ELECTRON LANDAU DAMPING OF RADIO-FREQUENCY WAVES IN A TOROIDAL PLASMA WITH SOLOV'EV EQUILIBRIUM

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Longitudinal permittivity elements are derived for RF waves in a toroidal plasma with Solov'ev type equilibrium by solving the drift-kinetic equation for untrapped and trapped particles. Our dielectric characteristics are suitable to estimate the wave dissipation by electron Landau damping in the frequency range of the Alfvén and fast magnetosonic waves, for both the large and low aspect ratio tokamaks with circular, elliptic and D-shaped magnetic surfaces. Contributions of the untrapped and usual trapped electrons to the imaginary part of the longitudinal permittivity are computed for a spherical tokamak plasma in the wide range of wave frequencies at the different magnetic surfaces.

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1. INTRODUCTION

Kinetic wave theory in tokamaks and stellarators should be based on the solution of Vlasov-Maxwell's equations [1-3]. However, this problem is not simple even in the scope of the linear theory since to solve the differential wave equations one should use the complicated integral dielectric characteristics valid in the given frequency range for realistic two- or three-dimensional plasma models. The form of the dielectric tensor components depends substantially on the geometry of an equilibrium magnetic field. In this paper, we analyze the contributions of untrapped and usual trapped particles to the imaginary part of the longitudinal permittivity elements for RF waves in an axisymmetric D-shaped toroidal plasma with Solov'ev type equilibrium under the condition when the additional groups of the so-called *d*-trapped particles are absent in the plasma volume, using an approach developed[4] for low aspect ratio tokamaks with concentric circular, elliptic and D-shaped magnetic surfaces.

2. PLASMA MODEL

The D-shaped transverse magnetic surface cross-sections corresponding to Solov'ev equilibrium[5] are not concentric and can be plotted by the following parametric equations for cylindrical (R,ϕ,Z) and quasi-toroidal coordinates (ρ,θ,ϕ):

$$R = \sqrt{R_0^2 + 2aR_1\rho\cos\theta}\ ,\ \phi = \phi\ ,\ Z = \frac{-\alpha\ aR_1\rho\ \sin\theta}{\sqrt{R_0^2 - \delta + 2aR_1\rho\cos\theta}},$$

where R_0 is the radius of the main magnetic axis; R_1 is the major radius of the external magnetic surface; a is the (minor) plasma radius; b/a is the elongation; d/a is the triangularity; ρ is the non-dimensional radius of a local magnetic surface ($0 \le \rho \le 1$); θ is the poloidal angle ($-\pi \le \theta \le \pi$); and the additional definitions are

$$\delta = \frac{(R_1 - a)^2(R_1 + a)^2 - (R_1 - d)^4}{2(R_0^2 - (R_1 - d)^2)},$$

$$\alpha^2 = \frac{2b^2}{R_0^2 - (R_1 - d)^2}.$$

In this case, the module of an equilibrium magnetic field is

$$H_0 = H_{0\phi}G(\rho,\theta) = H_{0\phi}\sqrt{\frac{1 + \dfrac{\beta_o\rho^2}{1+\alpha^2}g(\rho,\theta)}{1 + 2\dfrac{aR_1}{R_0^2}\rho\cos\theta}}, \qquad (1)$$

where p_0, $H_{0\phi}$ and $\beta_o = 8\pi p_0/H_{0\phi}^2$ are, respectively, the plasma pressure, toroidal magnetic field and central *beta*-value on the main magnetic axis (R=R_0 or ρ=0); and

$$g(\rho,\theta) = \sin^2\theta\left(1 + 2\frac{aR_1}{R_0^2}\rho\cos\theta\right) + \frac{\delta}{R_0^2}\cos^2\theta +$$

$$+\alpha^2\frac{1 + 2\dfrac{aR_1}{R_0^2}\rho\cos\theta}{\left(1 + \dfrac{2aR_1\rho}{R_0^2 - \delta}\cos\theta\right)^2}\left[\cos\theta + \frac{aR_1\rho}{R_0^2 - \delta}\left(1 + \cos^2\theta\right)\right]^2$$

After solving the drift-kinetic equation for the perturbed distribution functions

$$f(\rho,\theta,v_\parallel,v_\perp) = \sum_{s=\pm1} f_s(\rho,\theta,v,\mu)\exp(-i\omega t + in\phi), \qquad (2)$$

parallel current density components can be calculated as

$$j_\parallel(\rho,\theta) = \pi eG(\rho,\theta)\sum_s^{\pm1} s\int_0^\infty v^3\int_0^{1/G(\rho,\theta)} f_s(\rho,\theta,v,\mu)d\mu\, dv$$

where $v = \sqrt{v_\parallel^2 + v_\perp^2}$, $\mu = \dfrac{v_\perp^2}{v_\parallel^2 + v_\perp^2}\dfrac{1}{G(\rho,\theta)}$.

3. LONGITUDIONAL PERMITTIVITY

To evaluate the longitudinal permittivity elements we use the Fourier expansions of the current density and electric field over $\bar\theta$ (new poloidal angle where the $\mathbf{H}_0$ field lines are straight):

$$\frac{j_\parallel(\theta)}{G(\rho,\theta)}\left(1 + \frac{2aR_1\rho}{R_0^2}\cos\theta\right)\sqrt{1 + \frac{2aR_1\rho}{R_0^2 - \delta}\cos\theta} = \sum_m^{\pm\infty} j_\parallel^m e^{im\bar\theta}$$

$$\frac{E_\parallel(\theta)G(\rho,\theta)}{\sqrt{1 + \dfrac{2aR_1\rho}{R_0^2 - \delta}\cos\theta}} = \sum_{m'}^{\pm\infty} E_\parallel^{m'} e^{im'\bar\theta}. \qquad (3)$$

As a result,

$$\frac{4\pi i}{\omega} j_\parallel^m = \sum_{m'}^{\pm\infty} \varepsilon_\parallel^{m,m'} E_\parallel^{m'} = \sum_{m'}^{\pm\infty} (\varepsilon_{\parallel,u}^{m,m'} + \varepsilon_{\parallel,t}^{m,m'})E_\parallel^{m'} \qquad (4)$$

and the contributions of u-untrapped and usual t-trapped particles to $\varepsilon_\parallel^{m,m'}$ are

$$
\varepsilon_{\parallel,u}^{m,m'} = \frac{\omega_p^2 R_0^2 q_o^2 \left(1+\dfrac{2aR_1\rho}{R_0^2}\right)\sqrt{1+\dfrac{2aR_1\rho}{R_0^2-\delta}}}{4v_T^2\pi^2\,\overline{\Pi}\left(\pi/2,\rho\right)}\times \tag{5}
$$

$$
\times\sum_{p=-\infty}^{\infty}\int_0^{\mu_u}\frac{T_u A_p^m A_p^{m'}}{(p+nq)^2}\left[1+2u_p^2+2i\sqrt{\pi}\,u_p^3 W(u_p)\right]d\mu ,
$$

$$
\varepsilon_{\parallel,t}^{m,m'} = \frac{\omega_p^2 R_0^2 q_o^2 \left(1+\dfrac{2aR_1\rho}{R_0^2}\right)\sqrt{1+\dfrac{2aR_1\rho}{R_0^2-\delta}}}{4v_T^2\pi^2\,\overline{\Pi}\left(\pi/2,\rho\right)}\times \tag{6}
$$

$$
\times\sum_{p=1}^{\infty}\int_{\mu_u}^{\mu_t}\frac{T_t}{p^2} B_p^m B_p^{m'}\left[1+2v_p^2+2i\sqrt{\pi}\,v_p^3 W(v_p)\right]d\mu .
$$

Here we have used the following definitions:

$$
A_p^m = \int_0^{\pi}\cos\left[(m+nq)\bar{\theta}(\eta)-(p+nq)2\pi\frac{\tau(\eta)}{T_u}\right]d\eta ,
$$

$$
W(z) = \exp\left(-z^2\right)\left(1+\frac{2i}{\sqrt{\pi}}\int_0^{z}\exp\left(t^2\right)dt\right),
$$

$$
B_p^m = \int_0^{\theta_t}\cos\left[(m+nq)\bar{\theta}(\eta)-2\pi p\frac{\tau(\eta)}{T_t}\right]d\eta +
$$

$$
+(-1)^{p+1}\int_0^{\theta_t}\cos\left[(m+nq)\bar{\theta}(\eta)+2\pi p\frac{\tau(\eta)}{T_t}\right]d\eta ,
$$

$$
q(\rho) = \frac{q_o}{1+2\dfrac{aR_1}{R_0^2}\rho}\sqrt{\frac{1+\dfrac{\beta_o\delta\rho^2}{R_0^2(1+\alpha^2)}}{1+\dfrac{2aR_1\rho}{R_0^2-\delta}}}\times
$$

$$
\times\frac{2}{\pi}\Pi\left(\frac{\pi}{2},\frac{4aR_1\rho}{R_0^2+2aR_1\rho},\sqrt{\frac{4aR_1\rho}{R_0^2-\delta+2aR_1\rho}}\right),
$$

$$
q_o = q(0) = \frac{aR_1\sqrt{1+\alpha^2}}{\sqrt{\beta_o}R_0\sqrt{R_0^2-\delta}} ,
$$

$$
\Pi(\eta,\lambda,\kappa) = \int_0^{\eta}\frac{d\varphi}{\left(1-\lambda\sin^2\varphi\right)\sqrt{1-\kappa^2\sin^2\varphi}} ,
$$

$$
\bar{\theta}(\theta) = \pi\frac{\Pi\left(\dfrac{\theta}{2},\dfrac{4aR_1\rho}{R_0^2+2aR_1\rho},\sqrt{\dfrac{4aR_1\rho}{R_0^2-\delta+2aR_1\rho}}\right)}{\Pi\left(\dfrac{\pi}{2},\dfrac{4aR_1\rho}{R_0^2+2aR_1\rho},\sqrt{\dfrac{4aR_1\rho}{R_0^2-\delta+2aR_1\rho}}\right)},
$$

$$
\tau(\theta) = \int_0^{\theta}\frac{G(\rho,\eta)\ d\eta}{\sqrt{1-\mu G(\rho,\eta)}\sqrt{1+\dfrac{2aR_1\rho}{R_0^2-\delta}\cos\theta}} ,
$$

$$
\mu_u = \sqrt{\frac{1-2\dfrac{aR_1}{R_0^2}\rho}{1+\dfrac{\beta_o\rho^2}{1+\alpha^2}\left(\dfrac{\delta}{R_0^2}+\alpha^2\left(1-2\dfrac{aR_1}{R_0^2}\rho\right)\right)}} ,
$$

$$
\mu_t = \sqrt{\frac{1+2\dfrac{aR_1}{R_0^2}\rho}{1+\dfrac{\beta_o\rho^2}{1+\alpha^2}\left(\dfrac{\delta}{R_0^2}+\alpha^2\left(1+2\dfrac{aR_1}{R_0^2}\rho\right)\right)}} ,
$$

$$
v_T^2 = 2T/M\ ,\ \ T_u = 2\tau(\pi)\ ,\ \ T_t = 4\tau(\theta_t)\ ,
$$

$$
u_p = \frac{\omega T_u R_0 q_o}{2\pi\,|p+nq|\,v_T}\ ,\ \ v_p = \frac{\omega T_t R_0 q_o}{2\pi p v_T}\ ,\ \ \omega_p^2 = \frac{4\pi Ne^2}{M} .
$$

The reflection points $\pm\theta_t$ for the t-trapped particles should be defined numerically by solving the equation $1-\mu G(\rho,\theta) = 0$.

Note, Eqs. (5,6) describe the contributions of any kind of untrapped and trapped particles to $\varepsilon_{\parallel,u}^{m,m'}$ and $\varepsilon_{\parallel,t}^{m,m'}$. The corresponding expressions for electrons and ions can be obtained from (5,6) by replacing the temperature T, density N, mass M, charge e by the electron T_e, N_e, m_e, e_e and ion T_i, N_i, M_i, e_i parameters, respectively.

One of the main mechanisms of the RF plasma heating is the electron Landau damping of waves due to the Cherenkov resonance interaction of $E_\parallel$ with the trapped and untrapped electrons. As a result, after averaging in time and poloidal angle, the wave power absorbed by the trapped and untrapped electrons, $P = \mathrm{Re}(E_\parallel\cdot j_\parallel^*)/2$ can be estimated by the expression

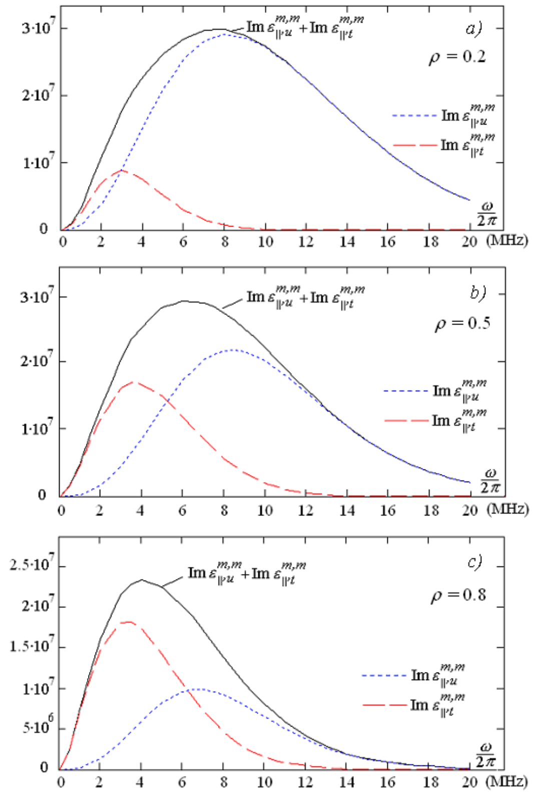
$$
P = \frac{\omega}{8\pi}\sum_{m}^{\pm\infty}\sum_{m'}^{\pm\infty}\mathrm{Im}\,\varepsilon_{\parallel}^{m,m'}\left(\mathrm{Re}\,E_\parallel^m\,\mathrm{Re}\,E_\parallel^{m'}+\mathrm{Im}\,E_\parallel^m\,\mathrm{Im}\,E_\parallel^{m'}\right), \tag{7}
$$

where $\mathrm{Im}\,\varepsilon_{\parallel,u}^{m,m'}$ and $\mathrm{Im}\,\varepsilon_{\parallel,t}^{m,m'}$ are the contributions of untrapped and t-trapped electrons to the imaginary part of $\varepsilon_{\parallel}^{m,m'}$: $\mathrm{Im}\,\varepsilon_{\parallel}^{m,m'} = \mathrm{Im}\,\varepsilon_{\parallel,u}^{m,m'}+\mathrm{Im}\,\varepsilon_{\parallel,t}^{m,m'}$. It means, under the same wave frequency and the electric field amplitude the wave dissipation by the untrapped and t-trapped electrons is differed by their different contributions to $\mathrm{Im}\,\varepsilon_{\parallel}^{m,m'}$.

4. NUMERICAL RESULTS

Now, let us consider the contributions of untrapped and usual t-trapped electrons to $\mathrm{Im}\,\varepsilon_{\parallel,u}^{m,m}$ and $\mathrm{Im}\,\varepsilon_{\parallel,t}^{m,m}$ in the spherical tokamak with $a = 50\,\mathrm{cm}$, $b = 90\,\mathrm{cm}$, $d = 15\,\mathrm{cm}$, $R_1 = 70\,\mathrm{cm}$, $R_0 = 86\,\mathrm{cm}$, $H_{0\phi} = 0.6\,\mathrm{T}$, $N(\rho) = 3\cdot 10^{13}\sqrt{1-\rho^2}$ cm^{-3}, $T(\rho) = 2000\sqrt{1-\rho^2}$ eV. Under these conditions (small elongation b/a=1.8 and $\beta_o = 0.067$), the additional groups of the d-trapped particles are absent in the plasma volume. Computations of the diagonal elements of the longitudinal permittivity elements are carried out for waves with poloidal and toroidal mode numbers $m=m'$=1 and n=2.

The dependence $\mathrm{Im}\,\varepsilon_{\parallel,u}^{m,m}$ and $\mathrm{Im}\,\varepsilon_{\parallel,t}^{m,m}$ on ω are presented in Fig. 1 for RF waves in our plasma model at the magnetic surfaces: *a)* ρ = 0.2, *b)* ρ =0.5 and *c)* ρ =0.8, respectively. As shown in these plots, the waves (usually, the low-frequency waves) interact effectively with the trapped electrons at the external magnetic surfaces, see e.g. Fig. 1*c*, where the fraction of the trapped particles increases and the fraction of untrapped particles decreases (and tends to zero in the spherical tokamaks if $a/R_1 \to 1$). By this reason, the effective heating of the trapped electrons is possible in tokamaks using the Alfvén waves, when the local Alfvén resonance condition is realized near the plasma boundary or in the region of the moderate radii. By comparing $\mathrm{Im}\,\varepsilon_{\parallel,u}^{m,m}$ and $\mathrm{Im}\,\varepsilon_{\parallel,t}^{m,m}$ in Figure, we see that the fast waves with a high phase velocity, $v_{ph} = \omega R_0 q_o T_u / [2\pi (m + nq)] > v_T$, dissipate mainly due to their transit-time resonant interaction with untrapped electrons. It means, using the fast waves, the favorable conditions can be created to transform the wave momentum into the momentum of the untrapped electrons leading the non-inductive current drive.



Contributions of untrapped and t-trapped electrons to $\mathrm{Im}\,\varepsilon_{\|,u}^{m,m}$ *and* $\mathrm{Im}\,\varepsilon_{\|,t}^{m,m}$ *versus* ω *at the different magnetic surfaces: a)* $\rho = 0.2$*, b)* $\rho = 0.5$*, c)* $\rho = 0.8$

CONCLUSIONS

Parallel permittivity elements are derived for RF waves in a 2D toroidal plasma with Solov'ev equilibrium and valid for both the large and low aspect ratio tokamaks in the wide range of the wave frequencies, mode numbers, and plasma parameters. Imaginary parts of the parallel permittivity are necessary to estimate the wave power absorbed by electron Landau damping, e.g., during the plasma heating and current drive generation.

The computations of $\mathrm{Im}\,\varepsilon_{\|,u}^{m,m'}$ and $\mathrm{Im}\,\varepsilon_{\|,t}^{m,m'}$ are carried out for RF waves in the D-shaped spherical tokamak plasma under the conditions when the *d*-trapped particles are absent in the plasma volume. In particular, it is shown that the main mechanism of the wave dissipation under the Alfvén wave heating is the bounce-resonance interaction of the RF waves with the trapped electrons.

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ЗАТУХАНИЕ ЛАНДАУ РАДИОЧАСТОТНЫХ ВОЛН НА ЭЛЕКТРОНАХ В ТОРОИДАЛЬНОЙ ПЛАЗМЕ С РАВНОВЕСИЕМ ПО СОЛОВЬЕВУ

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Элементы продольной диэлектрической проницаемости получены для радиочастотных волн в тороидальной плазме с равновесием по Соловьеву на основании решения дрейфово-кинетического уравнения для пролетных и запертых частиц. Наши диэлектрические характеристики могут быть использованы при вычислении поглощения волн за счет электронного затухания Ландау в диапазоне частот альфвеновских и быстрых магнитозвуковых волн в токамаках с круговыми, эллиптическими, D-образным сечениями магнитных поверхностей и произвольным аспектным отношением. Вклад пролетных и обычных запертых электронов в мнимую часть продольной проницаемости проанализирован численно в широком диапазоне частот на разных магнитных поверхностях в плазме сферического токамака.

ЗАГАСАННЯ ЛАНДАУ РАДІОЧАСТОТНИХ ХВИЛЬ НА ЕЛЕКТРОНАХ В ТОРОЇДАЛЬНІЙ ПЛАЗМІ З РІВНОВАГОЮ ЗА СОЛОВЙОВИМ

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Елементи повздовжньої діелектричної проникливості отримані для радіочастотних хвиль у тороідальній плазмі в умовах рівноваргі за Соловйовим на основі розв'язання дрейфово-кінетичного рівняння для частинок, що пролітають або заперті. Діелектричні характеристики, які отримані, можуть бути використані при розрахунках поглинання хвиль за рахунок електронного загасання Ландау в діапазоні частот альфвеновских та швидких магніто звукових хвиль в токамаках з перерізом магнітних поверхонь у вигляді кола, еліпсу, D-образним та довільним аспектним співвідношенням. Внесок електронів, що пролітають або заперті, в уявну частину повздовжньої проникливості проаналізовано чисельними методами в широкому діапазоні частот на різних магнітних поверхнях в плазмі сферичного токамаку.