SPATIAL ENERGY CHANNELLING AND STOCHASTIZATION OF FAST ION MOTION BY HIGH-FREQUENCY PLASMA INSTABILITIES

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The conditions are established when multiple Alfvén eigenmodes are able to withdraw a significant part of the energy of fast ions for possible transfer to another spatial region (spatial channelling). This can happen when the resonance islands of the instabilities overlap to form an extensive stochastic zone in the fast ion phase space. An analytical expression for the width of a resonance island induced by an Alfvén eigenmode in the phase space is derived. The number and amplitude of modes are estimated, which are required to form a stochastic zone in a given energy range. Two codes intended for numerical verification of these estimates are briefly described. First results of these codes are presented.

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INTRODUCTION

In many experiments on the spherical torus (spherical tokamak) NSTX, multiple high-frequency instabilities (the frequency $f = \omega / (2\pi)$~0.5...1 MHz) are observed when the neutral beam injection power is sufficiently high [1, 2]. These instabilities were identified as GAEs (global Alfvén eigenmodes) and CAEs (compressional Alfvén eigenmodes) driven by fast ions. It was pointed out [3] that the instabilities can channel energy and momentum of fast ions outside the region where these ions are located (Fig. 1) which explains the observed deterioration of the energy confinement. This channelling can lead, in particular, to cooling the plasma core. A necessary condition for the spatial energy channelling to occur is the ability of the instabilities to take away a significant fraction of the fast ion energy.

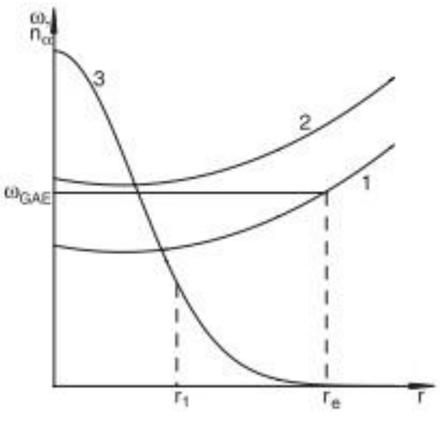

Fig. 1. GAE frequency (horizontal line), Alfvén continuum branches with the mode numbers m, n and $m+1$, n (curves 1, 2), and the radial profile of the beam ions (curve 3). This sketch demonstrates the energy and momentum channelling by a GAE mode: the mode receives energy and momentum of the beam ions mainly inside the region $r < r_1$ but gives the energy and momentum to electrons due to continuum damping at $r \approx r_e$

The aim of this work is to estimate the number and the amplitudes of the modes necessary to produce a wide stochastic region in the fast-ion phase space. In this case, the waves are able to receive a major part of the power of injected ions for transfer to another spatial region. Within this work, we obtain this estimate by analytical means and describe the numerical tools intended for the verification of our analytical results.

Our analysis here is restricted to GAE modes. In addition, we do not consider cyclotron resonances (playing probably a significant role in the excitation of high-frequency GAE instabilities in NSTX), which enables us to study the particle motion in the guiding-centre approximation. We believe that this is justified at the initial stage of our investigation; these limitations are to be removed in future works.

The structure of this paper is as follows. In Sec. 2, we obtain analytical estimates of the resonance island width and apply the Chirikov criterion [4] to obtaining the stochasticity condition. Our numerical tools and their first results are described in Sec. 3. Finally, in Sec. 4 we summarize and discuss our results.

1. RESONANCE WIDTH

The general resonance condition can be written as

$$\omega = s\omega_\vartheta - n\omega_\varphi = \Omega(J_\varphi, J_\vartheta)\,, \qquad (1)$$

where $\omega_\varphi = v_\parallel / R$; $\omega_\vartheta = v_\parallel /(qR)$ are frequencies of toroidal motion and poloidal motion, respectively; R is the major radius of the torus; the quantities s and n are integers, n being always equal to the toroidal wave number in axisymmetric geometry. We are interested in resonances of passing particles.

One can show that in linear approximation in wave amplitude, the guiding centre Lagrangian can be written as follows

$$L = J_\varphi d\varphi + J_\vartheta d\vartheta - W dt + \sum_{s,n} \mathrm{Re}[V_{sn} \exp(-i\omega t + is\vartheta - in\varphi)], \qquad (2)$$

where $J_\varphi, \varphi, J_\vartheta, \vartheta$ are the action-angle variables of the particle unperturbed motion, φ, ϑ being the toroidal and poloidal angles, respectively,

$$V_{sn} = \frac{ei}{2\pi^2\omega} \oint d\vartheta\, d\varphi (\vec{v} \cdot \tilde{\vec{E}}) \exp(i\omega t - is\vartheta + in\varphi) \qquad (3)$$

is the so-called matrix element of the wave-particle resonance. For interactions of fast ions with Alfvénic perturbations we can neglect the contribution of $\tilde{E}_{\parallel}$ and $\tilde{A}_{\perp}$, taking $\vec{v} \cdot \tilde{\vec{E}} \approx v_D \nabla \tilde{\Phi}$, where v_D is the drift velocity, $\tilde{\Phi}$ is the scalar potential of the electromagnetic field. Expanding the unperturbed part of the Hamiltonian at the resonance and keeping only the resonant term of the perturbation, we obtain the resonance width as follows:

$$(\Delta J_{\varphi})_{island} = 4 \left| \frac{nV_{sn}}{\Omega'} \right|^{1/2}. \qquad (4)$$

Using the relationship

$$\frac{\Delta W}{\omega} = -\frac{\Delta J_{\varphi}}{n} = \frac{\Delta J_{\vartheta}}{s} \qquad (5)$$

and evaluating V_{sn} for passing particles, we can write the following analytical expression for the width of the resonance islands of passing fast ions

$$\frac{(\delta W)_{island}}{W} = 4\chi \left| (1+\chi^2) \frac{1}{\kappa R k_s} \frac{\tilde{B}_r}{B} \right|^{1/2} \cdot \gamma^{-1/2}, \qquad (6)$$

where

$$\gamma = 1 - \frac{\omega_{\varphi}^2 s^2}{\omega^2} \frac{\chi \rho R €}{r^2 \kappa q}, \qquad (7)$$

W is the particle energy; r is the radial coordinate; ρ is the Larmor radius; ω_B is the cyclotron frequency; $\tilde{B}_r$ and B are the wave and equilibrium magnetic fields; respectively; *q(r)* is the safety factor; € is the magnetic shear; $k_s=(s/q-n)$; κ is the plasma cross-section ellipticity; $\chi = v_{\parallel}/v$ is the pitch angle cosine with *v* the velocity.

Let us use Eq. (6) to evaluate the number of modes needed to stochastize the ion motion in a certain energy range. We take the plasma parameters of the experiments described in Ref. [2] (B=0.45 T; the energy of injected ions W_{inj} = 90 keV; the electron density $n_e \approx (5...6) \cdot 10^{19} m^{-3}$). Then the instabilities with $f \sim 1$ MHz correspond to $k_{\parallel} \sim 6$. Taking, in addition, χ=0.8, κ= 1.5, *a*=0.85 m, *R*=1 m , we find that the width of a resonance island is $(\delta W)_{island}/W$~0.1 for the mode amplitude $\tilde{B}_r / B \sim 5 \cdot 10^{-3}$ and $(\delta W)_{island}/W$~0.03 for $\tilde{B}_r / B \sim 5 \cdot 10^{-4}$. This means that about 5 modes with the amplitude of $5 \cdot 10^{-3}$ or 15 modes with the amplitude of $5 \cdot 10^{-4}$ are sufficient to create a stochastic motion zone reaching from W_{inj} to $W_{inj}/2$. These numbers seem plausible for NSTX experiments.

The slowing-down process caused by the waves is accompanied by the radial motion of the ion. The estimates above are based on the assumption that the radial position of the beam ions does not change during slowing down. The ion displacement is, indeed, small at high ω, as follows from Eq. (5) (which, in fact, describes the characteristics of the quasilinear equation [5]):

$$\Delta r^2 = \frac{m}{\omega_B \omega} \Delta v_{\parallel}^2. \qquad (8)$$

We take the inequalities $\Delta v_{\parallel}^2 / v_{\parallel 0}^2 > 1/2$ (the subscript '0' corresponds to the birth energy) and $\Delta r^2 < a^2/4$ as conditions that the particles are not lost while losing over a half of their energies. Then Eq. (8) yields $\omega > m \cdot 130$ kHz, which is satisfied for most high-frequency GAE modes observed in NSTX.

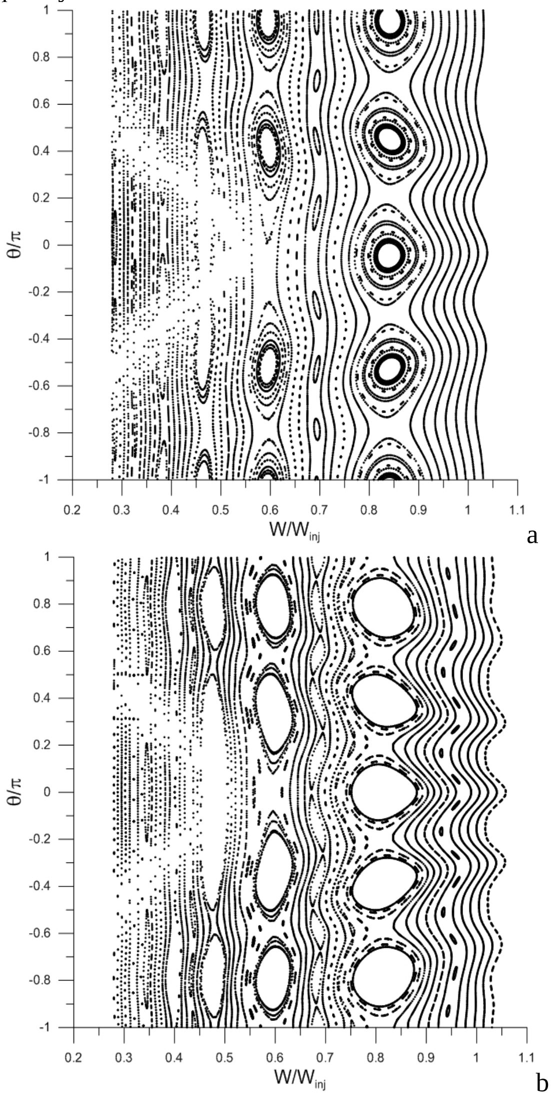

Fig. 2. Poincare plots for the modes $(m,n) = (-3,-5)$ (a) and (m,n) = (-4; -6) (b). Several chains of resonance islands are seen, the number of the islands in the chain being equal to the poloidal resonance number s

2. NUMERICAL SIMULATIONS

The analytical results presented above are based on the assumption that an interval of energies is stochastic when the total width of all resonance islands is sufficient to cover it. Although this statement seems credible, being similar to the well-known Chirikov criterion [4], it needs to be verified by numerical experiment. Having this in mind, we developed two guiding centre codes.

The first code constructs Poincaré maps to study the width of resonance islands induced by a single Alfvén mode. The second one is developed to study the overlap of resonances of multiple waves and reveal the resulting stochastic domains. The idea is that we seed a set of test particles with different energies at a certain radius and adiabatically (i.e., sufficiently slowly) turn on the perturbation, then we adiabatically turn off the wave and find the phase space regions where the energies of the particles have changed [6]. Energy intervals in which the particles become randomly displaced after the process indicate the stochasticity and/or resonance islands. Regions with negligible energy displacement of the particles correspond to domains where KAM-surfaces between islands exist. This gives us a possibility to find the energy spaces in which the particles are able to lose energy rapidly.

For our simulations we took the plasma parameters described above and the safety factor profile of the form $q(r) = q_0 - (q_a - q_0)\,(r/a)^2$, where $q_a = 3.1$, $q_0 = 0.1$, $a = 0.85$ m is the minor radius of the torus. In the experiments [2], modes with frequencies in the range of 0.5 to 1.1 MHz, and toroidal wave numbers n from -2 to -7 were observed. For our numerical simulation we took modes with these parameters, choosing m so that $\omega \approx k_{||} v_A$, where $k_{||} = (m/q - n)/R$ and v_A is the Alfvén velocity, and, at the same time, the waves were resonant with particles of interest.

In Fig. 2, Poincaré plots for different modes in NSTX are shown to illustrate the islands produced by GAE modes with $\tilde{B}_r / B$ of order of 10^{-3} and the frequency $f = 0.97$ MHz. The resonance width evaluated from Eq. (6) is in agreement with the code results (with discrepancy of 20...30%).

In Fig. 3, the energy intervals calculated by the second (adiabatic) code for the same modes are shown. The results of the two codes for a single mode are in good agreement. We observe that energy intervals in which the particles wander due to the mode are at the same places as the resonance islands in Fig. 2.

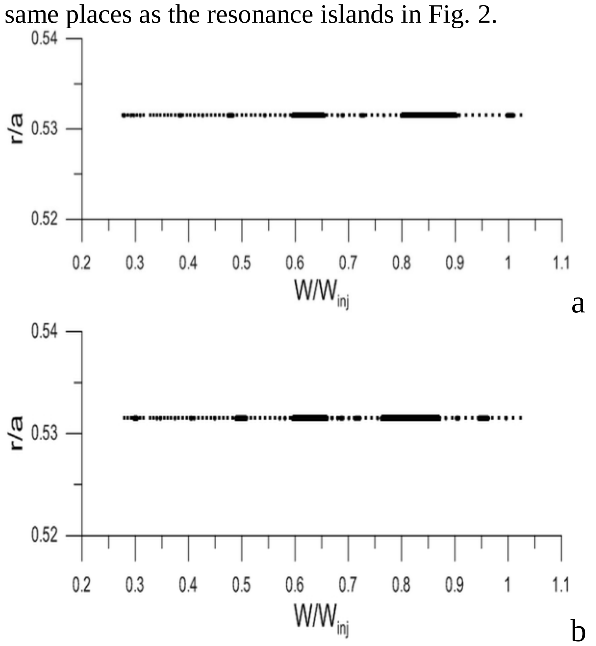

Fig. 3. Consequences of adiabatic switching on and off a mode for particles at a certain radius. Horizontal bars show energy intervals in which the particles become randomly displaced after the process. Dots show particles with negligible displacements. The calculations were carried out for the same modes and the same mode amplitudes as in Fig. 2

Theory predicts that the widths of these intervals should be $\sqrt{2}$ times less than the island widths (in order to provide the same phase space volume). We observe that this is really the case (an exception is the resonance at $W \approx 0.8 W_{inj}$ in Fig. 3,b, which is somewhat wider). Thus, the results of the two codes for single modes are in reasonable agreement, which proves that the approach implemented in the adiabatic code is viable.

A typical example of calculations for several (two) modes is shown in Fig. 4. Comparing this figure with Figs. 2 and 3, we observe that a most part of the energy interval in Fig. 4 is covered by horizontal bars. These bars are extensive energy intervals where particles wander, which appear due to the overlap of the islands shown in Figs. 2, 3. Detailed verification of our conclusions on the channelling threshold amplitudes is yet to be done with this code.

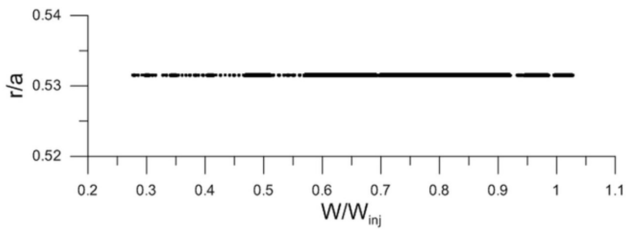

Fig. 4. Domains of stochastic motion produced by simultaneous adiabatic switching on and off the modes (m,n)=(-3,-5) and (m,n) = (-4,-6). The mode amplitudes are the same as in Figs. 2 and 3

DISCUSSION AND CONCLUSIONS

Our analytical estimates show that about 5 GAE modes with the amplitudes of $\tilde{B}_r / B \sim 5\cdot 10^{-3}$ are sufficient to make stochastic the motion of injected ions in NSTX in the energy interval from 90 to 50 keV. This means that these modes are capable to extract a half of the fast ion energy. The radial displacements of the particles in the course of the energy extraction are moderate and should not result in premature losses of the particles. The normalized mode amplitudes of order of $5\cdot 10^{-3}$ are usually considered as realistic in NSTX, and spectrograms of instabilities in NSTX (see, e. g., [1, 2]) show that 5…10 unstable modes are often observed simultaneously. Thus, our estimates confirm the conclusions of Ref. [3].

The numerical simulations have shown that our two codes agree with each other and with analytical estimates. It is planned to use them for detailed verification of our analytical results.

It should be mentioned that our analysis does not take account of cyclotron resonances, which are known to be an important mechanism of the excitation of high-frequency instabilities in NSTX [7]. This means that the stochasticity threshold found above is, most probably, overestimated – even lower mode amplitudes are sufficient to take away the energy from the particles. To include the cyclotron resonances into our consideration (we are planning to do this in the future), we will have to abandon the guiding-centre approximation. This will require more time-consuming numerical simulations. In addition, it will be difficult to use the Poincaré map approach in this case. However, we expect that the approach based on the adiabatic variation of the perturbation amplitude will remain efficient.

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ПРОСТРАНСТВЕННОЕ КАНАЛИРОВАНИЕ ЭНЕРГИИ И СТОХАСТИЗАЦИЯ ДВИЖЕНИЯ БЫСТРЫХ ИОНОВ ВЫСОКОЧАСТОТНЫМИ НЕУСТОЙЧИВОСТЯМИ ПЛАЗМЫ

М.Г. Тищенко, Ю.В. Яковенко

Устанавливаются условия, при которых множественные альфвеновские собственные моды способны отнять значительную долю энергии у быстрых ионов для возможной передачи в другую область пространства (пространственного каналирования). Это может происходить, если резонансные острова неустойчивостей перекрываются, образуя обширную стохастическую зону в фазовом пространстве быстрых ионов. Выводится аналитическое выражение для ширины резонансного острова, образованного альфвеновской собственной модой в фазовом пространстве. Оцениваются количество и амплитуда мод, необходимые для возникновения стохастической зоны в заданном диапазоне энергий. Кратко описываются два кода, предназначенные для численной проверки этих оценок. Представлены первые результаты этих кодов.

ПРОСТОРОВЕ КАНАЛЮВАННЯ ЕНЕРГІЇ ТА СТОХАСТИЗАЦІЯ РУХУ ШВИДКИХ ІОНІВ ВИСОКОЧАСТОТНИМИ НЕСТІЙКОСТЯМИ ПЛАЗМИ

М.Г. Тищенко, Ю.В. Яковенко

Встановлюються умови, за яких множинні альфвенові власні моди є здатними забрати значну частку енергії в швидких іонів для можливої передачі в іншу область простору (просторового каналювання). Це може траплятись, якщо резонансні острови нестійкостей перекриваються і утворюють широку стохастичну зону в фазовому просторі швидких іонів. Виводиться аналітичний вираз для ширини резонансного острова, створеного альфвеновою власною модою в фазовому просторі. Оцінюються кількість та амплітуда мод, що потрібні для утворення стохастичної зони в певному діапазоні енергій. Стисло описуються два коди, призначені для числової перевірки цих оцінок. Представлено перші результати цих кодів.