TRANSPORTATION OF HIGH-CURRENT CHARGED-PARTICLE BEAM IN COAXIAL UNDULATOR WITH PARTIALLY SHIELDED CATHODE IN GUIDE HOMOGENEOUS MAGNETIC FIELD

***A.S. Tishchenko*[1], *T. Yatsenko*[1], *K. Ilyenko*[2]**

[1]Institute for Radiophysics and Electronics of NAS of Ukraine, Kharkiv, Ukraine;
[2]Department of Electrical and Computer Engineering MSC01, 1 University of New Mexico, Albuquerque, NM, 87131, USA

E-mail: yatsenko.tetyana.yu@gmail.com

We derive the system of transcendental equations, which gives the equilibrium radii of charged-particle beam and the equilibrium value of square azimuthal momentum of beam boundary in a coaxial magnetic undulator with partially shielded cathode in guide homogeneous magnetic field. We formulate the dynamics equations, which describe the transportation of charged-particle beam. We receive an analytical estimate of the azimuthal and longitudinal momentum of beam boundary. We study the equilibrium and differential equation numerically, compare results received for a coaxial magnetic undulator with shielded and non-shielded cathode and different induction of guide magnetic field.

PACS: 84.30.Jc

INTRODUCTION

Many authors investigate limiting current and transportation of charged-particle beam in the ideally conducting grounded drift tube in the guide (longitudinal homogeneous) strong (infinite) or finite magnetic fields [1-3]. Authors of papers [4, 5] discuss the limiting current of charged-particle beam in the magnetostatic pump filed of the hybrid coaxial free electron laser/maser (FEL/FEM). In papers [6, 7] the limiting current of charged-particle beam in the strong (infinite) longitudinal homogeneous magnetic field is estimated for a thin beam in coaxial drift tube. The results of papers [8, 9] show that the propagation current in coaxial undulator are smaller than those in the longitudinal homogeneous guide magnetic field. We arrive at the conclusion that the degree of cathode shielding has a substantial influence on the charged-particle beam transportaion in the conducting grounded drift tube in the hybrid external guide magnetic field [10, 11].

In Section 1 we present the problem setup and the equilibrium and dynamics equations which describe the transportation of charged-particle beam in coaxial undulator with partially shielded cathode in guide homogeneous magnetic field. We numerically study the dependence of the equilibrium radii on the magnetic induction of guide magnetic field in Section 2. Also numerically, we study the transportation of high-current charged-particle beam in a coaxial undulator.

1. PROBLEM SETUP

We obtain the dynamic equations, which describe the transportation of high-current relativistic charge-particle beam in coaxial undulator with partially shielded cathode in assumption of laminar flow. We also suppose that an angular beam velocity ω_b is constant ("rigid rotor" model $\omega_b \equiv \upsilon_\theta(r)/r = const$, where $\upsilon_\theta(r)$ is the θ-component of beam velocity, r is the radial coordinate, the overbar "–" denotes the averaging over spatial period l_w of the periodic undulator magnetostatic system) and we assume that charged-particle beam has the constant beam density n_0. The dynamics equations and initial conditions under those assumptions have the form

$$\frac{d\pi_r}{d\zeta} = \left(1 - \frac{\pi_z^2(\rho,\zeta)}{\gamma_0^2}\right)\bar{e}_r^{self}(\rho) + \frac{1}{\gamma_0}\Big[\pi_\theta(\rho,\zeta)\times \\ \times(b_z^{ext}(\rho,\zeta) + \bar{b}_z^{self}(\rho)) + \frac{\pi_\theta(\rho,\zeta)}{\rho}\times \\ \times(\pi_\theta(\rho,\zeta) + \rho b_{z0}^{ext})\Big], \qquad (1)$$

$$\pi_r(\rho,\zeta_0) = 0,$$

where $\zeta = z/r_2$ is the dimensionless longitudinal coordinate; z is the longitudinal coordinate; $\zeta_0 = 3\pi/(4\kappa_w)$; $\kappa_w \equiv 2\pi r_2/l_w$; r_2 is the radius of the external tube; ρ is the dimensionless radial coordinate; $\pi_r = p_r/(m_q c)$, $\pi_\theta = p_\theta/(m_q c)$, $\pi_z = p_z/(m_q c)$; p_r, p_θ, p_z are the r-, θ-, z-components of beam relativistic momentum; m_q is the mass of particles (for electrons $m_q = m_e$); c is the light velocity in vacuum; we assume that the relativistic factor $\gamma = \sqrt{1 + (p_r^2 + p_\theta^2 + p_z^2)/(m_q c)^2}$ is constant i.e. $\gamma \equiv \gamma_0 = \sqrt{1 + p_{z0}^2/(m_q c)^2}$, where p_{z0} is z-the component of dimensionless initial momentum; $\vec{b}_{z0}^{ext} = \vec{B}_{z0}^{ext}(r,z) q r_2/(m_q c^2)$; $\vec{B}_0^{ext} = (0,0,B_{z0}^{ext})$ is the external homogeneous static magnetic field produced by a solenoid; $\vec{b}_\perp^{ext}(r,z) = \vec{B}_\perp^{ext}(r,z) q r_2/(m_q c^2)$; $\vec{B}_\perp^{ext}(r,z) = (B_r^{ext}(r,z), 0, B_z^{ext}(r,z))$ is the periodic undulator magnetostatic field produced by a system of permanent magnets [8, 15, 16]; $\bar{e}_r^{self} \equiv \bar{E}_r^{self}(r)\times(q r_2/m_q c^2)$; $\bar{b}_\theta^{self} \equiv \bar{B}_\theta^{self}(r)(q r_2/m_q c^2)$; $\bar{b}_z^{self} \equiv \bar{B}_z^{self}(r)(q r_2/m_q c^2)$ are the radial component of the dimensionless self-electric and the θ- and z-components of the dimensionless self-magnetic fields of the beam, respectively [10, 11]. Those assumptions are natural for thin beams, which can be proved by an expansion in the small parameter $\Delta r = r_o - r_i$ of the expression of self-electric and self-magnetic fields [10, Eqs. (9)–(11)] that were obtained for the general case. Under above assumptions the components of beam relativistic momentum have such analytical estimates

$$\pi_z(\rho,\zeta) \approx \sqrt{\gamma_0^2 - 1 - \pi_r^2(\rho,\zeta) - \pi_\theta^2(\rho,\zeta)}\,; \qquad (2)$$

$$\pi_\theta(\rho,\zeta) \approx -\left[1 - \frac{i_0 Q(\rho)}{2\pi_{z0}(\rho)(1-\rho_1^2)}\right]^{-1} \times \left[\frac{\rho b_{z0}^{ext}}{2}\left(1 - \delta_c \frac{\rho_c^2}{\rho^2}\right) + a_\theta^{ext}(\rho,\zeta)\right], \qquad (3)$$

where $i_0 = I_0 / I_A$ is the dimensionless beam current, $I_0 \approx q n_0 \upsilon_0 \pi (r_o^2 - r_i^2)$ is the beam current, $I_A = m_q c^3 / q$ is the Alfvén current ($I_A =$ –17,05 kA for electrons); $\upsilon_0 = c\sqrt{\gamma_0^2 - 1}$; $\pi_{z0}(\rho) = \sqrt{\gamma_0^2 - 1 - \pi_{\theta 0}^2(\rho)}$;

$$Q(\rho) = \frac{(\rho_o^2 - \rho^2)(1-\rho_1^2)}{(\rho_o^2 - \rho_i^2)} - (1-\rho^2)(\rho_i^2 + \rho_o^2 - 2\rho_1^2)\,;$$

$\rho_c = r_c / r_2$; r_c is the cathode radius; δ_c is the degree of cathode shielding; $\rho_{i(o)} = r_{i(o)} / r_2$ and $\rho_1 = r_1 / r_2$; $r_{i(o)}$ and r_1 are the inner and outer radii of charged-particle beam and the inner radius of the drift tube;

$$a_\theta^{ext}(\rho,\zeta) = \frac{b_\perp^m}{\kappa_w} \sum_{k=0}^{\infty} C_{2k+1} \cos((2k+1)(\kappa_w \zeta - \pi/4)) \times F_{2k+1}^{(1)}((2k+1)\kappa_w \rho), \quad \zeta_0 < \zeta; \qquad (4)$$

$$a_\theta^{ext}(\rho,\zeta) = 0, \quad \zeta \le \zeta_0;$$

$$C_{2k+1} = 4\sin(\pi(2k+1)/4)/(\pi(2k+1))\,;$$

$$F_{2k+1}^{(1)}((2k+1)\kappa_w\rho) = f_{2k+1} I_1((2k+1)\kappa_w\rho) + g_{2k+1} K_1((2k+1)\kappa_w\rho)\,;$$

$$f_{2k+1} = \frac{K_0((2k+1)\kappa_w\rho_1) + K_0((2k+1)\kappa_w\rho_2)}{\Delta_{2k+1}}\,;$$

$$g_{2k+1} = \frac{I_0((2k+1)\kappa_w\rho_1) + I_0((2k+1)\kappa_w\rho_2)}{\Delta_{2k+1}}\,;$$

$$\Delta_{2k+1} = I_0((2k+1)\kappa_w\rho_1) K_0((2k+1)\kappa_w\rho_2) - I_0((2k+1)\kappa_w\rho_2) K_0((2k+1)\kappa_w\rho_1),$$

$I_{0(1)}(\cdot)$ are the Bessel function of 0 (1) order; $K_{0(1)}(\cdot)$ are the modified Bessel function of 0 (1) order.

The system of equilibrium equations under above assumptions has the form

$$\overline{\pi_\theta^2(\rho_{i(o),eq},\zeta)}\left[1 - \frac{i_0 Q(\rho_{i(o),eq})}{2\sqrt{\gamma_0^2 - 1 - \overline{\pi_\theta^2(\rho_{i(o),eq},\zeta)}}\,(1-\rho_1^2)}\right]^2 - \left[\frac{\rho_{i(o),eq}^2 (b_{z0}^{ext})^2}{4}\left(1 - \delta_c \frac{\rho_c^2}{\rho_{i(o),eq}^2}\right) + \frac{(b_\perp^m)^2}{2\kappa_w^2} C_1^2 (F_1^{(1)}(\kappa_w \rho_{i(o),eq}))^2\right] = 0\,; \qquad (5)$$

$$\frac{\overline{\pi_\theta^2(\rho_{i(o),eq},\zeta)}}{\rho_{i(o),eq}} + \gamma_0 \left(1 - \frac{\overline{\pi_z^2(\rho_{i(o),eq},\zeta)}}{\rho_{i(o),eq}}\right) \times \bar{e}_r^{self}(\rho_{i(o),eq}) + \overline{\pi_\theta(\rho_{i(o),eq},\zeta) b_z^{ext}(\rho_{i(o),eq},\zeta)} + \overline{\pi_\theta(\rho_{i,eq},\zeta)}(\bar{b}_z^{self}(\rho_{i,eq}) + b_{z0}^{ext}) = 0, \qquad (6)$$

where $\rho_{i(o),eq}$ are the dimensionless equilibrium radii;

$$\overline{\pi_z^2(\rho_{i(o),eq},\zeta)} = \gamma_0^2 - 1 - \overline{\pi_\theta^2(\rho_{i(o),eq},\zeta)};$$

$$\overline{\pi_\theta(\rho_{i(o),eq},\zeta) b_z^{ext}(\rho_{i(o),eq},\zeta)} = -\frac{(b_\perp^m)^2}{2\kappa_w}\left[1 - \frac{i_0 Q(\rho_{i(o),eq})}{2\sqrt{\overline{\pi_z^2(\rho_{i(o),eq},\zeta)}}\,(1-\rho_1^2)}\right]^{-1} \times C_1^2 F_1^{(0)}(\kappa_w \rho_{i(o),eq}) F_1^{(1)}(\kappa_w \rho_{i(o),eq});$$

$$\overline{\pi_\theta(\rho_{i(o),eq},\zeta)} = -\left[1 - \frac{i_0 Q(\rho_{i(o),eq})}{2\sqrt{\overline{\pi_z^2(\rho_{i(o),eq},\zeta)}}\,(1-\rho_1^2)}\right]^{-1} \times \frac{\rho_{i(o),eq} b_{z0}^{ext}}{2}\left(1 - \delta_c \frac{\rho_c^2}{\rho_{i(o),eq}^2}\right).$$

2. NUMERICAL SOLUTIONS OF EQUILIBRIUM EQUATIONS

In [14, Fig. 2] we study the transportation of electron beam in the absence of a longitudinal magnetic field over a wide range of values of the longitudinal component of the magnetic induction on the cylindrical surfaces of the permanent magnets of the undulator, $B_\perp^m$, and beam current, I_0. In this paper we consider the dependence the transportation of electron beam on the induction of longitudinal homogenous magnetic field.

In Fig. 1 the dependence of equilibrium radii, the azimuthal and longitudinal momenta of electron beam boundaries transported in a coaxial drift tube in a hybrid magnetostatic pumping filed FEL/FEM on the induction of external longitudinal magnetic field is shown. It is seen that with increasing the induction of longitudinal external field the external and internal equilibrium radii of the electron beam tend to cathode radius, the equilibrium values the angular momentum tend asymptotically to zero, and the equilibrium values of longitudinal one tend to a stationary value. Consequently, the limit to the model of "strong" (infinite) external longitudinal magnetic field is correct (the azimuthal momentum tends to zero).

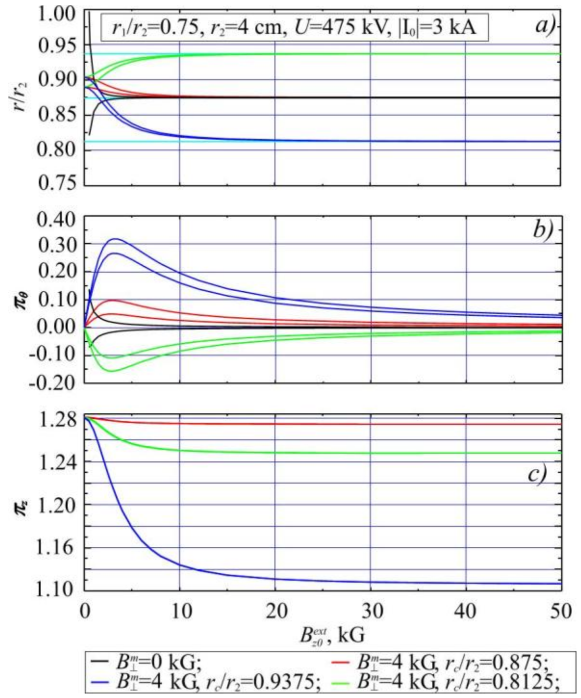

Fig. 1. Dependence of the equilibrium radii, azimuthal and longitudinal momenta on the induction of external longitudinal magnetic field for Strathclyde FEL/FEM [4, 9]

In Fig. 2 the dependence of the longitudinal coordinate radii of the various layers of electron and the radial, azimuthal and longitudinal components of the dimensionless momenta in the presence of longitudinal homogeneous (guide) magnetic fields obtained by numerically solving the system of equations and initial conditions (1) is presented for Strathclyde FEL/FEM [4, 9]. One can see that the longitudinal magnetic (guide) field contributes to a significant increase in the radial component of the momentum π_r that will lead to improved interaction with the amplified/generated electromagnetic wave, although at the same time enhances the oscillatory longitudinal component of momentum π_z .

CONCLUSIONS

The transportation and equilibrium steady state of high-current charged-particle beam in a coaxial drift tube with a shielded and non-shielded cathodes in the hybrid longitudinal homogeneous (guide) magnetic and magnetostatic undulator fields is studied in the approximation of constant beam density and “rigid rotor” type of rotation. It is shown that the equilibrium steady state of charged-particle beam in a coaxial drift tube can be controlled by choosing the cathode radius and the value of the longitudinal homogeneous (guide) magnetic field so that the beam can be transported in any suitable position and can amplify efficiently arbitrary mode of the coaxial drift tube.

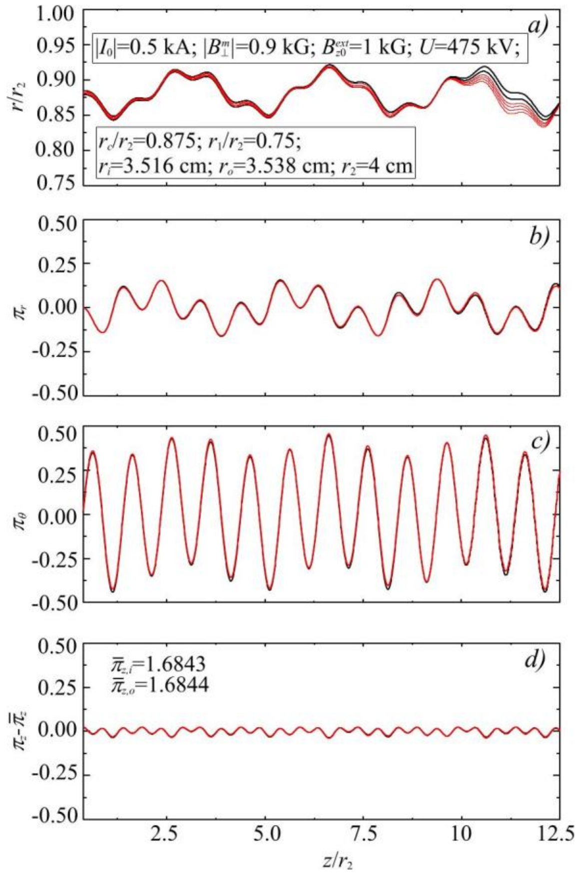

Fig. 2. Dependence of the radii of various beamlets (beam layers) on the longitudinal coordinate in the presence of guide magnetic field for Strathclyde FEL/FEM [4, 9]. Initially (in the injection plane) inner and outer beamlets are shown in black; initially internal beamlets are shown in red

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Article received 15.10.2014

ТРАНСПОРТИРОВКА СИЛЬНОТОЧНОГО ПУЧКА ЗАРЯЖЕННЫХ ЧАСТИЦ В КОАКСИАЛЬНОМ ОНДУЛЯТОРЕ С ЧАСТИЧНО ЭКРАНИРОВАННЫМ КАТОДОМ В ВЕДУЩЕМ ОДНОРОДНОМ МАГНИТНОМ ПОЛЕ

А.С. Тищенко, Т. Яценко, К. Ильенко

Получена система трансцендентных уравнений, решениями которой являются равновесные радиусы и равновесные значения квадратов азимутальных импульсов границ пучка заряженных частиц в коаксиальном магнитном ондуляторе с частично экранированным катодом в ведущем однородном магнитном поле. Сформулированы динамические уравнения, которые описывают транспортировку пучка заряженных частиц в камере дрейфа. Получены аналитические оценки азимутального и продольного импульсов границ пучка. Численно исследованы уравнения равновесия и динамические уравнения, проведены сравнения результатов в случае экранированного и неэкранированного катодов, а также зависимость от напряженности ведущего магнитного поля.

ТРАНСПОРТУВАННЯ СИЛЬНОСТРУМОВОГО ПУЧКА ЗАРЯДЖЕНИХ ЧАСТИНОК У КОАКСІАЛЬНОМУ ОНДУЛЯТОРІ З ЧАСТКОВО ЕКРАНОВАНИМ КАТОДОМ У ВЕДУЧОМУ ОДНОРІДНОМУ МАГНІТНОМУ ПОЛІ

А.С. Тіщенко, Т. Яценко, К. Ільєнко

Отримана система трансцендентних рівнянь, рішеннями якої є рівноважні радіуси та рівноважні значення квадратів азимутальних імпульсів меж пучка заряджених частинок у коаксіальному магнітному ондуляторі с частково екранованим катодом у ведучому однорідному магнітному полі. Сформульовані динамічні рівняння, які описують транспортування пучка заряджених частинок у камері дрейфу. Отримані аналітичні оцінки азимутального і повздовжнього імпульсів меж пучка. Чисельно вивчені рівняння рівноваги та динамічні рівняння, проведено порівняння результатів у випадку екранованого та неекранованого катодів, а також залежність від напруги ведучого магнітного поля.