A NUMERICAL MODEL FOR SECOND HARMONIC ION CYCLOTRON HEATING OF SLOSHING IONS IN MIRROR TRAP

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A numerical model calculating the distribution of radio-frequency fields and absorption of their energy by sloshing ions is presented. The model solves time-harmonic Maxwell's equations written in terms of the electric field and accounts for fundamental and second harmonic ion cyclotron wave damping. It uses a 2D grid, Fourier series in the 3rd coordinate and is based on a non-staggered mesh not aligned along the steady magnetic field. Numerical stability of the scheme is demonstrated, and the convergence analysis is presented.
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INTRODUCTION

The aim of this paper is to present a simple and computationally efficient numerical model that describes heating in non-axisymmetric mirror geometry. This model is implemented in the code NAMICRF aimed to investigate ICRF heating in non-axisymmetric mirrors. It has briefly been introduced in paper [1] and was used there for studies of scenario of sloshing ions heating in a reactor scale straight field line mirror. The computational efficiency of the model is achieved by using a zero electron mass approximation that reduces the number of unknown components of the electric field by one and also by using a finite difference analogue (the non-staggered scheme) of the uniform finite element method [2]. This is the major innovation incorporated to the model as compared to other ICRF calculations (see [3] and references therein).

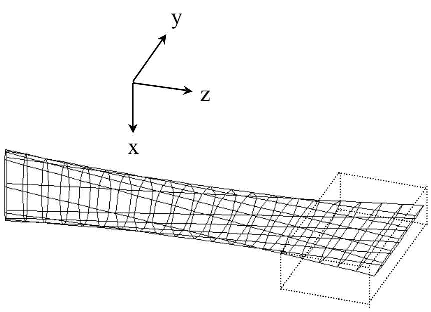

Fig.1. Straight field line mirror. Calculation domain is show by dotted line

The uniform finite element method has a pronounced advantage in accuracy compared to the standard numerically stable finite element method of the same order in the case of stiff problems, e.g. near the ion cyclotron resonance where the plasma dielectric responses to the right and left polarized electric field are substantially different. The introduced non-staggered scheme does not require a mesh alignment to the steady magnetic field which simplifies its implementation. This scheme, in contrast to the standard finite difference scheme, does not produce spurious solutions.

1. A MODEL FOR ICRF HEATING

Since the conversion and damping of the waves occur near the end of the mirror trap (see Fig. 1) we consider the rectangular calculation domain that occupies only the right end of the trap (see Fig. 2) and put a dissipative boundary condition at its left boundary. Since the gradients of stationary quantities in the x direction is much larger than along y, uniformity along y is assumed. The slow wave is not involved in the scenario under consideration. This allows us to simplify the model assuming high plasma conductivity along the magnetic field lines [4].

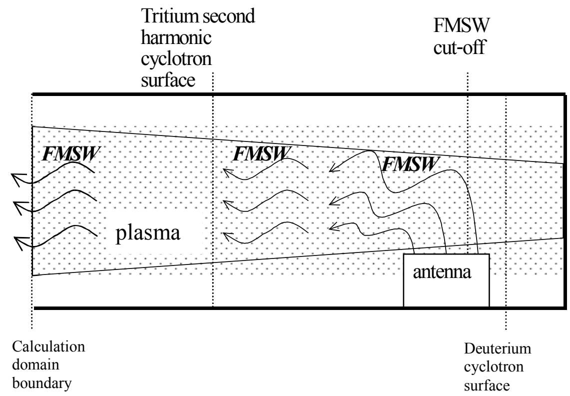

Fig.2. Sketch of second harmonic heating scheme

The corresponding Maxwell's equations are:

$$\nabla\times\nabla\times\left(E - e_{\|}e_{\|}\cdot E\right) - \omega^2\mu_0 D = i\omega\mu_0 j_{ext} \qquad (1)$$

where $e_{\|} = B/B$ is the unit vector along the steady magnetic field and j_{ext} is the current in antenna. We introduce circular unit vectors $e_{\pm} = (e_n \mp e_y)/\sqrt{2}$ with $e_n = e_y \times e_{\|}$. In the formula for the electric displacement D we use local expressions for the non-resonant terms and non-local for the resonant term:

$$\vec{D}/\varepsilon_0 = \vec{e}_+\int_{-L}^{L}\tilde{\varepsilon}_{++}(l,l')E_+(l')dl' + \vec{e}_-\varepsilon_{--}E_- + \vec{e}_{\|}\varepsilon_{\|\|}E_{\|} \quad (2)$$

where $l\in(-L,L)$ is the coordinate along the magnetic field line. Our ICRF heating scheme is formulated in WKB terms assuming small wavelengths along the plasma column. Application of the WKB approximation to the non-local component of the dielectric tensor in (2) results in [5]

$$\tilde{\varepsilon}_{++} = \varepsilon_{++}(l')\delta(l-l') + 2i\,\mathrm{Im}\,\varepsilon_{++}(l')\delta(l+l'-l_{res})\times \eta(\vec{k}\cdot\vec{e}_{\|}\nabla\vec{B}\cdot\vec{e}_{\|}) \quad (3)$$

where l_{res} is the coordinate of the cyclotron resonance point, η is the Heaviside function and k is the wavevector. The local part of the expression (3) (the first term) contains the corresponding component of the dielectric tensor of hot uniform plasma

$$\varepsilon_{++} = 1 - \sum_{\alpha}\frac{\omega_{p\alpha}^2}{\omega\left|k_{\|}v_{T\alpha}\right|}\left[F(\beta_\alpha) - i\frac{\sqrt{\pi}}{2}\exp(-\beta_\alpha^2)\right], \quad (4)$$

where F is the Dawson integral, $\beta_\alpha = (\omega-\omega_{c\alpha})/\left|k_{\|}v_{T\alpha}\right|$, α denotes the plasma species, $\omega_{p\alpha}$ and $\omega_{c\alpha}$ are the plasma and cyclotron frequencies, and $v_{T\alpha}$ is the thermal velocity. The second term in formula (3) is the contribution from the "mirror" point and would be non-zero if the wave would propagate to stronger magnetic field. Since in our scenario the wave propagates in the opposite direction this term vanishes.

The second harmonic ion cyclotron resonance reveals up if the electromagnetic field is non-uniform across the magnetic field lines. In the WKB limit the corresponding contribution to the resonant component of the displacement vector can be calculated by the finite Larmor radius expansion

$$\delta D_+ = \varepsilon_0\frac{\partial}{\partial r_+}\tilde{\varepsilon}_{+2}\frac{\partial}{\partial r_-}E_+ \quad (5)$$

where $r_\pm = \tilde{x}\pm iy$, $\tilde{x}$ is a local coordinate perpendicular to the magnetic field, $\nabla\tilde{x}\approx e_n$,

$$\tilde{\varepsilon}_{+2} = \sum_\alpha\frac{4\omega_{p\alpha}^2 v_{T\perp\alpha}^2}{\omega\left|k_{\|}v_{T\|\alpha}\right|\omega_{c\alpha}^2}\left[F(\beta_{2\alpha}) - \frac{i\sqrt{\pi}}{2}\exp(-\beta_{2\alpha}^2)\right]\times \left[1+(1-2\omega_{c\alpha}/\omega)(v_{T\perp\alpha}^2/v_{T\|\alpha}^2-1)\right] \quad (6)$$

and $\beta_{2\alpha} = (\omega-2\omega_{c\alpha})/\left|k_{\|}v_{T\|\alpha}\right|$. The expression (6) differs from the corresponding expression in [5] by the last term in square brackets. This term is caused by ion velocity distribution anisotropy. For an isotropic velocity distribution that term is unity.

The Fourier series $E(x,y,z) = \sum_m E_m(x,z)\exp(2\pi imy/L_y)$ along y is used. However, in our problem the boundary conditions couple the Fourier harmonics. This is accounted for in the way described in paper [1].

For discretization of Eq.(1) we use the non-staggered scheme equivalent to the low-order uniform finite element scheme[2]. For a uniform rectangular 2D mesh its template could be written in the form (the index m is omitted):

$$\frac{1}{2}\left(S_x^{(i-1/2,k)} + S_x^{(i+1/2,k)}\right) + \omega^2\mu_0 D_x = i\omega\mu_0 j_{ext,x}$$
$$S_y^{(i,k)} + \omega^2\mu_0 D_y = i\omega\mu_0 j_{ext,y} \quad (7)$$
$$\frac{1}{2}\left(S_z^{(i,k-1/2)} + S_z^{(i,k+1/2)}\right) + \omega^2\mu_0 D_z = i\omega\mu_0 j_{ext,z}$$

where the i and k indices enumerate mesh nodes in the x and z directions and

$$S_x^{(i+1/2,k)} = \frac{1}{2}\left[k_y^2(E_x^{(i+1,k)} + E_x^{(i,k)}) - \Delta_{zz}^{(i+1,k)}E_x - \Delta_{zz}^{(i,k)}E_x\right] + ik_y\Delta_x^{(i+1/2,k)}E_y + \Delta_{xz}^{(i+1/2,k)}E_z$$
$$S_y^{(i,k)} = ik_y\Delta_x^{(i,k)}E_x - \Delta_{xx}^{(i,k)}E_y - \Delta_{zz}^{(i,k)}E_y + ik_y\Delta_z^{(i,k)}E_z \quad (8)$$
$$S_z^{(i,k+1/2)} = \Delta_{xz}^{(i,k+1/2)}E_x + ik_y\Delta_z^{(i,k+1/2)}E_y + \frac{1}{2}\left[k_y^2(E_z^{(i,k+1)} + E_z^{(i,k)}) - \Delta_{xx}^{(i,k+1)}E_z - \Delta_{xx}^{(i,k)}E_z\right]$$

Here $k_y = 2\pi m/L_y$, the operators Δ denote the central finite differences, e.g.

$$\Delta_{zz}^{(i+1,k)}E_x = (E_x^{(i+1,k+1)} - 2E_x^{(i+1,k)} + E_x^{(i+1,k-1)})/h_z^2 .$$

Since for this scheme the condition

$$\Delta_x^{(i,k)}S_x + ik_yS_y + \Delta_z^{(i,k)}S_z = 0\,, \quad (9)$$

which is a discrete re-production of the identity $\nabla\cdot\nabla\times\nabla\times\left(E - e_{\|}e_{\|}\cdot E\right) = 0$, is met at each internal mesh point, the degeneracy of the operator $\nabla\times\nabla\times$ is accounted for, providing numerical stability of the scheme.

The discretization of Maxwell's equations results in a system of linear equations for the unknowns $\left(E_n^{(i,k)}, E_y^{(i,k)}, D_{\|}^{(i,k)}\right)$, from which $D_{\|}^{(i,k)}$ could be eliminated directly.

2. CONVERGENCE AND NUMERICAL STABILITY

To investigate convergence and stability properties of the code we have reproduced the numerical solution for sencod ion cyclotron harmonic heating from paper [6]. In the convergence calculations we vary the number of mesh nodes in the x and z directions proportionally and analyze the relative error δ in the total power emitted by the antenna. This dependence is shown in Fig. 3.

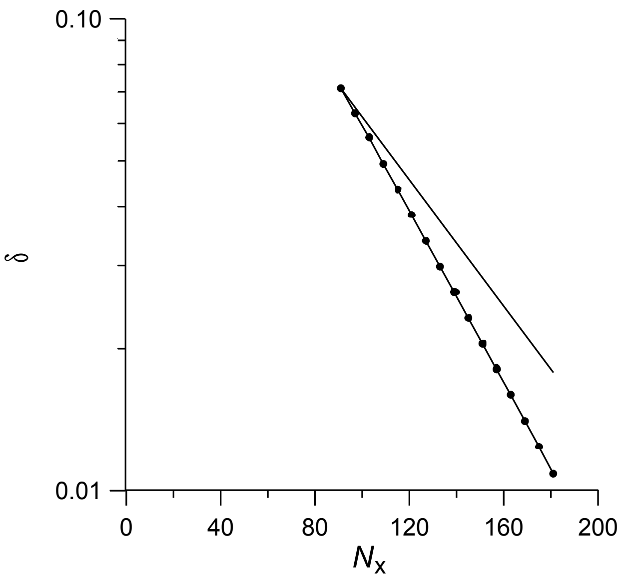

Fig. 3. The dependence of the relative error δ in the antenna emitted power (curve with circle markers) on the number of mesh nodes N_x in the x direction. The solid line is the curve C / N_x^2

The monotonous behavior of δ demonstrates a uniform convergence to the exact solution, which is pertinent to numerically stable schemes. The rate of convergence is faster than the square of the mesh nodes number. This rate is in agreement with the accuracy of the template.

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ЧИСЛЕННАЯ МОДЕЛЬ НАГРЕВА ПЛЕЩУЩИХСЯ ИОНОВ НА ВТОРОЙ ЦИКЛОТРОННОЙ ГАРМОНИКЕ

В.Е. Моисеенко, О. Агрен

Представлена численная модель для расчета распределения электромагнитных полей и поглощения энергии плещущимися ионами. В рамках модели решаются уравнения Максвелла для электрического поля при учете ионно-циклотронного поглощения на первой и второй гармониках. Она использует двумерную сетку и разложение Фурье по третьей координате и базируется на не качающейся, не связанной с направлением магнитного поля сетке. Обеспечена численная устойчивость этой схемы и приведен анализ сходимости.

ЧИСЕЛЬНА МОДЕЛЬ НАГРІВУ ІОНІВ, ЩО ПЛЕСКАЮТЬСЯ, НА ДРУГІЙ ЦИКЛОТРОННІЙ ГАРМОНІЦІ

В.Є. Моісеєнко, О. Агрен

Подана числова модель для розрахунку розподілу електромагнітних полів та поглинання енергії іонами, що плескаються. В рамках моделі розв'язуються рівняння Максвела для електричного поля з урахуванням іонно-циклотронного поглинання на першій та другій гармоніках. Вона використовує двовимірну сітку і розлад Фур'є по третій координаті та базується на сітці, що не пов'язана з напрямком магнітного поля та не гойдається. Забезпечено числову стійкість цієї схеми та наведений аналіз збіжності.