EFFECT OF A NONCIRCULAR SHAPE OF THE TORUS ON THE MAGNETIC SURFACES OF *l*=1 TORSATRON

V. G. Kotenko
Institute of Plasma Physics of the NSC "Kharkov Institute of Physics and Technology", Kharkov, Ukraine

The paper presents the numerical calculation results on the magnetic field produced by one helical winding laying on the surface of a noncircular torus. The influence of the chosen poloidal cross-section shape of the torus on the parameters of the closed magnetic surface configuration has been investigated. The calculation results show that the change of a torus with circular cross-section by a torus with noncircular cross-section in the *l*=1 torsatron under consideration decreases the value of the mirror ratio on the magnetic surfaces by a factor of ~8. And other parameters of magnetic surfaces were changed no more than by a factor of 1.5 to 2. A feasibility of the simplest torsatron with central stochastic magnetic field line region is discussed.

PACS: 52.55.Hc

INTRODUCTION

As is well known [1] the maximally simplified magnetic system for a stellarator-type closed magnetic plasma trap, namely, *l*=1 torsatron, can be formed by one helical coil wound on the torus. To create a magnetic surface configuration, i.e. a plasma confinement region, there is no need in any additional magnetic field if the condition $\alpha m \approx 3$, relatively to the helical coil geometry is observed. Here $\alpha = a/R_0$, R_0 is the major radius of the torus, *a* is the minor radius of the torus, *m* is the number of helical coil pitches along the torus length. Provided that the magnetic plasma trap is realized without any massive and bulky structures designed for creation of additional magnetic fields, one can expect that the access to the working volume of the trap will be substantially easier and its cost will be decreased.

However, under conditions of total absence of additional magnetic fields such as an additional toroidal magnetic field and/or compensating vertical magnetic field, some parameters of magnetic surfaces in the *l*=1 torsatron are unattractive in the view of stellarator experiment carrying out. First of all, it concerns an extremely high value of the mirror ratio on the magnetic surfaces, $\gamma \sim 10…40$ ($\gamma = B_{max}/B_{min}$, B_{max}, B_{min} are the maximum and minimum values of the magnetic field on the magnetic surface) [2]. At the same time, minimization of the γ-value in the closed magnetic plasma traps is one of the main recommendations following from the neoclassical plasma confinement theory [3].

In this paper, we use, as an initial model, a calculation model for the toroidal magnetic system of the *l*=1 torsatron to investigate the structure of the magnetic field which arises as a result of change from the initial circular poloidal torus cross-section to a noncircular one. The aim of the study is to find a possibility for a radical decrease of the mirror ratio on the magnetic surfaces of the *l*=1 torsatron.

CALCULATION MODEL OF THE *l*=1 TORSATRON WITH A CIRCULAR TORUS

As an initial calculation model with a circular torus ($a = a_c$=const.), we use a magnetic system whose main geometric characteristics can improve access to the working volume of a closed toroidal magnetic plasma trap:

- *l*=1 is the polarity;
- toroidicity $\alpha = a_c/R_0 = 0.375$;
- number of helical coil pitches along the torus length *m*=8, i.e., the pitch parameter of the helical coil $\alpha m = 3$;
- the system does not contain toroidal magnetic field coils and compensating solenoids;
- the helical coil is made of a thin conductor wound on the torus by the equi-inclined law generalized for the case of significant toroidicity [4]:

$$\theta = 2\mathrm{arctg}(((1+\alpha)/(1-\alpha))^{0.5}\mathrm{tg}(m\varphi/2)). \qquad (1)$$

The helical coil projection (thick line) of the initial calculation model of the *l*=1 torsatron onto the equatorial torus plane is shown in Fig. 1,*a*,*b*.

The thin line in Fig.1,*a* shows the calculated projection of the spatial magnetic axis of the magnetic surface configuration onto the equatorial torus plane. The projection has the shape of helical line projection, its number of pitches along the torus length is equal to that of the helical coil (*m*=8). The helical line lies at the surface of an imaginary torus whose major radius (major radius of the magnetic axis) $R_{0ax}/R_0 = 1.305$. The poloidal cross-section of the imaginary torus has an elliptic shape, its minor radius (minor radius of the magnetic axis) varies within $a_{ax}/R_0 = 0.045…0.11$ (Fig. 2, cross section $\varphi = 0°$).

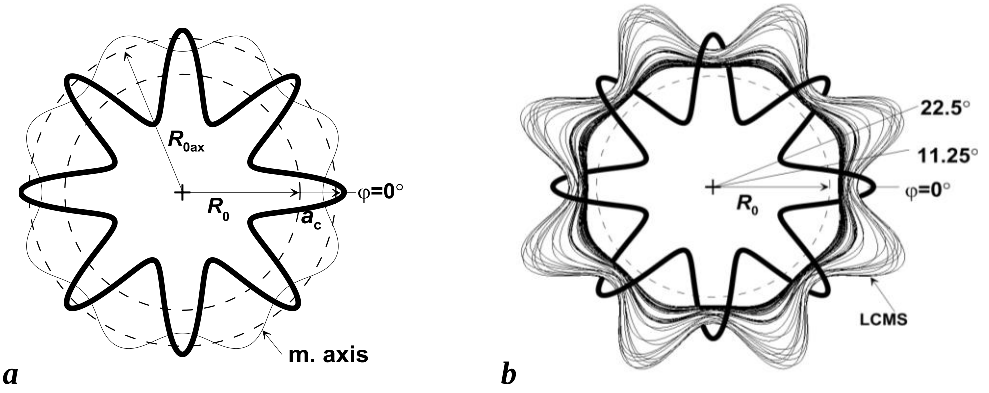


Fig. 1. Top view of the helical coil and the magnetic axis (a) and the region of magnetic surface existence LCMS (b) in the initial calculation model of the l=1 torsatron with a circular torus. The toroidal azimuths of characteristic poloidal cross-sections are indicated (see Fig. 2, 3)

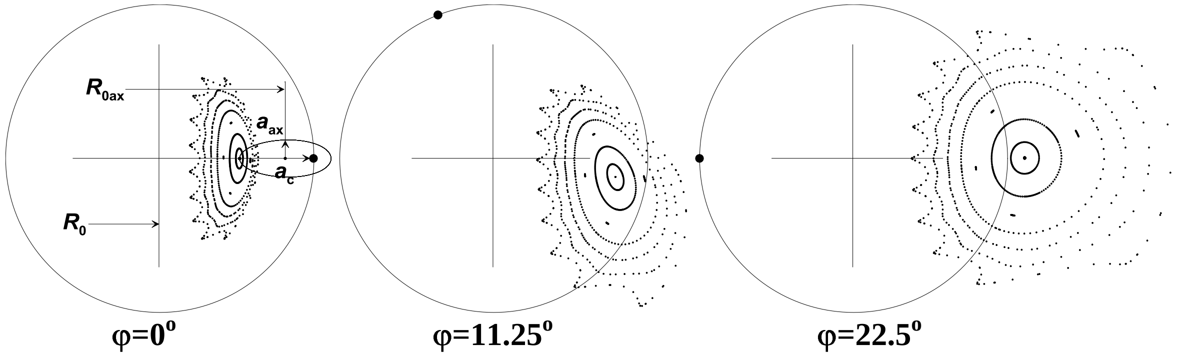


Fig. 2. Cross-sections of magnetic surfaces in the initial calculation model with a circular torus. In the cross-section φ=0° one can see the imaginary torus cross-section where the magnetic axis is disposed

Fig. 1,*b* presents the projection onto the equatorial torus plane of the last closed magnetic surface (LCMS), i.e., the region of magnetic surface existence. It is seen that the magnetic surface configuration along the total torus length is in the region of $R>R_0$, i.e., in the rather weak magnetic field.

Fig. 2 presents the calculated magnetic surface poloidal cross-sections in the l=1 torsatron with a circular torus. The cross-sections are spaced round the toroidal angle φ (see Fig. 1,*b*) within the limits of magnetic field half-period, φ=0°, 11.25°, 22.5°. In the figures the circle represents the torus cross-section of a minor radius a_c with traces of the helical coil (thick dots).

From the figure one can see that the size of the region of magnetic surface existence depends, to a great extent, on the cross-section toroidal azimuth. So, the value of the LCMS average radius is $r_{lc0}/R_0=0.11$ in the cross-section φ=0° and reaches the maximum $r_{lc22}/R_0=0.31$ in the cross-section φ=22.5° (also see Fig. 1,*b*). The rotational transformation angle is $\iota_{axis}\rightarrow\iota_{lcms}=0.73\rightarrow0.8$ (in 2π units) on the magnetic surfaces, a large magnetic hill U=0.26 takes place, and the value of mirror ratio $\gamma_{axis}\rightarrow\gamma_{lcms}=6.2\rightarrow30.5$. The "effective" value of the mirror ratio within the LCMS limits $\gamma_{eff}=(r_{lc22})^2/(r_{lc0})^2\approx8$ was determined by the magnetic flow conservation law.

CALCULATION MODEL OF THE *l*=1 TORSATRON WITH A NONCIRCULAR TORUS

The equation for the torus surface in the Cartesian system, where the z axis is directed along the straight axis of torus rotation, can be written in the parametric form convenient for numerical calculations:

$$x=(R_0+a\cos(\theta))\cos(\varphi),$$
$$y=(R_0+a\cos(\theta))\sin(\varphi),$$
$$z=a\sin(\theta), \qquad (2)$$

Here, the parameters θ and φ denote the poloidal angle and the toroidal angle, respectively. If $\theta=\theta(\varphi)$, then equations (2) describe the line on the torus.

Unlike an ordinary circular torus, in a noncircular torus the poloidal cross-section has a noncircular shape and the minor radius of the torus is the explicit function of the poloidal angle. The present study is aimed to the magnetic system with noncircular torus for which the minor radius value, according to one of the version considered in [5], is determined by the formula:

$$a=a_c(1-\delta|\sin(\theta_1)|), \qquad (3)$$

here $\theta_1=m\varphi$, $a_c=0.375$, $\delta=0.5$. The poloidal azimuth of the minor radius of the noncircular torus has been calculated from the condition that the helical conductor winding was made in compliance with the combined law [6]:

$$\theta=\theta_1-k(\theta_2-2\mathrm{arctg}(\mathrm{tg}(\theta_1/2))). \qquad (4)$$

Here $\theta_2=2\mathrm{arctg}(((1+\alpha)/(1-\alpha))^{0.5}\mathrm{tg}(m\varphi/2))$ is in the complete compliance with equation (1), $\alpha=a_c/R_0=0.375$, and the coefficient value $k=-1.95$.

After substituting the values a and θ obtained from Eq. (3) and (4) into Eq. (2), the helical coil on the circular torus is transformed into the helical coil lying on the surface of the noncircular torus, the poloidal cross-section is in the form represented in Fig.3*a* (thick line). The same figure presents the cross-section of the imaginary torus on which the spatial magnetic axis of the closed magnetic surface configuration is wound. The major radius of this torus (the major radius of the magnetic axis) $R_{0ax}/R_0=1$ is provided by the above-given values of δ and k. The minor radius of this torus (the minor radius of the magnetic axis) changes within the limits of $a_{ax}/R_0=0.036\ldots0.051$, i.e., the poloidal cross-section of the imaginary torus has the shape of an ellipse elongated along the vertical.

The top view of the helical coil is shown in Fig. 3*b*. In the same figure one can see the calculated projection of magnetic axis (thin line) onto the equatorial torus plane. The projection has the shape of the helical line projection having the same number of pitches on the torus length as in the helical coil ($m=8$). Fig. 3,*c* presents the projection onto the equatorial torus plane of the last closed magnetic surface (LCMS), i.e., the region of magnetic surface existence. It is seen that the magnetic surface configuration along the total torus length is in the region $R\sim R_0$, i.e., within the range of higher values of the magnetic field strength.

Fig. 4 shows the calculated magnetic surface poloidal cross-sections in the $l=1$ totsatron with a noncircular torus. The cross-sections are spaced round the toroidal angle φ (see Fig. 3,*c*) within the limits of magnetic field half-period, $\varphi=0^\circ$, 11.25°, 22.5°. The poloidal cross-section of the noncircular torus is shown with traces of the helical coil (thick dots).

It is seen from Fig. 4 that the size of the region of magnetic surface existence is in a lesser dependence on its toroidal azimuth. So, the value of the LCMS average radius is $r_{lc0}/R_0=0.12$ in the cross-section $\varphi=0^\circ$ and reaches the maximum, $r_{lc22}/R_0=0.17$, in the cross-section $\varphi=22.5^\circ$. The rotational transform angle is $\iota_{axis}\rightarrow\iota_{lcms}=0.61\rightarrow0.79$ on the magnetic surfaces, a large magnetic hill is observed, $U=0.38$, and the mirror ratio $\gamma_{axis}\rightarrow\gamma_{lcms}=1.63\rightarrow4.1$. In compliance with the law of magnetic flow conservation the "effective" value of the mirror ratio within the limits of the LCMS is $\gamma_{eff}=(r_{lc22})^2/(r_{lc0})^2\approx2$.

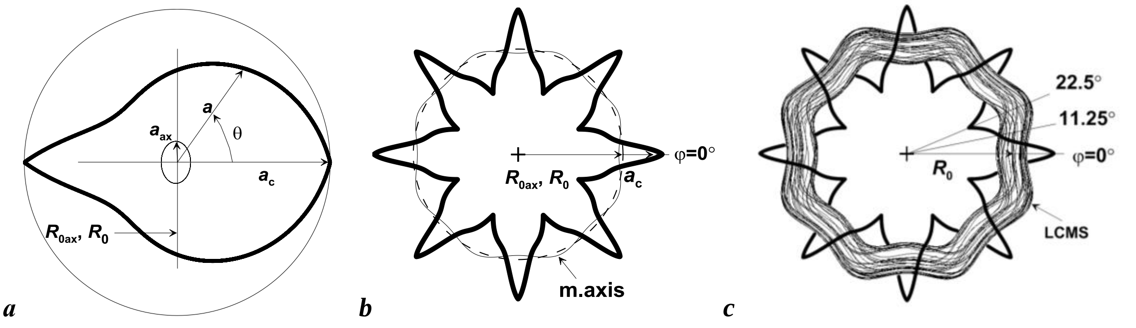


Fig. 3. Poloidal cross-section of the noncircular torus and magnetic axis imaginary torus (a) and top view of the helical coil and the magnetic axis (b) and the region of magnetic surface existence LCMS (c) in the calculation model of the l=1 torsatron with a noncircular torus

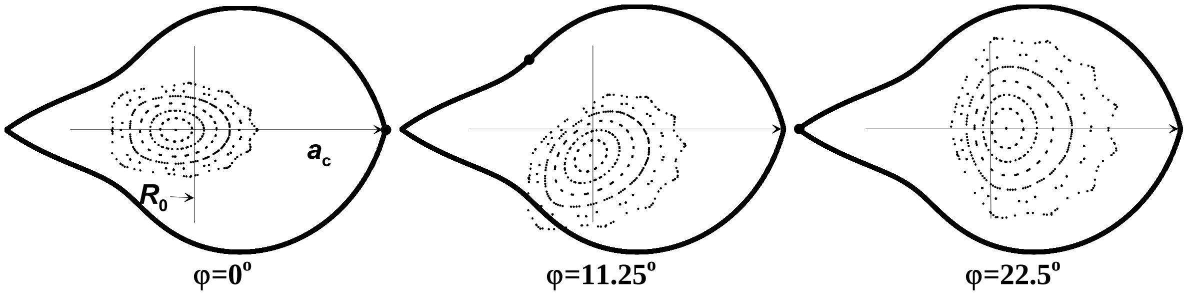


Fig. 4. Cross-sections of magnetic surfaces in the calculation model of the l=1 torsatron magnetic system with a noncircular torus

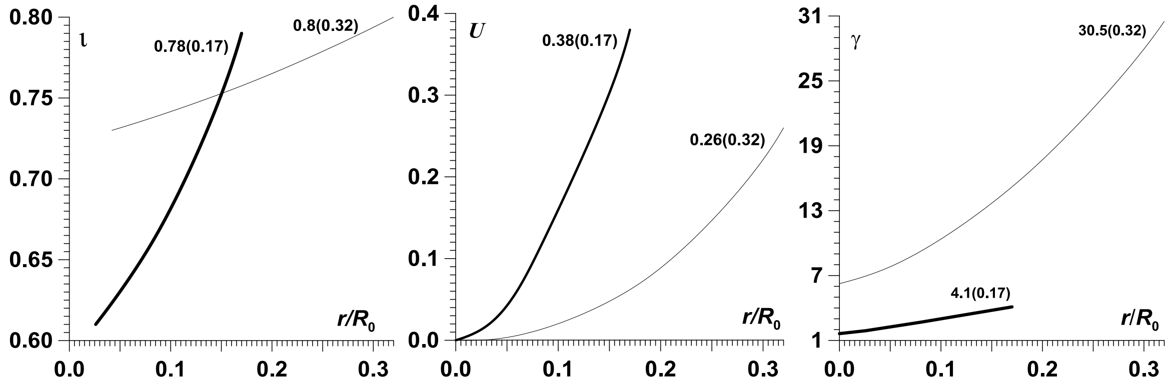


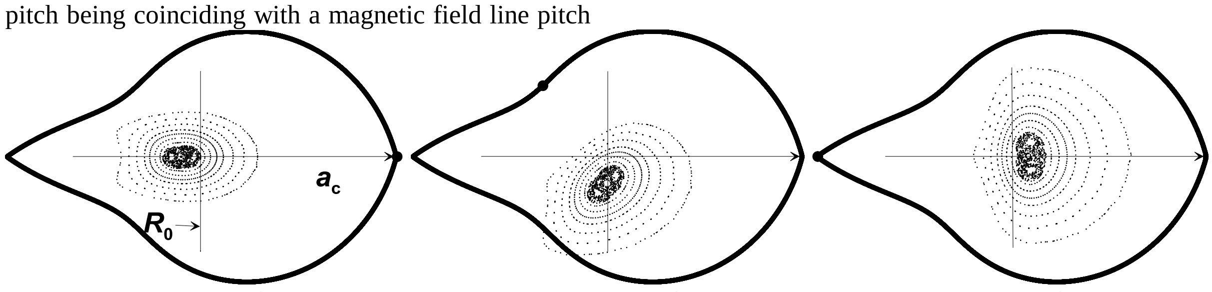
Fig. 5. The rotational transform angle (ι in 2π units), magnetic hill U, mirror ratio γ versus the value of the magnetic surface average radius r/R₀ in the cross-section φ=22.5° by the calculation model of the initial circular torus (thin lines) and noncircular torus (thick lines)

For comparison, the magnetic surface parameters as a function of their average radius in the cross-section φ=22.5° are presented in Fig. 5. The values of LCMS parameters together with its average radius value (in brackets) are indicated by the inscriptions near the curves. From the figures one can see that the change from the circular to the noncircular torus leads to well-centering of the magnetic surface configuration and to decreasing in the high value of the mirror ratio on the magnetic surfaces by a factor of 4 to 8. Other magnetic surface parameters change no more than by a factor of 1.5 to 2.

THE SIMPLEST TORSATRON WITH A CENTRAL STOCHASTIC REGION

In paper [7] it has been supposed that an efficient plasma confinement can be reached by applying a magnetic field the structure of which contains a stochastic (“turbulent”) region of magnetic field lines. This region should be enclosed within a rather thick layer of “laminar” field lines forming, for example, closed magnetic surfaces. It is expected that the stochastic behavior of field lines can significantly decrease the plasma density and temperature gradients and thus promote suppression of various plasma instabilities. In paper [8] is shown that for creating a stochastic region on the radius *r* (counted off, for example, from the magnetic axis) on it a special “resonance” winding should be placed, a winding pitch being coinciding with a magnetic field line pitch on the same radius. Consequently, a central (*r*=0) stochastic region can be created with an additional magnetic field that arises as a result of electric current passing through the special “resonance” winding the shape of which coincides with the magnetic axis one.

To make sure that the supposed opportunity exists the calculations were carried out on the magnetic surface configuration in the *l*=1 torsatron, comprising a noncircular torus with an electrical current flowing along its magnetic axis. The geometry of the special “resonance” winding in this case coincides with the geometry of the magnetic axis shown in Fig. 3*a*, *b*. The main calculation result is represented in Fig. 6. It is seen that under action of the axial current magnetic field, in the central part of the magnetic field structure a stochastic region of magnetic field lines arises being enclosed within a thick layer of regular magnetic surfaces. The stochastic region size is confined by the magnetic surface of an average radius r_{st}/R_0=0.047 (in the cross section φ=22.5°). According to calculations the “stochastized” field line is not outside this surface along many hundreds (~10^3) of turns around the torus length. The magnetic field structure obtained is observed when the axial current value $I_{ax}=0.01I_h$, and its direction coincides with the direction of I_h current in the helical winding. Here the parameters of regular magnetic surfaces are changed slightly. Some features and dynamics of central stochastic region formation may be a subject of further investigations.



φ=0°   φ=11.25°   φ=22.5°

Fig. 6. Cross-sections of the magnetic surface configuration in the calculation model of the l=1 torsatron with an electrical current along the magnetic axis

It is not possible to place a special “resonance” winding inside the plasma column. Therefore, to create the stochastic region one can use advanced physical and engineering techniques of non-inductive electric current generation in plasma by means of concentrated flows of short-wave electromagnetic radiation (ECCD) and/or charged particle beams (N-NBCD). But it is obvious that in practice the realization of this suggestion can require a high level of localization of the current (~10^4A) flowing in the plasma along the chosen magnetic field line.

CONCLUSIONS

The paper presents the numerical calculations of the magnetic field created by a single helical winding laying on the torus surface having a noncircular poloidal cross-section.

The calculation results show that using the fitted values of coefficients δ and k in Eq. (3) and (4) for the l=1, m=8 torsatron with a noncircular torus one can obtain a well-centered configuration of closed magnetic surfaces. A high value of the mirror ratio on the magnetic surfaces, observed in the l=1, m=8 torsatron with a circular torus, decreases by a factor of 4 to 8 in the torsatron with noncircular torus. The values of other magnetic surface parameters, namely, a maximum average radius of the closed magnetic surface, rotational transformation angle, magnetic hill are changing no more than by a factor of 1.5 to 2. A feasibility of the simplest torsatron with central stochastic magnetic field line region is discussed.

So, the observed features are a good reason for further comprehensive analysis in the stellarator-type magnetic systems with a noncircular torus. At the next stage the effect of helical coil finite sizes on the magnetic surface parameters and the ponderomotive force action on the helical coils can be studied.

REFERENCES

1. C. Gourdon, P. Hubert, D. Marty. Ultimate simplification des bobinages creant une configuration magnetique du genre stellarator // C. *R. Hebd. Seances Acad. Sci. Paris.* 1970, v. 271, p. 843-845.

2. V.Z. Amelin, A.V. Georgievskij, V.E. Ziser, V.G. Peletminskaya, D.G. Pogozhev, M.N. Skoblik. The structure of the magnetic field of an l=1 toroidal torsatron // *Nucl. Fusion*. 1975, v. 15, N 6, p. 1133-1141.

3. A.A. Galeev, R.Z. Sagdeev. “Neoclassical” theory of diffusion // *Voprosy teorii plasmy (Reviews of Plasma Physics).* Moscow: “Atomizdat”, 1973, v. 7, 205 p. (in Russian).

4. E.D. Volkov, V.A. Suprunenko, A.A. Shishkin. *Stellarator*. Kiev: “Naukova Dumka”, 1983 (in Russian).

5. V.G. Kotenko. Magnetic field of helical currents flowing over the surface of a non-circular torus // *Problems of Atomic Science and Technology. Ser.* “*Termoyaderny Sintez*”,. Moscow, 2013, v. 36, №4, p. 64-69 (in Russian).

6. V. Kotenko, E. Volkov, K. Yamazaki. Field ripple behavior in helical systems // *Plasma Devices and Operations*. 2004, v. 12, N 2, p.143-152.

7. G.V. Skornyakov // *Zhurnal Tekhnicheskoj Fiziki*. 1962, v. 32, p. 261, 777 (in Russian).

8. B.V. Chirikov. *Doklady Academii Nauk USSR*. 1967, v. 174, № 6, p. 1313-1316 (in Russian).

Article received 16.11.2014

ВЛИЯНИЕ НЕКРУГОВОЙ ФОРМЫ ТОРА НА МАГНИТНЫЕ ПОВЕРХНОСТИ ОДНОЗАХОДНОГО (l=1) ТОРСАТРОНА

В. Г. Котенко

Проведены численные расчеты магнитного поля, создаваемого одной винтовой обмоткой, лежащей на поверхности некругового тора. Изучено влияние выбранной формы полоидального сечения тора на параметры конфигурации замкнутых магнитных поверхностей. Расчетами показано, что в рассмотренном варианте l=1 торсатрона переход от круговой к некруговой форме тора приводит к уменьшению величины пробочного отношения на магнитных поверхностях в ~8 раз. При этом другие параметры магнитных поверхностей изменяются не более, чем в 1,5…2 раза. Обсуждается возможность реализации l=1 торсатрона с центральной стохастической областью силовых линий магнитного поля.

ВПЛИВ НЕКРУГОВОЇ ФОРМИ ТОРА НА МАГНІТНІ ПОВЕРХНІ ОДНОЗАХОДНОГО (l=1) ТОРСАТРОНА

В.Г. Котенко

Виконані чисельні розрахунки магнітного поля, що створюється однією гвинтовою обмоткою, що лежить на поверхні некругового тора. Вивчено вплив обраної форми полоїдального перерізу тора на параметри конфігурації магнітних поверхонь. Розрахунками виявлено, що в розглянутому варіанті l=1 торсатрона перехід від кругової до некругової форми тора призводить до зменшення величини коркового відношення на магнітних поверхнях у ~ 8 разів. При цьому інші параметри магнітних поверхонь змінюються не більше, ніж в 1,5…2 рази. Дискутується можливість створення l=1 торсатрона з центральною стохастичною областю силових ліній магнітного поля.