-3} \text{ eV}$ ; e) filled squares is  $T_e = 10^{-4} \text{ eV}$ . Speed is measured in units of  $V_0 = 10^6 \text{ c m/s}$ .

The Fig. 3 shows that energy losses increase and their maximum displaces toward lower velocities with lowering of the temperature of the electron gas. An axis of abscise is logarithm. The value of a plasma frequency equal  $\omega_p = 10^8 \text{ s}^{-1}$  [1].

## 2. ANISOTROPIC VELOCITY DISTRIBUTION

### 2.1. POLARIZATION OPERATOR

In the case of an anisotropic velocity distribution of the electron gas it is convenient to introduce the concept of the transverse and longitudinal temperature (velocity spread):  $T_{e\perp}$  is a transverse temperature,  $T_{e\parallel}$  is a longitudinal temperature. If assume that these two components are independent, then the distribution can be written as [7].

$$n_p = \exp\left(\frac{\mathcal{E}_{p\perp}}{T_{e\perp}}\right) \exp\left(\frac{\mathcal{E}_{p\parallel}}{T_{e\parallel}}\right). \quad (20)$$

Let choose an axis 0 along the direction of the electron beam as a whole then

$$n_p = \exp\left(\frac{p_x^2 + p_y^2}{2m_e T_{e\perp}}\right) \exp\left(\frac{p_z^2}{2m_e T_{e\parallel}}\right). \quad (21)$$

Analytical expressions of the real part and imaginary one of the dielectric susceptibility in the anisotropic case are calculated in the same way to the isotropic distribution:

$$\begin{aligned} \text{Im } \kappa(g, \omega) &= \sqrt{\frac{\pi}{2\tau_{\perp}}} \frac{1}{2\delta_0} \frac{1}{g'} \frac{1}{g^2} \sum_{j=1}^2 (-1)^j \exp(-\xi_j^2), \quad (22) \\ \text{Re } \kappa(g, \omega) &= \\ &= \sqrt{\frac{\pi}{2\tau_{\perp}}} \frac{1}{2\delta_0} \frac{1}{g'} \frac{1}{g^2} \sum_{j=1}^2 (-1)^{j+1} \exp(-\xi_j^2) \text{erfi}(\xi_j), \quad (23) \end{aligned}$$

where

$$\xi_j = \frac{1}{g' \sqrt{2\tau_{\perp}}} (\omega + (-1)^{j+1} \delta_0 g^2), \quad (24)$$

$$|g'| = \sqrt{g_x^2 + g_y^2 + g_z^2 \frac{\tau_{\parallel}}{\tau_{\perp}}}. \quad (25)$$

Equations (22) and (23) in the case of equal transverse and longitudinal temperature  $\tau_{\perp} = \tau_{\parallel} = \tau$  reduce to equations (11) and (12), respectively.

### 2.2. LOW TEMPERATURE CASE

When the projectile particle moves with velocity  $V_i$ , which significantly exceeds the average thermal velocity of the electron gas one can expand the expression (23) of the parameter  $v_{e\perp, \parallel} / V_i$ .

$$\begin{aligned} \varepsilon(q, \omega) &= \\ &= 1 - \frac{\omega_p^2}{\omega^2} \left[ 1 + 3 \frac{q^2}{\omega^2} \left( v_{e\perp}^2 + \frac{\omega^2}{(qV_i)^2} (v_{e\parallel}^2 - v_{e\perp}^2) \right) \right]. \quad (26) \end{aligned}$$

Let obtain the energy loss of the projectile particle moving along the direction of anisotropy by substituting equation (26) into equation (10). With accurate to terms of order  $v_{e\perp, \parallel}^2 / V_i^2$  longitudinal losses are:

$$\begin{aligned} -\frac{dE}{dt} &= \\ &= \frac{e^2 \omega_p^2}{V_i} \left( \left( 1 - 3 \frac{T_{e\perp}}{m_e V_i^2} + 3 \frac{T_{e\parallel}}{m_e V_i^2} \right) L_c - \frac{3}{2} \frac{T_{e\parallel}}{m_e V_i^2} \right). \quad (27) \end{aligned}$$

### 2.3. NUMERICAL CALCULATION

By using equation (10) one can numerical calculate the dependence of the longitudinal energy losses from velocity of the projectile particle. Let compare obtained results with equation (27) and with results of isotropic velocity distribution.

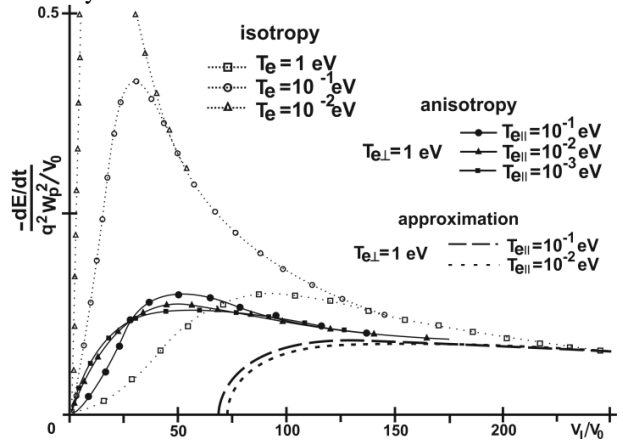


Fig. 4. The dependence of energy losses, normalized by  $e^2 \omega_p^2 / V_0$ , in the electron gas for different longitudinal electron temperatures (anisotropy) compared to their approximation Eq. (27) and to isotropic electron velocity distribution

Points lines correspond to an isotropic distribution: line with a) open squares is  $T_e = 1 \text{ eV}$ ; b) open circles is  $T_e = 10^{-1} \text{ eV}$ ; c) open triangles is  $T_e = 10^{-2} \text{ eV}$ .

The solid lines correspond to the case with an anisotropic velocity distribution: a) filled circles is  $T_{e\parallel} = 10^{-1} \text{ eV}$ ; b) filled triangles is  $T_{e\parallel} = 10^{-2} \text{ eV}$ ; c) filled squares is  $T_{e\parallel} = 10^{-3} \text{ eV}$ . Long-dashed and short-dashed lines correspond  $T_{e\parallel} = 10^{-1} \text{ eV}$ , Eq. (27) and  $T_{e\parallel} = 10^{-2} \text{ eV}$ , eq.(27), respectively.

Transverse temperature is  $T_{e\perp} = 1\text{eV}$ . Speed is measured in units of  $V_0 = 10^6\text{ cm/s}$ .

### CONCLUSIONS

In the frame of the quantum field method it is possible to consider the influence of electron gas temperature on the energy loss of the massive charged particle.

As consequence of acceleration of electron gas its longitudinal temperature is significantly lower than transverse one. The presence of a lower longitudinal electron gas temperature in an anisotropic distribution leads to a shift of the maximum of energy losses compared to the isotropic case for the same values of transverse temperature to the lower value of the projectile particle velocity.

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### REFERENCES

1. An International Accelerator Facility for Beams of Ions and Antiprotons, FAIR Baseline Technical Report, GSI Darmstadt, ISBN 3-9811298-0-6 (2006)
2. G.I. Budker, A.N. Skrinskii. Electron cooling and new possibilities in elementary particle physics // *UFN*. 1978, v. 21, p. 277-296.
3. V.V. Parkhomchuk, A.N. Skrinsky. Electron cooling: physics and prospective application // *Rep. Prog. Phys.* 1991, v. 54, p. 919-947.
4. I.N. Meshkov. Electron Cooling: Status and Perspectives // *Physics of Elementary Particles and Atomic Nuclei*. 1994, v. 25, 6, p. 1487-1560.
5. A.I. Larkin. Passage of particles through plasma // *Sov. Phys. JETP*. 1960, v. 10, p. 186-191.
6. A. Akhiezer // *Sov. Phys. JETP*. 1961, v. 13, p. 667.
7. D.A. Frank-Kamenetskii. *Lectures on Plasma Physics*. Moscow: "Atomizdat", 1968.

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### ВЛИЯНИЕ АНИЗОТРОПНОЙ ТЕМПЕРАТУРЫ ЭЛЕКТРОННОГО ГАЗА НА ЭНЕРГЕТИЧЕСКИЕ ПОТЕРИ ЗАРЯЖЕННОЙ ЧАСТИЦЫ В ПЛАЗМЕ

*А.В. Хелемеля, Р.И. Холодов*

В рамках квантово-полевого метода получено аналитическое выражение для реальной и мнимой частей диэлектрической восприимчивости электронного газа с анизотропным распределением. Подсчитаны энергетические потери массивной заряженной частицы в электронном газе в случае изотропного распределения по скоростям и в случае движения налетающей частицы в направлении анизотропии. Результаты численного счета энергетических потерь подтверждаются аналитическими расчетами в приближении малых температур.

### ВПЛИВ АНІЗОТРОПНОЇ ТЕМПЕРАТУРИ ЕЛЕКТРОННОГО ГАЗУ НА ЕНЕРГЕТИЧНІ ВТРАТИ ЗАРЯДЖЕНОЇ ЧАСТИНКИ В ПЛАЗМІ

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У рамках квантово-польового методу отримано аналітичний вираз для реальної та уявної частин діелектричної сприйнятливості електронного газу з анізотропним розподілом за швидкостями. Підраховано енергетичні втрати масивної зарядженої частинки в електронному газі у випадку ізотропного розподілу за швидкостями та у випадку руху налітаючої частинки в напрямку анізотропії. Результати чисельного обчислення енергетичних втрат підтверджуються аналітичними розрахунками в наближенні малих температур.