FOUR MOTIONAL INVARIANTS IN ADIABATIC EQUILIBRIA

O. Ågren, V.E. Moiseenko[1]

Uppsala University, Ångström laboratory, SE-751 21 Uppsala, Sweden;
[1] Institute of Plasma Physics, NSC "Kharkov Institute of Physics and Technology", 61108 Kharkov, Ukraine

Recently published derivations of four stationary motional invariants in adiabatic equilibria are presented. The four invariants $(\varepsilon, \mu, I_r, I_{\|})$ contains a radial drift invariant I_r, which determines the density radial profile and the diamagnetic drift, and an additional parallel invariant $I_{\|}$ that determines the plasma current along the magnetic field. Thus, there are in general more than three stationary invariants for the adiabatic motion of a gyrating particle. The result is valid to first order in the gyro radius, and is applicable to geometries with adiabatic fields, including toroidal as well as open trap geometry. In axisymmetric tori, the toroidal invariant can replace the longitudinal invariant in the analysis and the radial invariant can be determined from the projected gyro center motion. The four invariants is determined for passing as well as trapped particles. For equilibria with sufficiently small banana widths, the radial invariant can to lowest order be approximated by the gyro center value $I_r \approx \overline{r}_0(\mathbf{x},\mathbf{v})$ of the radial Clebsch coordinate. To this lowest order, the gyro centers drift on a magnetic flux surface.

PACS: 52.50.Lp, 52.55.Jd, 52.55.Ez

THREE, FOUR OR FIVE INVARIANTS?

The standard opinion is that most three independent stationary constants of motion can be found for a point charge. A reason for this is that the theorem on integrable systems by Liouville, which states that three independent invariants in involution is sufficient to integrate the equations of motion, often has been erroneously interpreted as implying that a fourth invariants cannot exist. Although the trajectories are determined by three constants of motion in involution (their Poisson brackets are zero), the misinterpretation origins from the claim that the trajectory would be a function of the three separating invariants only. The true conclusion is that, in addition to the three invariants, the trajectory also depends on a curve parameter, since the orbit is determined by a first order differential equation. If there exists a way of eliminating the curve parameter by a function of the form $I_4(\mathbf{x},\mathbf{v})$, a fourth independent stationary invariant is found, and a Vlasov equilibrium can be liberated from a handicapped treatment with only three constants of motion.

In tokamaks, the set $(\varepsilon, \mu, p_\varphi)$ is often used to describe the kinetic system. However, a confined particle cannot escape from the confining magnetic field region, and this implies that the average along the guiding center of the radial Clebsch coordinate $I_r \approx \overline{r}_0(\mathbf{x},\mathbf{v})$ of the particle has to be constant. It can be demonstrated that I_r determines a fourth independent stationary invariant in Vlasov equilibria [1,2]. The invariance of the radial coordinate is not restricted to fields in axisymmetric tori, and the result with the fourth independent invariant has applications also to mirrors and stellarators.

Invariants are useful in a variety of plasma studies, and it is not possible to foresee in detail all areas where an application of a fourth invariant could be important. It is well known that MHD and kinetic stability, transport and heating are profile sensitive, and a complete set of invariants could be required to develop reliable models with realistic profiles.

The existence of a radial invariant has a range of implications. The dependence on the radial invariant determines the radial profiles of the density and temperature and this gives a diamagnetic drift in a direction perpendicular to the magnetic field. In tokamaks, where the standard set $(\varepsilon, \mu, p_\varphi)$ of three invariants is insufficient to model a poloidal current, the poloidal current can directly be determined by using the radial invariant in the distribution function. It is also possible to establish a bridge between Vlasov equilibria and ideal MHD with the use of the radial invariant [2].

A key goal of the studies presented here has been to understand how many invariants are required to get a complete description of adiabatic Vlasov equilibria. This is not a trivial question, since intuitive arguments point in different directions for the number of stationary invariants. The equations of motions give the six invariants $x_i(0) = f_i(\mathbf{x},\mathbf{v},t)$ and $\mathrm{v}_i(0) = g_i(\mathbf{x},\mathbf{v},t)$ for the initial position and velocity, and the task is to identify combinations of these invariants which reduce to time independent invariants. A suggestion by G. Schmidth, based on an argument that only a single phase variable is needed to specify the orbit, is that as much as five stationary invariants may be found, see p. 70 in Ref. 3.. Second, it is claimed in Ref. 4 (p. ix) that the number of adiabatic invariants are less or equal to the degree of freedom for a point charge, which would imply at most three independent invariants for a point charge. Finally, the theorem by Liouville states that three independent invariants in involution are sufficient to integrate the equations of motion of a point charge1 [5]. This may indicate that no more than three independent stationary invariants could be found. However, this is a false conclusion, since the specific case of a point charges moving in a constant magnetic field, i.e.

$$\frac{1}{\Omega_0}\frac{d\mathbf{v}}{dt} = \mathbf{v}\times\hat{\mathbf{z}}$$

gives the four independent and exact invariants $\left(\mathrm{v}_\parallel, \mathrm{v}_\perp, \bar{x}_0, \bar{y}_0\right)$, where

$$\bar{x}_0(\mathbf{x},\mathbf{v}) = x + \frac{\mathrm{v}_y}{\Omega_0}$$

$$\bar{y}_0(\mathbf{x},\mathbf{v}) = y - \frac{\mathrm{v}_x}{\Omega_0}$$

are the guiding center variables of the x and y coordinates. In view of the theorem of Liouville, this set of four stationary invariants may seem like a “quasi-paradox”, but both results are correct, as pointed out in Ref. 3.

Several other systems with four, or even five, stationary invariants can be identified. A particularly simple case is the field free case with a constant particle velocity where the orbit is a straight line and the three initial values of the Cartesian coordinates are timedependent motional invariants;

$$I_x(\mathbf{x},\mathbf{v},t) = x - \mathrm{v}_x t$$

$$I_y(\mathbf{x},\mathbf{v},t) = y - \mathrm{v}_y t$$

$$I_z(\mathbf{x},\mathbf{v},t) = z - \mathrm{v}_z t$$

The task is to construct new invariants by eliminating the time dependence. A simple calculation yield the three time independent invariants

$$I_4(\mathbf{x},\mathbf{v}) = \mathrm{v}_y x - \mathrm{v}_x y$$

$$I_5(\mathbf{x},\mathbf{v}) = \mathrm{v}_z x - \mathrm{v}_x z$$

$$I_6(\mathbf{x},\mathbf{v}) = \mathrm{v}_z y - \mathrm{v}_y z$$

where the last invariant depends on the other invariants, since $\mathrm{v}_z I_4 - \mathrm{v}_y I_5 + \mathrm{v}_x I_6 = 0$. In this simple case, $(\mathrm{v}_x, \mathrm{v}_y, \mathrm{v}_z, I_4, I_5)$ provides a set of five independent stationary invariants.

The five invariants may seem like an exceptional case. However, a large number of studies of gyrating particle motion have been devoted to find a canonical transformation (by some asymptotic expansion) to action angle variables, where the transformed Hamiltonian $H = H(J_1, J_2, J_3)$ is a function of the action variables only [6]. The idea behind the action angle formalism is to transform the resulting orbit equations into the straight lines $Q_i(t) = Q_i(0) + \omega_i(\mathbf{J})t$ for the transformed canonical coordinates, where the frequencies $\omega_i(\mathbf{J}) = \partial H/\partial J_i$ are constant. Five independent stationary invariants can be instructed in this case in exactly the same manner as for the constant velocity case.

This raises the natural question if there is a fifth independent stationary invariant for particles gyrating in a constant magnetic field. A check shows that the fifth invariant is the gyro angle dependent quantity

$$I_5(\mathbf{x},\mathbf{v}) = \mathrm{v}_\parallel \varphi_g - \Omega_0 z$$

Although this is indeed a time independent invariant, the equilibrium distribution function does not depend on this invariant in the typical cases where the equilibrium distribution functions do not dependent on the gyro angle. For this reason, four independent stationary invariants are required to construct Vlasov equilibria in representative situations.

THE STRAIGHT FIELD LINE MIRROR

There are some realistic systems where it is possible to express the four “useful” invariants in closed form. Our interest in the problem arose from the simple form of four constants of motion in the “straight field line mirror” [7], which is a marginal minimum B. It is well known that the minimum B producing field has the drawback of producing a strong ellipticity of the flux-tube near the mirrors. The optimal choice which combines MHD stability with the smallest possible ellipticity ought to this reason be a marginal minimum B field. The unique solution for this magnetic field reads in the near paraxial approximation

$$\frac{\mathbf{B}}{B_0} = \frac{\nabla s}{1 - s^2/c^2} = \nabla x_0 \times \nabla y_0.$$

where s is the arc length of the magnetic field lines, x_0 and y_0 are Clebsch coordinates and c and B_0 are constants. To leading orders in a/c, where a is the mid plane radius of the flux tube, the arc length is

$$\bar{s}(x,y,z) = \bar{z} + \frac{1}{2}\left(\frac{\bar{x}^2}{1+\bar{z}} - \frac{\bar{y}^2}{1-\bar{z}}\right)$$

where $\bar{s} = s/c$ and $\bar{z} = z/c$ and the Clebsch coordinates are $x_0 = x/(1+\bar{z})$ and $y_0 = y/(1-\bar{z})$, which describe straight nonparallel field lines with focal lines at $z = \pm c$, see Fig. 1. The flux tube boundary is determined by

$$a^2 = \left(\frac{x}{1+\bar{z}}\right)^2 + \left(\frac{y}{1-\bar{z}}\right)^2$$

which gives the ellipticity $\varepsilon_{ell} = (\sqrt{R_m} + \sqrt{R_m - 1})^2$, where $R_m(z)$ is the local mirror ratio. For a mirror ratio of 4, $\varepsilon_{ell} = 13.9$ and this seems acceptable for a mirror reactor.

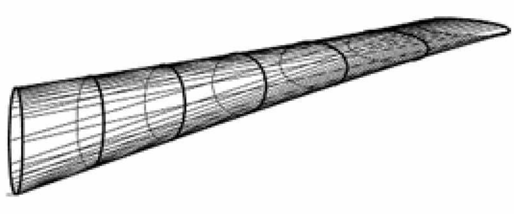

Fig. 1. The straight nonparallel magnetic field lines in the marginal minimum B field. Each gyro center bounces back and forth on a single field line in this particular field

A check shows that to leading orders $|\nabla s| = 1$ and thus $B = B(s)$ is a marginal minimum B field. From this follows that the guiding center magnetic drift is zero, since

$$\mathbf{v}_\perp \sim \mathbf{B}\times\nabla B(s) = 0$$

This implies that each ion moves back and forth on a single magnetic field line, whereby the guiding center values of the Clebsch coordinates are constant2:

$$x_{0,gc} = x_0 + (1-\bar{s})^2 \dot{y}_0 / \Omega_0$$

$$y_{0,gc} = x_0 - (1+\bar{s})^2 \dot{x}_0 / \Omega_0$$

The set $(\varepsilon, \mu, x_{0,gc}, y_{0,gc})$ provides four constants of motion and Vlasov equilibria to first order in the plasma beta can be described with distribution functions of the form $F(\varepsilon, \mu, x_{0,gc}, y_{0,gc})$. The resulting magnetic field is

$$\mathbf{B} = (1-\frac{\beta}{2})(\frac{B_0 \nabla s}{1-s^2/c^2} + \nabla \phi_{m,pl})$$

$$\phi_{m,pl}(\mathbf{x}) = -\frac{B_0}{8\pi} \int \frac{dV'}{1-\bar{s}'^2} \frac{\partial\beta/\partial s'}{|\mathbf{x}-\mathbf{x}'|}$$

where $\beta = 2\mu_0 P_\perp / B_v^2(s)$. This leads to $j_\parallel = 0$, which is a sufficient criterion to obtain exactly omnigenous equilibria [8], i.e. the radial drift is zero and the gyro center moves on a magnetic flux surface to first order in beta. There is therefore no neoclassical enhancement of the radial transport, and this is achieved without an axisymmetrization of the confining field.

GYRO CENTER MOTION

In general geometry, toroidal as well as open traps, we intend to determine a radial drift invariant for the confined particles. To lowest order in the radial drift the invariant can be identified with the guiding center radial Clebsch coordinate. To determine the dependence of $I_r(\mathbf{x},\mathbf{v})$ on the phase space coordinates $(\mathbf{x},\mathbf{v})$, it is necessary to carry out an adiabatic expansion of the equations of motions. First, a transformation $(x,y,z) \to (r_0, \theta_0, s)$ to flux coordinates is introduced by, see [1] and [2],

$$\frac{\mathbf{B}}{B_0} = \nabla r_0 \times r_0 \nabla \theta_0$$

$$\nabla s = \hat{\mathbf{B}} + \kappa \nabla r_0 + \eta r_0 \nabla \theta_0$$

where B_0 is a constant, r_0 and θ_0 are the radial and angle Clebsch coordinates, s is the arc length along the magnetic field lines and $\kappa(\mathbf{x})$ and $\eta(\mathbf{x})$ are "geometric" functions associated with the magnetic field geometry. The particle velocity $\mathbf{v} = \mathrm{v}_\parallel \hat{\mathbf{B}} + \mathbf{v}_\perp$ is

$$\mathbf{v}_\perp = \frac{(\dot{r}_0 r_0 \nabla\theta_0 - r_0 \dot{\theta}_0 \nabla r_0) \times \hat{\mathbf{B}}}{B/B_0}$$

$$\mathrm{v}_\parallel = \dot{s} - (\kappa \dot{r}_0 - \eta r_0 \dot{\theta}_0) = \dot{s} - \mathbf{v}_\perp \cdot \nabla s \equiv \frac{ds}{d\tau_\parallel}$$

and dots stand for time derivatives. The last formula shows that $\mathrm{v}_\parallel$ is the rate of change of the arc length projected on the flux lines, and the parallel velocity is zero at locations where the motion does not generate a change of the arc length coordinate of the particle. The first order difference between $\mathrm{v}_\parallel$ and $\dot{s}$ must be included to arrive at exact energy conservation for the gyro center motion in a stationary field.

The motion is split into a rapidly gyrating part and a slowly varying gyro center motion. Bars denote gyro center quantities, and the radial position of the particle is of the form

$$r_0 = \bar{r}_0(\mathbf{x},\mathbf{v}) + r_{0,gyro} \quad (1)$$

where $r_{0,gyro}$ is a "gyro ripple" associated with the gyrations, see [1] and [2], and this gyro ripple is responsible for the diamagnetic current. In a stationary field, the velocity of the guiding center is to first order in the gyro radius determined by the four equations

$$\varepsilon = U_{gc}(\bar{\mathbf{x}}) + m\mathrm{v}_\parallel^2/2 = const \quad (2a)$$

$$\frac{d\bar{s}}{dt} = \bar{\mathrm{v}}_\parallel + \bar{\kappa}\frac{d\bar{r}_0}{dt} + \bar{\eta}\bar{r}_0\frac{d\bar{\theta}_0}{dt} \equiv \bar{\mathrm{v}}_\parallel + \mathbf{v}_\perp \cdot \nabla\bar{s} \quad (2b)$$

$$m\Omega_0 \frac{d\bar{r}_0}{dt} = -\frac{1}{\bar{r}_0}\frac{\partial U_{gc}}{\partial\bar{\theta}_0} - \bar{\eta}\frac{\partial U_{gc}}{\partial\bar{s}} + 2(\varepsilon - U_{gc})\frac{\partial\bar{\eta}}{\partial\bar{s}} \quad (2c)$$

$$m\Omega_0 \bar{r}_0 \frac{d\bar{\theta}_0}{dt} = \frac{\partial U_{gc}}{\partial\bar{r}_0} + \bar{\kappa}\frac{\partial U_{gc}}{\partial\bar{s}} - 2(\varepsilon - U_{gc})\frac{\partial\bar{\kappa}}{\partial\bar{s}} \quad (2d)$$

where $U_{gc}(\bar{\mathbf{x}}) = q\phi + \mu B$ and $\bar{\mathbf{v}}_\perp(\bar{\mathbf{x}})$ is the guiding center perpendicular drift,

$$\bar{\mathbf{v}}_\perp(\bar{\mathbf{x}}) = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{\mu}{q}\frac{\mathbf{B}\times\nabla B}{B^2} + \frac{m}{q}\frac{\mathrm{v}_\parallel^2}{B^4}\mathbf{B}\times[(\mathbf{B}\nabla)\mathbf{B}]$$

For a periodic guiding center motion, a longitudinal invariant, which is an exact (not only adiabatic) constant to first order in the gyro radius, can be defined as an integral along the gyro center orbit, i.e.,

$$J_\parallel = \sqrt{2m}\oint\sqrt{\varepsilon - U_{gc}(S)}\,dS \quad (3)$$

where S is a curve parameter for the guiding center orbit [1]. This definition differs somewhat from the parallel action integral introduced by Northrop [4] (which is integrated along a magnetic field line), see also [1,2]. For confined particles, the drift orbit average $I_r = \langle\bar{r}_0\rangle$ must be constant, and $(\varepsilon, \mu, I_r, J_\parallel)$ is a set of four independent invariants for the first order adiabatic motion.

A remark on confinement could be necessary: The general system of equations (2a-d) does not guarantee the existence of a radial invariant, since there is no imposed difference on the meaning of the Clebsch coordinates r_0 and $-r_0\theta_0$ by the relation $\mathbf{B} = B_0 \nabla r_0 \times r_0 \nabla\theta_0$. Even if collisions are neglected, confinement cannot be assured without some additional constraint on the fields, such as

$$\nabla r_0 \cdot \mathbf{j} = 0,$$

compare Catto and Hazeltine. One illustration of this possibility is fields in nonaxisymmetric tori (i.e. certain stellarator fields or tokamak fields with too strong field ripples), where the field lines may trace out from the toroidal confinement region. Since the dominant component of the guiding center velocity is along the magnetic field, particles would escape from the confinement region after sufficiently many revolutions around the torus if the fields are not constrained to be inside the torus.

It seems plausible that the condition $\nabla r_0 \cdot \mathbf{j} = 0$ would lead to confinement of the single particle motion, and by this constraint a clear physical distinction is made between the two Clebsch coordinates. In nearly omnigenous equilibria, $d\bar{r}_0 / dt$ would be close to zero, while a finite $d\bar{\theta}_0 / dt$ would be associated with the gyro center drift.

AXISYMMETRIC TORI

In axisymmetric tori fields, the symmetry gives a toroidal invariant p_φ, which can replace the parallel invariant in the analysis, and the drift orbit can be projected on a closed curve in the (r, z) plane. The projected gyro center motion determines the radial drift invariant. These properties are seen by introducing the poloidal angle ζ_0,

$$\zeta_0(r_0,\theta_0,s) = -\frac{\pi}{2} + \theta_0 \frac{B_0}{\tilde{B}_t(r_0)} + \frac{s}{r_0}\frac{\tilde{B}_p(r_0)}{\tilde{B}(r_0)} = \zeta(r,z), \quad (4)$$

where $\tilde{B}^2 = \tilde{B}_p^2 + \tilde{B}_t^2$ and $\zeta(r,z)$ is specified in [1] and [2]. The relation $\zeta_0(r_0,\theta_0,s) = \zeta(r,z)$ is associated with the linear dependence of $\theta_0(r,\varphi,z)$ and $s(r,\varphi,z)$ on the toroidal angle and the toroidal symmetry of the fields, which implies $U_{gc} = U_{gc}(r_0,\zeta)$, see [1] and [2]. The projected guiding center motion is determined by

$$\frac{d\bar{r}_0}{dt} = -\frac{\tilde{B}_t}{qB\tilde{B}}\frac{1}{\bar{r}_0}\left[\frac{\partial U_{gc}}{\partial\bar{\zeta}} + 2(\varepsilon - U_{gc})\frac{1}{B}\frac{\partial B}{\partial\bar{\zeta}}\right], \quad (5a)$$

$$\frac{d\bar{\zeta}}{dt} = \frac{\tilde{B}_p}{\tilde{B}}\frac{\mathrm{v}_\parallel}{\bar{r}_0} + \frac{\tilde{B}_t}{qB\tilde{B}}\frac{1}{\bar{r}_0}\left[\frac{\partial U_{gc}}{\partial\bar{r}_0} - 2(\varepsilon - U_{gc})\frac{c_0}{\bar{r}_0}\right], \quad (5b)$$

where $\mathrm{v}_\parallel = \sigma\sqrt{2/m}\sqrt{\varepsilon - U_{gc}}$, $\sigma = \pm 1$ determines the direction of the parallel velocity and $c_0(\bar{r}_0,\bar{\zeta})$ is defined in [2]. In straight systems with rotational symmetry, $\bar{r}_0$ is a motional invariant and such equilibria are omnigenous with constant values of $c_0 = \tilde{B}_p^2 / B^2$ and $d\bar{\zeta} / dt$.

In axisymmetric tori, passing particles far from a trapped state have a nearly constant $d\bar{\zeta} / dt$, while $\bar{\zeta}$ oscillates for a trapped particle, providing a finite banana orbit width. For each constant values of ε and μ, the solution of Eqs. (5a,b), for passing as well as trapped particles, is a closed curve $\bar{r}_0(\bar{\zeta}) \equiv \bar{r}_0(\sigma,\varepsilon,\mu,\bar{\zeta})$ in the (r,z) plane, where $\bar{\zeta}$ is the curve parameter. For trapped particles, two orbit portions with opposite signs $\mathrm{v}_\parallel$ of connect at the points where σ changes sign.

The system of guiding center equations provides a radial invariant $I_r = \bar{r}_0(\mathbf{x},\mathbf{v}) + I_r^{(1)}$ and a toroidal invariant $\bar{p}_\varphi$ for the guiding center, which has the same value as the toroidal invariant of the particle [7].

The general solution of the stationary Vlasov equation is a function of four (not only three, as often stated) invariants. In axisymmetric equilibria, a nearly local Maxwellian distribution function, expressed in terms of the invariants and thereby as a solution of the Vlasov equation, can be written

$$F(\varepsilon,\mu,p_\varphi,I_r) = n_0(I_r)\left[\frac{2/m}{\pi k_B T_0(I_r)}\right]^{3/2} e^{-\varepsilon / k_B T_0(I_r)} + F^{(1)},$$

where the correction $F^{(1)}$, and its contribution to the toroidal current, has to be determined from detailed considerations of the transport and heating. With inclusion of the first order finite radial drift excursions and neglecting the contribution from $F^{(1)}$ to the radial force balance, we obtain $\mathbf{j}_\perp \times \mathbf{B} \approx \nabla P(r_0)$, which provides a bridge between Vlasov equilibria and ideal MHD, see [2].

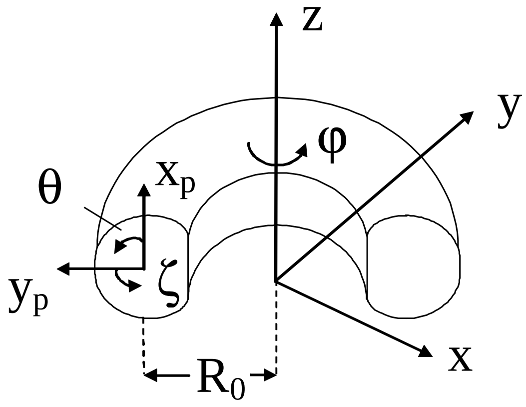

Fig. 2. Outline of the pseudo-toroidal coordinates (x_p, y_p, φ) and the projected poloidal angle ζ. A fraction of the particles are mirror trapped in the weaker field region at the outer part of the torus

ACKNOWLEDGEMENT

Prof. Mats Leijon is acknowledged for support. The Swedish Institute has provided Vladimir Moiseenko with a grant to do studies at Uppsala University.

REFERENCES

1. O. Ågren, V. Moiseenko, C. Johansson, N. Savenko// *Phys. Plasmas* 2005, v.12, p.122503.
2. O. Ågren and V. Moiseenko// *Phys. Plasmas.* 2006, v.13, p. 052501.
3. G. Schmidt. *Physics of high temperature plasmas.* London: Academic Press Inc. 1966.
4. T. Northrop. The adiabatic motion of charged particles // *Interscience Tracts on Physics and Astronomy Library of Congress Catalog*. Vol 21. Card number 63-22462, New York: Wiley, 1963.
5. V. I. Arnold. *Mathematical methods of Classical Mechanics.* Berlin: SpringerVerlag, 1979.
6. M. Kruskal// *J. Math. Phys.* 1962, v. 4, p.806.
7. O.Ågren, V. Moiseenko and N. Savenko// *Phys. Rev. E* 2005, v. 72, p. 026408.
8. P. J. Catto and R. D. Hazeltine// *Phys. Fluids* 1981, 24(9), p. 1663.

ЧЕТЫРЕ ИНВАРИАНТА ДВИЖЕНИЯ В АДИАБАТИЧЕСКИХ РАВНОВЕСНЫХ СОСТОЯНИЯХ

О. Агрен, В.Е. Моисеенко

Представлены недавно опубликованные исследования по поиску четырех стационарных инвариантов движения в адиабатических равновесных плазменных конфигурациях. Найденные четыре инварианта $(\varepsilon, \mu, I_r, I_{\|})$ включают в себя радиальный дрейфовый инвариант I_r, который отвечает за радиальный профиль плотности плазмы и за диамагнетизм, и дополнительный параллельный инвариант $I_{\|}$, который определяет продольный ток. Таким образом, существует более чем три стационарных инварианта для адиабатического движения частицы в магнитном поле. Этот результат является приближением первого порядка по малому гирорадиусу и применим к геометриям с адиабатически меняющимся магнитным полем, включая тороидальные системы и открытые ловушки. В осесимметричном торе параллельный инвариант может быть замещен тороидальным инвариантом, и радиальный инвариант может быть найден из спроецированных уравнений движения. Четыре инварианта существуют как для пролетных, так и для запертых частиц. Для равновесных состояний с достаточно малой шириной банановых траекторий радиальный инвариант в первом приближении представляет собой радиальную Клебш-координату центра Ларморовской орбиты частицы $I_r \approx \overline{r}_0(\mathbf{x}, \mathbf{v})$. В этом приближении частицы дрейфуют вдоль магнитных поверхностей.

ЧОТИРИ ІНВАРІАНТА РУХУ В АДІАБАТИЧНИХ РІВНОВАЖНИХ СТАНАХ

О. Агрен, В.Є. Моісеєнко

Подані результати нещодавно опублікованих досліджень з пошуку чотирьох стаціонарних інваріантів руху в адіабатичних рівноважних плазмових конфігураціях. Знайдені чотири інваріанти $(\varepsilon, \mu, I_r, I_{\|})$ включно з радіальним дрейфовим інваріантом I_r, що відповідає за радіальний розподіл густини плазми і за діамагнетизм, та додатковий паралельний інваріант $I_{\|}$, який зазначає уздовжній струм. Таким чином, існує більш ніж три стаціонарних інваріанта для адіабатичного руху частинки в магнітному полі. Цей результат є наближенням першого порядку за малим гірорадіусом і може бути застосований до геометрій, де магнітне поле змінюється адіабатично, включно з тороідальними системами та відкритими пастками. В вісесиметричному торі паралельний інваріант може бути замінений на тороідальний, і радіальний інваріант може бути знайдений із спроектованих рівнянь руху. Чотири інваріанти існують як для пролітних, так і для замкнених частинок. Для рівноважних станів з достатньо малою шириною бананових траєкторій радіальний інваріант у першому наближенні є радіальною Клебш-коордінатою Ларморовського центру орбіти частинки $I_r \approx \overline{r}_0(\mathbf{x}, \mathbf{v})$. В цьому наближенні частинки дрейфують уздовж магнітних поверхонь.