NONLINEAR DECAY OF LANGMUIR WAVES INTO COUNTER-PROPAGATING SURFACE ONES

Yu.A. Akimov, N.A. Azarenkov

Karazin Kharkiv National University, 31 Kurchatov av., 61108 Kharkiv, Ukraine,
e-mail: akimov.yury@mail.ru

This paper presents a study of the nonlinear excitation of surface waves involving a Langmuir wave decay at a plasma-dielectric interface. The Langmuir wave is considered to be incident perpendicularly upon the interface, where it decays into a couple of counter-propagating surface ones. The excitation is analyzed for both the undamped and damped Langmuir waves.

PACS: 52.35.Mw, 52.40.Db

1. INTRODUCTION

At present, properties of surface waves (SWs) in bounded plasma-like structures are a subject of intensive theoretical and experimental research. Directions of the nonlinear effect studies determining properties of SWs in plasma waveguides are wide enough. Analysis of the SW dynamics in three-wave interactions is main among of them.

High-frequency volume waves, such as Langmuir waves, are well known to be essential for unmagnetized plasma. Their propagation in bounded plasma plays an important role in SW dynamics. They are responsible for the resonant damping of the SWs in inhomogeneous transient layers, as well as for their nonlinear damping, caused by the generation of a SW second harmonic at frequencies, close to the plasma one. Moreover, the Langmuir waves incident upon a dielectric surface can effectively decay into two SWs [1].

It should be marked that the results presented in paper [1] are restricted by the consideration of a dielectric with the permittivity $\varepsilon_d = 3$, when the Langmuir waves decay into a couple of the potential SWs. Thus, the decay of the Langmuir waves into the electromagnetic SWs that takes place at $\varepsilon_d < 3$ has been left without appropriate attention. Our paper is devoted to this aspect of the parametric interaction of the Langmuir and surface waves. We consider a resonant parametric instability of the counter-propagating electromagnetic SWs at their interaction with the Langmuir wave incident normally upon a dielectric surface.

2. LINEAR SWS

Let us consider a semibounded homogeneous dissipative plasma bounded by a dielectric. Let the plasma occupy the half-space $x > 0$, whereas the dielectric occupies the $x < 0$ region. The wavenumber k_z and frequency ω of the SWs propagating along the plasma-dielectric border (the z-axis) are well known to be connected by the following relation [2]

$$k_z^2 = k^2 \frac{\varepsilon_p \varepsilon_d}{\varepsilon_p + \varepsilon_d}. \qquad (1)$$

In this expression, $k = \omega/c$ is the vacuum wavenumber, c is the speed of light in vacuum, $\varepsilon_p = 1 - \omega_{pe}^2/\omega^2$ is the dielectric permittivity of the plasma with ω_{pe} being the electron plasma frequency, and ε_d is the permittivity of the dielectric.

According to linear dispersion relation (1), the SWs are reciprocal in the considered structure. It means that two counter-propagating SWs may exist with the same frequency ω and opposite wavenumbers $\pm k_z$. Their fields can be represented in the following form

$$\mathbf{W}_\pm = \frac{1}{2}\left[\overline{\mathbf{W}}_\pm \exp(-i\omega t) + \overline{\mathbf{W}}_\pm^* \exp(i\omega t)\right], \qquad (2)$$

where $\mathbf{W} = (E_x, E_z, H_y)$.

Spatial distribution of the SW fields in the plasma and dielectric is given by [2]

$$\begin{aligned}
x > 0: \quad & \overline{E}_{\pm x} = \pm i \frac{k_z}{\kappa_p} E_\pm \exp(-\kappa_p x \pm i k_z z), \\
& \overline{E}_{\pm z} = E_\pm \exp(-\kappa_p x \pm i k_z z), \\
& \overline{H}_{\pm y} = i \frac{k\varepsilon_p}{\kappa_p} E_\pm \exp(-\kappa_p x \pm i k_z z), \\
x < 0: \quad & \overline{E}_{\pm x} = \mp i \frac{k_z}{\kappa_d} E_\pm \exp(\kappa_d x \pm i k_z z), \\
& \overline{E}_{\pm z} = E_\pm \exp(\kappa_d x \pm i k_z z), \\
& \overline{H}_{\pm y} = -i \frac{k\varepsilon_d}{\kappa_d} E_\pm \exp(\kappa_d x \pm i k_z z),
\end{aligned} \qquad (3)$$

where E_+ and E_- are the $E_z(x=0)$-field amplitudes both the waves, propagating in the positive and negative directions of the z-axis. Here, $\kappa_{p,d}^2 = k_z^2 - k^2 \varepsilon_{p,d}$ characterize penetration depths of the wave-fields into the plasma and dielectric.

3. NONLINEAR DISPERSION EQUATION

We study the parametric excitation of the electromagnetic SWs by the Langmuir wave propagating perpendicularly to the plasma-dielectric interface. In the hydrodynamical approach of a cold plasma, such a Langmuir wave is described by

$$\begin{aligned}
& \mathbf{E}_0 = \frac{1}{2}\left[\overline{\mathbf{E}}_0 \exp(-i\omega_0 t) + \overline{\mathbf{E}}_0^* \exp(i\omega_0 t)\right], \omega_0 = \omega_{pe} \\
& \overline{\mathbf{E}}_0 = \{E_0[\exp(-ik_0 x) - \exp(ik_0 x)], 0, 0\},
\end{aligned} \qquad (4)$$

with $\omega_0 = \omega_{pe}$. Efficiency of this interaction is provided by the reciprocity of the SWs under study and by the spatial synchronism of the three waves, $0 = k_z + (-k_z)$. Another condition characterizing the efficiency of such an interaction is the temporary synchronism of the considered waves, $\omega_0 = \omega + \omega$. It is provided by the fact that the maximum frequency, $\omega_{max} = \omega_{pe}/\sqrt{1+\varepsilon_d}$, above which the SW existence is impossible, exceeds the half-frequency of the Langmuir wave, $\omega_{max} \geq \omega_0/2$, in the range $\varepsilon_d \leq 3$. It enables the effective interaction of the surface and Langmuir waves.

Based on nonlinear Maxwell's equations and the equation of plasma electron motion in the fields of weakly nonlinear SWs, we can write the following set of equations for the SW fields in the plasma region

$$\left.\begin{aligned} \nabla \times \overline{\mathbf{E}}_\pm - ik\overline{\mathbf{H}}_\pm = 0, \\ \nabla \times \overline{\mathbf{H}}_\pm + ik\varepsilon_p \overline{\mathbf{E}}_\pm = (4\pi/c)\mathbf{J}_\pm, \end{aligned}\right\} \tag{5}$$

where the right-hand sides of Eqs. (5) is governed by a nonlinear current

$$\mathbf{J}_\pm = \frac{ie\omega_{pe}^2}{8\pi m\omega^2\omega_0}\Big[(\overline{\mathbf{E}}_0\nabla)\overline{\mathbf{E}}^*_\mp + (\overline{\mathbf{E}}^*_\mp\nabla)\overline{\mathbf{E}}_0 - \overline{\mathbf{E}}_0 \times (\nabla \times \overline{\mathbf{E}}^*_\mp) + \frac{\omega\omega_0}{\omega_{pe}^2}\overline{\mathbf{E}}^*_\mp(\nabla\overline{\mathbf{E}}_0)\Big]. \tag{6}$$

It can be easily shown that, since, in the system under consideration, the Langmuir wave possesses the perpendicular to the medium interface electric field-component only, a surface current to second order is equal to zero.

Substituting linear fields of the SWs (3) and pump wave (4) into nonlinear current (6) and solving then equation set (5) together with the boundary conditions, consisting of the continuity of $H_{\pm y}$ and $E_{\pm z}$ fields at the interface $x = 0$, one can derive the nonlinear dispersion equation of the considered SWs

$$k\left(\frac{\varepsilon_p}{\kappa_p} + \frac{\varepsilon_d}{\kappa_d}\right)E_\pm = \frac{2ek_0(\kappa_p^2 + k_z^2)}{cm\omega\kappa_p(k_0^2 + 4\kappa_p^2)}E_0E^*_\mp. \tag{7}$$

The left-hand side of Eq. (7) is a dispersion relation for the linear SWs, while its right-hand side is a response of the nonlinear current $\mathbf{J}_\pm$, caused by the SW interaction with the Langmuir wave.

4. SW AMPLITUDE DYNAMICS

We consider the Langmuir wave amplitude to be small, when the SW amplitudes undergo a little change over their period, $|\partial \ln E_\pm/\partial t| \ll |\omega|$. In this approach, the dynamical equations [3] for the excited wave amplitudes can be written, using (7), as follows

$$\frac{\partial E_\pm}{\partial t} = \alpha E_0 E^*_\mp \exp(-\gamma_0 t) - \gamma E_\pm, \tag{8}$$

$$\alpha = -\frac{2e}{cm}\frac{\varepsilon_d\varepsilon_p^2 kk_0}{(\varepsilon_d + \varepsilon_p^2)[4\varepsilon_p^2k^2 - (\varepsilon_p + \varepsilon_d)k_0^2]},$$

where the coefficient α characterizes efficiency of the SW interaction with the pump wave. The factor $\exp(-\gamma_0 t)$, in (8), describes the Langmuir wave attenuation. In the cold plasma considered here, it is mainly caused by electron collisions:

$$\gamma_0 = \nu/2, \tag{9}$$

where ν is the the electron collision frequency. The item, $-\gamma E_\pm$, in (8), describes the linear attenuation of the SWs, $E_\pm \propto \exp(-\gamma t)$, with a damping rate γ. The rate of collisional and resonant attenuation of the SWs [2] is

$$\gamma = \frac{\nu}{2}\frac{\varepsilon_d(1-\varepsilon_p)}{\varepsilon_p^2 + \varepsilon_d} - \sqrt{-\frac{\varepsilon_p^2}{\varepsilon_p + \varepsilon_d}}\frac{\pi\eta k\omega\varepsilon_p^2\varepsilon_d^2}{\varepsilon_d^2 - \varepsilon_p^3 - \varepsilon_p\varepsilon_d(1-\varepsilon_p)}, \tag{10}$$

The parameter $\eta = (d\varepsilon_p/dx)^{-1}_{x=x_0}$ characterizes the plasma density inhomogeneity in the narrow transient layer at the resonant point, x_0, where $\varepsilon_p(x_0) = 0$.

4.1 LANGMUIR WAVE OF A CONSTANT AMPLITUDE

When there is a steady source maintaining the Langmuir wave amplitude to be constant, $\gamma_0 = 0$. In this case, the solution of Eq. (8) is threshold in nature,

$$E_\pm = E_\pm(0)\exp(-\gamma t) \times \left[\mathrm{ch}(\beta t) + \frac{\alpha E_0}{\beta}\frac{E^*_\mp(0)}{E_\pm(0)}\mathrm{sh}(\beta t)\right], \ \beta = |\alpha E_0|. \tag{11}$$

So, a threshold value of the pump wave amplitude, above which the SW excitation is possible, can be written as

$$|E_0|_{th} = \gamma/|\alpha|. \tag{12}$$

When the pump wave amplitude exceeds threshold value (12), a simultaneous growth of both the SW amplitudes appears with the rate

$$\gamma_{NL} = \gamma\left(|E_0|/|E_0|_{th} - 1\right). \tag{13}$$

Thus, an increase in the pump wave amplitude, as well as a decrease of the linear SW damping rate, leads to an increase of the nonlinear growth rate, γ_{NL}.

Mark that influence of k_0 on the nonlinear growth rate, γ_{NL}, and threshold, $|E_0|_{th}$, is governed by the dependence $\alpha(k_0)$. It can be easily shown that the parameter $\alpha(k_0)$ is negative. Its absolute value reaches the maximum at the following wavenumber of the Langmuir wave

$$k_{0(max)} = k\sqrt{-\frac{4\varepsilon_p^2}{\varepsilon_p + \varepsilon_d}}, \tag{14}$$

At this wavenumber, threshold (12) has the minimum, while nonlinear growth rate (13) attains the maximum. The numerical analysis (fig.1) shows that, with an increase in the permittivity of the dielectric, ε_d, the minimum of $|E_0|_{th}$ decreases and shifts towards larger wavenumbers of the pump wave. At that, the SW excitation is the most effective at wavenumbers of the Langmuir wave close to $5\,\omega_{pe}/c$.

Mark, the obtained results are fair at the initial stage, $t < 1/\gamma_0$, of a weakly damped pump wave decay also, when $\gamma_0 \ll \gamma_{NL}$. At this stage, the amplitude $|E_0|$ can be treated as a constant.

4.2 DAMPED LANGMUIR WAVE

Account for the Langmuir wave damping complicates solution of coupled equation set (8),

$$E_\pm = \left\{C_{1\pm}I_{1/2}(\xi) + C_{2\pm}K_{1/2}(\xi)\right\} \times \exp[-(\gamma_0/2 + \gamma)t], \ \xi = \frac{|\alpha E_0|}{\gamma_0}\exp(-\gamma_0 t). \tag{15}$$

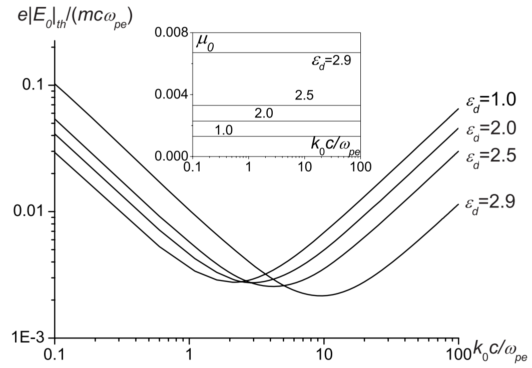

Fig.1. Influence of k_0 and ε_d on threshold $|E_0|_{th}$ (12) for the plasma with $(d\omega_{pe}^2/dx)^{-1}_{x=x_0}\omega_{pe}^3/c = 0.001$ and $\nu/\omega_{pe} = 0.001$

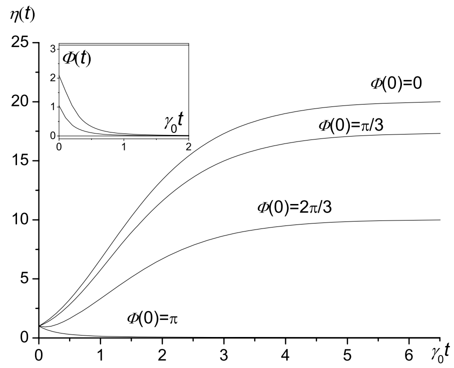

Fig.2. Evolution of the normalized amplitudes, η, and phases, Φ, under $|\alpha E_0|/\gamma_0 = 3.0$

Here, $I_{1/2}$ and $K_{1/2}$ are the modified cylindrical Bessel and McDonald functions of the order $1/2$. The constants $C_{1\pm}$ and $C_{2\pm}$ are determined by initial values of the SW amplitudes and their derivatives.

Analysis of the last expression demonstrates that interaction of the counter-propagating SWs with the damped pump wave does not result in an unlimited growth of the SW amplitudes. Moreover, after the complete damping of the Langmuir wave, the SW amplitudes damp with the linear rate, $|E_\pm(t \to \infty)| \to \eta_\pm(\infty)|E_\pm(0)|\exp(-\gamma t)$. The parameters $\eta_\pm(t) = \exp(\gamma t)|E_\pm(t)|/|E_\pm(0)|$ are the ratios of the nonlinear SW amplitudes, $|E_\pm(t)|$, to the amplitude values of linear waves, $|E_\pm(0)|\exp(-\gamma t)$, and characterize efficiency of the three-wave interaction. By virtue of the considered SW equivalence and their reciprocity, it is naturally to set $|E_+(0)| = |E_-(0)|$ and $\eta_\pm(t) \equiv \eta(t)$.

Analysis of the parameter $\eta(t)$ shows that dynamics of the SW amplitudes significantly depends on initial phases of the interacting waves. The temporal dependences of $\eta(t)$ and $\Phi(t) = \Phi_0(t) - \Phi_+(t) - \Phi_-(t)$, with $\Phi_{0,\pm} = \arg E_{0,\pm}$, are presented in fig.2 for different initial values $\Phi(0)$. One can see that a SW amplitude saturation level, $\eta(\infty)$, decreases with an increase of the initial phase, $\Phi(0)$, deviation from zero. Moreover, at $\Phi(0) \to \pi$, the interacting waves have opposite phases and $\eta(\infty) \to 0$. It means that, at $\Phi(0) \to \pi$, nonlinear interaction of the SWs and pump wave does not result in a growth of the SW amplitudes. On the contrary, it leads to their damping.

In the considered case of a damped Langmuir wave, the influence of the wavenumber k_0 on the decay instability dynamics is also governed by the dependence $\alpha(k_0)$. Thus, one can conclude that, at the damped Langmuir wave decay, the greatest efficiency of the SW excitation is reached at $k_0 = k_{0(max)}$, when the coupling coefficient of the SWs with the pump wave, $\alpha(k_0)$, has the maximum.

REFERENCES

1. N.B. Aleksić, A.G. Zagorodny, V.I. Zasenko. Resonant interaction of electron waves in semibounded plasma: *Preprint*. Kiev: Institute of Theoretical Physics, ITP-90-28P, 1990.
2. A.N. Kondratenko. *Plasma Waveguides*. Moscow: “Atomizdat”, 1976.
3. J. Weiland, H. Wilhelmsson. *Coherent Nonlinear Interaction of Waves in Plasmas*. Oxford: “Pergamon”, 1976.

НЕЛИНЕЙНЫЙ РАСПАД ЛЕНГМЮРОВСКИХ ВОЛН НА ДВЕ ВСТРЕЧНЫЕ ПОВЕРХНОСТНЫЕ ВОЛНЫ

Ю.А. Акимов, Н.А. Азаренков

Исследовано нелинейное возбуждение двух встречных поверхностных волн при распаде ленгмюровской волны, падающей перпендикулярно на границу раздела плазма-диэлектрик. Проанализирована динамика возбуждения поверхностных волн незатухающей и затухающей ленгмюровскими волнами.

НЕЛІНІЙНИЙ РОЗПАД ЛЕНГМЮРІВСЬКИХ ХВИЛЬ НА ДВІ ЗУСТРІЧНІ ПОВЕРХНЕВІ ХВИЛІ

Ю.О. Акімов, М.О. Азарєнков

Досліджено нелінійне збудження двох зустрічних поверхневих хвиль при розпаді ленгмюрівської хвилі, що падає перпендикулярно на межу розподілу плазма-діелектрик. Проаналізована динаміка збудження поверхневих хвиль незагасаючою та загасаючою ленгмюрівськими хвилями.