NONQUASINEUTRAL CURRENT STRUCTURES IN PLASMAS WITH A ZERO NET CURRENT

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A nonquasineutral vortex structure with a zero net current is described that arises as a result of electron drift in crossed magnetic and electric fields, the latter being produced by charge separation on a spatial scale of about the magnetic Debye radius $r_B = |B|/(4\pi e n_e)$. In such a structure with radius r ~ r_B, the magnetic field maintained by a drift current on the order of the electron Alfven current $J_{Ae} = m_e c^3/(2e)$ and can become as strong as $B \cong \sqrt{4\pi n_e m_e c^2}$. The system with closed current that is considered in the present paper can also serve as a model of hot spots in the channel of a Z-pinch.
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1. INTRODUCTION

In the resent years, investigations have been carried out with the nonquasineutral current structures whose size varies from a few microns in pinches to billions of kilometers in cosmic space and in which charges are separated on spatial scales of about the magnetic Debye radius $r_B \sim B/(4\pi e n_e)$ and an electric field is generated due to the Hall effect – the factor that set electrons into relativistic drift motion [1-3]. An important feature of the resulting quasi-equilibrium is the onset of crossed electric and magnetic fields $|E| \sim |B|$. However, in the structures considered theoretically and numerically in [1-3], the net current in the quasi-equilibrium states under analysis was nonzero. The question to be answered is then how laser pulses or other extreme energy inputs (e.g., in Z-pinches) can drive a nonzero net current in such isolated structures, The obtained in the further investigations result that the net current in such quasi-neutral structures is zero substantially simplifies the construction of the scenario for relaxation to them. The nonquasineutral current structures in question could serve as a model of X-ray-emitting hot plasma spots on spatial scales of c/ω_{pe} at electron densities of $n_e \sim 10^{20} - 10^{23}$ cm^{-3}, which have been achieved in experiments with Z – pinches [4].

2. THE MAIN EQUATIONS

We describe the electron plasma by equation of motion for cold relativistic electrons in the following modified form [1-3]

$$\frac{\partial p_e}{\partial t} + \nabla(\gamma m_e c^2) = -e\overset{\circ}{E} - \frac{e}{c}[\overset{\circ}{v}_e \times \overset{\circ}{\Omega}], \qquad (1)$$

$$\overset{\circ}{\Omega} = \overset{\circ}{B} - \frac{c}{e}[\nabla \times \overset{\circ}{p}_e], \qquad (2)$$

and Maxwell equations

$$[\nabla \times \overset{\circ}{B}] = \frac{4\pi e}{c}(z_i n_i \overset{\circ}{v}_i - n_e \overset{\circ}{v}_e) + \frac{1}{c}\frac{\partial E}{\partial t}, \qquad (3)$$

$$-\frac{1}{c}\frac{\partial B}{\partial t} = [\nabla \times \overset{\circ}{E}], \ \nabla \cdot E = 4\pi e(z_i n_i - n_e). \qquad (4)$$

Here $v_e, \bar{v}_i$ are the electron and the ion velocities, $p_e = \gamma m_e v_e$, $\gamma = 1/\sqrt{1 - \overset{\circ}{v}_e^2/c^2}$, n_e and n_i are the electron and the ion densities, z_i is the ion charge number, E is the electric field, B is the magnetic field.
At first, we use the spherical coordinates (r, θ, φ).
Making use the stationary equations (1) and introducing the vector potential A, one can obtain the expressions for the electric and magnetic fields components

$$eE_r = -\frac{\partial}{\partial r}(\gamma m_e c^2) - \frac{e}{c}\frac{v_{e\varphi}}{r}\frac{\partial}{\partial r}\left[r\left(A_\varphi - \frac{c}{e}p_{e\varphi}\right)\right] \qquad (5)$$

$$eE_\theta = -\frac{1}{r}\frac{\partial}{\partial\theta}(\gamma m_e c^2) - \frac{e}{c}\frac{v_{e\varphi}}{r\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\left(A_\varphi - \frac{c}{e}p_{e\varphi}\right)\right], \qquad (6)$$

$$B_r = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\varphi), \ B_\theta = -\frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi). \qquad (7)$$

Further on, it will be assumed that the ion velocity is equal to zero and the ion density is constant.
Now, inserting expressions (5) – (7) into the stationary Eq.(3) and the second Eq.(4) and eliminating the electron density n_e, one can obtain, after going to the function b = a r sinθ and the variable s = r sinθ, the following final equation

$$\frac{d}{ds}\left(\frac{1}{\gamma}\frac{db}{ds}\right) - \frac{\gamma}{s}\frac{db}{ds} = N_i\beta\gamma s - \beta\gamma\frac{d}{ds}(\beta^2\gamma). \qquad (8)$$

Here $a = \frac{eA_\varphi}{m_e c^2}$, $\beta = \frac{v_{e\varphi}}{c}$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$,

$$N_i = \frac{4\pi z_i^2 e^2 n}{m_e c^2}.$$

At last, the potentiality condition for the electric field $\mathbf{E}$ results in the additional connection between functions b and β

$$b-\beta\gamma s=f\left(\frac{\beta}{s}\right), \qquad (9)$$

where f(x) is the arbitrary function of its argument.

In order to obtain the localized configuration, it is necessary that β(s) should vanish both for s = 0 and for $s\to\infty$. This is why it is necessary to further analyze Eq. (8).

The asymptotic behavior of the function β(s) at s = 0 and $s\to\infty$ can be examined by rewriting Eq.(8) in a somewhat different form. We substitute the expression b=f+βγs, which follows from (9), into Eq.(8) and differentiate the function f(ξ) in the resulting equation with respect to the argument ξ = β/s to obtain

$$F_1\frac{df}{d\xi}+F_2\frac{d^2f}{d\xi^2}+\frac{d^2}{ds^2}(\beta\gamma)-\beta\frac{d^2\gamma}{ds^2}+\frac{1}{s}\gamma\frac{d\beta}{ds}-$$

$$-\frac{1}{s^2}\beta\gamma=N_i\beta, \qquad (10)$$

$$F_1=\frac{1}{s^2\gamma^2}\frac{d^2\beta}{ds^2}-\frac{1}{s^3}\left(1+\frac{2}{\gamma^2}\right)\frac{d\beta}{ds}+\frac{1}{s^4}\left(1+\frac{2}{\gamma^2}\right)\beta-$$

$$\frac{1}{s^2}\beta\left(\frac{d\beta}{ds}\right)^2+\frac{1}{s^3}\beta^2\frac{d\beta}{ds},$$

$$F_2=\frac{1}{s\gamma^2}\left(\frac{1}{s}\frac{d\beta}{ds}-\frac{\beta}{s^2}\right)^2.$$

For f=0, i.e. for an electron fluid that is free of vorticity, there not exists the solution which meets the physically reasonable conditions at s = 0 and $s\to\infty$ together. For $f\neq 0$, the asymptotic behavior of the function β(s) at s = 0 differs radically from that in the previous case. If $df/ds\neq 0$, then, in the vicinity of the point s = 0, the function β(s) satisfies the equation

$$\frac{d^2\beta}{ds^2}-\frac{3}{s}\frac{d\beta}{ds}+\frac{3}{s^2}\beta=0, \qquad (11)$$

which has the solutions

$$\beta=C_1s+C_2s^3 . \qquad (12)$$

Now, these two solutions near the point s = 0 are physically allowed. In addition, the constant C_1 is an eigenvalue of nonlinear equation (6) and should be found from the condition for β to vanish at $s\to\infty$ by the taking into account the asymptotic (12).

3. THE FILAMENT STRUCTURE AND THE ESTIMATE OF THE MAGNETIC FIELD VALUE

Let us examine in more detail the structure of the current equilibrium state under consideration, namely, the state that arises as a result of the balance between the electric and magnetic forces and also the centrifugal force

$$\frac{\gamma\beta^2}{s}=\frac{eE_\rho}{m_ec^2}+\beta\frac{eB_z}{m_ec^2}. \qquad (13)$$

Here

$$\frac{eB_z}{m_ec^2}=\frac{1}{s}\frac{db}{ds} \qquad (14)$$

is the z component of the magnetic field. The expression for the radial electric field component in the (x,y) plane follows from Eq.(13)

$$\frac{eE_\rho}{m_ec^2}=\frac{\beta}{s}\left(\beta\gamma-\frac{db}{ds}\right). \qquad (15)$$

Now, we change the spherical coordinates to the cylindrical. The use of the spherical coordinates at the first stage of the equation transformation allows to obtain the additional condition (9).

In the stationary case, it is convenient to convert Eq. (3) and the second Eq.(4) into the form:

$$N_e\beta=\frac{d}{ds}\left(\frac{1}{s}\frac{db}{ds}\right),\quad N_e=\frac{4\pi e^2n_e}{m_ec^2}, \qquad (16)$$

$$N_e-N_i=\frac{1}{s}\frac{d}{ds}\beta\left(\frac{db}{ds}-\beta\gamma\right). \qquad (17)$$

From Eq. (16), we can see that the dimensionless electron current density N_e β is expressed in terms of the derivative of the magnetic field component B_z with respect to s. At the point s_0, at which the magnetic field has maximum, the dimensionless current density N_e β vanishes, which corresponds to the change in the sign of the velocity component $v_{e\varphi}$.

Hence, since the electron velocity equals zero at the axis of the vortex structure and at the point s = s_0, it has a maximum at a certain intermediate point s = s_1 for $C_1 > 0$. According to Eq.(15), the electric field component E_ρ vanishes at the point s = s_0. Therefore, by integrating Eq.(17), we can show that the total charge in the region $0\le s\le s_0$ is equal to zero.

It is easy to see that, in the region s > s_0, the electron velocity is negative, has a minimum at a certain point s = s_2, and tends to zero at infinity.

For $C_1 < 0$ the signs of the velocity and the magnetic field are changed, but the sign of the electric field is conserved according to Eq. (13).

From Eq. (17), if the estimate of $db/ds\propto s^3$ is taken into account, over the region of the small values s one can obtain

$$N_e(0)=N_i-2C_1^2. \qquad (18)$$

Thus, near the axis there exists the excess of ions and the electric field is positive. Therefore, these ions expand towards the periphery in the considered quasi-equilibrium.

The estimation of the value of the magnetic field in the filament from Eq. (14) by the account $N_i s^2\sim 1$ gives:

$$B \cong \frac{m_e c^2}{e}\sqrt{N_i} \cong \sqrt{4\pi n_e m_e c^2}\,. \qquad (19)$$

In accordance with [5], we set $n_e \cong 1.5 \cdot 10^{22}\,cm^{-3}$ to obtain $B \cong 4 \cdot 10^8$ G. This estimate is reasonable close to the value $B \cong 0.7 \cdot 10^9$ G, which was measured in the experiments on the irradiation of a plasma by the high-power laser pulses [5]. Note that such strong magnetic field is maintained by the drift current that flows on micron scales and whose magnitude is on the order of the electron Alfven current $J_{Ae} = m_e c^3/(2e) \cong 8.5$ kA.

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НЕКВАЗИНЕЙТРАЛЬНЫЕ ТОКОВЫЕ СТРУКТУРЫ В ПЛАЗМЕ С РАВНЫМ НУЛЮ ПОЛНЫМ ТОКОМ

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Описана неквазинейтральная вихревая структура с равным нулю полным током, которая возникает в результате дрейфа электронов в скрещённых магнитном и электрическом полях, причём последнее создаётся при разделении зарядов на пространственном масштабе порядка магнитного дебаевского радиуса $r_B = |B|/(4\pi e n_e)$. В такой структуре с размером r ~ r_B магнитное поле поддерживается дрейфовым током порядка электронного альфвеновского тока J_{Ae} =m_e c^3/(2e) и может достигать значений $B \cong \sqrt{4\pi n_e m_e c^2}$. Система с замкнутым током, подобная рассмотренной может также служить моделью горячих точек в канале Z-пинча.

НЕКВАЗІНЕЙТРАЛЬНІ ТОКОВІ СТРУКТУРИ В ПЛАЗМІ З РІВНИМ НУЛЮ ПОВНИМ ТОКОМ

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Описано неквазінейтральну вихрову структуру з рівним нулю повним струмом, що виникає в результаті дрейфу електронів у схрещених магнітному й електричному полях, причому останнє створюється при поділі зарядів на просторовому масштабі порядку магнітного дебаєвського радіуса $r_B = |B|/(4\pi e n_e)$. У такій структурі з розміром r ~ r_B магнітне поле підтримується дрейфовим струмом порядку електронного альфвенівського струму J_{Ae} =m_e c^3/(2e) і може досягати значень $B \cong \sqrt{4\pi n_e m_e c^2}$. Система з замкнутим струмом, подібна до тієї, що розглянуто, може також служити моделлю гарячих крапок у каналі Z-пинча.