UDK 533.9

NONLINEAR BEAM INTERACTION WITH ELECTROMAGNETIC WAVES IN INFINITE PLASMA

A. S. Volokitin[1], *C. Krafft*[2]

[1] *Institute of Terrestrial Magnetism, Ionosphere and Radiowave Propagation, Academy of Sciences, Troitsk, Moscow Region 142190, Russia (volokiti@mail.sitek.ru)*

[2] *Laboratoire de Physique des Gaz et des Plasmas,Université Paris Sud, Orsay Cedex, France*

In the presence of strong dissipation, the nonlinear interaction of a monoenergetic cylindrical or spiraling electron beam with a packet of whistlers or lower hybrid waves differs considerably from the nondissipative case. The evolution of the beam-waves system exhibits a strong tendency to self-organization and leads to the formation of electron bunches continuously decelerated by waves. Strong dissipation prevents the phase mixing required for the quasilinear theory, keeps waves' phases in the packet correlated and allows the formation of dynamically stable bunches. The nonlinear model developed here considers whistler and lower hybrid waves emission through Cherenkov and cyclotron resonances by a density modulated thin electron beam.

1. Introduction

The study of energetic beam interaction with electromagnetic waves in magnetized plasmas is motivated by experiments involving electron beam injection in laboratory and space plasmas [1-4]. For example, emissions of VLF (Very Low Frequency) waves by modulated and pulsed electron beams injected by satellites in the Earth's ionosphere and magnetosphere were observed by several space active experiments [1,2].

Beam-whistler interaction in boundless magnetized space plasmas and in finite size plasma experiments can be very different. This interaction becomes more specific if the beam radius is comparable to or less than the perpendicular wavelength of the radiated whistler, what is the typical situation in space experiments involving beam injection. In previous theoretical studies considering such a thin beam [5], whistler emissions through Cherenkov resonance have shown the crucial role of wave radiation out of the bounded beam volume to infinity as an effective dissipation mechanism, which strongly modifies the energy exchange between resonant beam particles and waves (wave packet or quasi-monochromatic wave) during the electron trapping process and throughout all the system's nonlinear evolution. In the presence of strong dissipation, the nonlinear interaction of a monoenergetic electron beam with a single wave differs considerably from the classical nondissipative picture [6]: it was shown that the beam-wave system exhibits a strong tendency to self-organization [7-9]. Indeed, dissipative effects - due to collisions in plasma or to effective wave radiation out of the thin beam volume - make the system not conservative and, as a result of the irreversible loss of wave momentum and energy, prevent the periodic energy exchange between beam and wave. At the same time, the nonlinear evolution of resonant particles is characterized by the formation of dynamically stable electron bunches that are continuously decelerated and supply energy to the wave through resonant Cherenkov interaction owing to a self-adjusted nonlinear shift of the parallel wave number [7,8].

In the case of a wave packet, bunched particles exchange energy with several waves and one could expect that the beam-waves system should evolve according to the quasilinear theory (diffusion to lower velocities and plateau formation). However, presented theoretical models and related numerical simulations show that, due to the strong losses of wave energy, the phases of all waves can become strongly correlated and thus can prevent the stochastic phase mixing required for the validity of quasilinear theory.

In theoretical models presented below, electromagnetic wave fields are calculated in a semi-analytical way as functions of a given electron beam current distribution; at the same time, the full nonlinear dynamics of particles moving in the background magnetic field and the calculated wave fields is considered. This approach allows to avoid expensive Particle-In-Cell (PIC) simulations (e.g., [10]) and to develop clear physical effects.

The paper studies the interaction of a modulated spiraling or cylindrical electron beam with electromagnetic waves at Cherenkov and cyclotron resonances (normal and anomalous Doppler) in a magnetized plasma. The beam is injected along the ambient magnetic field $\mathbf{B}=B_0\,\mathbf{z}$ or with a pitch-angle θ_p. All beam electrons are magnetized and supposed not to leave the magnetic tube of radius R. Outside this tube, waves are propagating free as cylindrical waves out of the beam volume to infinity. Inside the tube, waves' amplitudes are supposed to evolve slowly along the magnetic field, in the z direction. The beam is supposed to keep a spiral structure due to Larmor rotation for $\theta_p>0$; its initial thickness is much less than the spiral radius. Such structure was often considered theoretically [7,11] and observed in laboratory experiments [2,12] as well as in space [13].

2. Models of wave-beam interaction

We present here nonlinear models developed to study resonant wave emission by a density modulated thin electron beam of small radius r_b and fixed modulation frequency ω. The evolution of the beam current modulation is considered self-consistently as the result of

ВОПРОСЫ АТОМНОЙ НАУКИ И ТЕХНИКИ 2000. №1.
Серия: Плазменная электроника и новые методы ускорения (2), с. 248-252.

nonlinear beam particles' motion in waves' fields. All nonlinearity is held in electrons' motion. As the waves are excited by an electron beam filling a cylindrical magnetic tube, the system is supposed to have cylindrical or, more generally, spiral symmetry, especially in the region outside the beam. Moreover, because resonant emission by a beam modulated at a fixed frequency is considered, main features of waves' structure outside the beam are determined by the resonance condition. Namely, wave fields outside the beam are described as cylindrical waves whose wave numbers are determined by dispersion properties and resonance condition $k_z v_{bz} = \omega - n\omega_c$ (where n is the cyclotron harmonic number, k_z the parallel wave number and v_{bz} the parallel beam velocity; ω and ω_c are the wave and the electron cyclotron frequencies). Slow changes in beam velocity provide slow changes in waves' phase velocities, wave vectors and amplitudes. If waves' parameters are known at the boundary of sources suited inside the cylindrical tube, then, outside the beam region, waves can be readily described as linear cylindrical waves propagating freely to infinity. The more difficult problem to obtain a compact description of waves' behavior in the interaction region is considered below.

We consider electromagnetic waves in the frequency range $\omega_{lh} << \omega < max\{\omega_c, \omega_p\}$ (ω_{lh} and ω_p are the lower hybrid and the plasma frequencies) propagating almost normally to the ambient magnetic field, that is, $k_z << k_\perp$, where $k_\perp \approx k$ is the perpendicular wave number. These waves are so-called sheared whistlers. In the electrostatic limit $k^2c^2 > \omega_p{}^2$, they become lower hybrid waves. The emitted waves evolve self-consistently with the beam modulation and their spatial structure is characterized by slow variations along the distance z from the injector. Partial simulation is used to describe the slow evolution of their amplitudes and phases.

As first case, let us present the so-called electrostatic model. Ions are supposed to be motionless and the fluid plasma description provides the main equation describing the evolution of the wave's scalar potential φ

$$\frac{\partial^2}{\partial t^2}\left(1+\frac{\omega_p^2}{\omega_c^2}\right)\nabla^2\varphi + \omega_p^2\frac{\partial^2\varphi}{\partial z^2} = \frac{\partial^2}{\partial t^2}4\pi n_b \quad (1)$$

where n_b is the beam density. In the case of steady state modulated beam injection and for a quasi-monochromatic cylindrical wave $\varphi=\Sigma\varphi_n exp(ik_z z+in\theta-i\omega t)$, the balance between wave field energy outflow to infinity and energy gain resulting from the beam-wave interaction can be written as

$$\frac{\partial P_z}{\partial z} + P_r = \frac{\omega}{n\omega_c - \omega}\langle j_{bz}E_z\rangle, \quad (2)$$

where the right hand side represents the work of the beam on the wave; P_z and P_r are the parallel and radial fluxes of wave energy. If r exceeds the radius of any beam electron ($r > r_b$), P_r does not depend on r.

Wave equations are linear and their general solution for an arbitrary beam electrons distribution can be obtained in integral form with the help of Green functions as

$$\varphi_n(r) = \frac{2ek_z\omega_c^2}{\omega_c^2+\omega_p^2}\int d^2r'dz'\cdot n_b(r',z',t)G_n(r,r')e^{-i\eta'} \quad (3)$$

$$G_n(r,r') = \frac{i}{4}\begin{bmatrix} J_n(k_* r')H_n^{(2)}(k_* r) & r > r' \\ H_n^{(2)}(k_* r')J_n(k_* r) & r < r' \end{bmatrix} \quad (4)$$

$\eta = k_z z + n\theta - \omega t$ is the phase along the particle trajectory. Potentials and fields can be calculated if the beam particles' distribution is known. Using the conservation of phase space volume along the electrons' trajectories, one can express the integral term of (3) as a summation on N macro-particles as

$$\varphi_n = \frac{4\pi^2 e n_b r_b^2\omega_c^2}{\omega_c^2+\omega_p^2}\frac{1}{N}\sum_{r'}G_n(r,r')e^{-i\eta'} \quad (5)$$

During the nonlinear evolution of the beam-wave system, this equation is solved together with the Newton-Lorentz equation for each beam electron moving in the electric fields of the lower hybrid wave.

A similar approach can be used for whistlers in the general electromagnetic case when $k^2c^2 \le \omega_p{}^2$. Then, wave fields are described with the help of the electrostatic potential and the electromagnetic potential A_z ($A_\perp << A_z$). For steady state modulated beam injection, the set of equations for sheared whistler waves is

$$\left(1-\frac{c^2\nabla^2}{\omega_p^2}\right)A_z + i\frac{c}{\omega}\frac{\partial\varphi}{\partial z} = \frac{4\pi c}{\omega_p^2}j_{bz}$$

$$\nabla_\perp^2\left\{\left[\frac{\omega_p^2}{\omega_c^2}-\frac{\omega_{pi}^2}{\omega^2}\right]\varphi + i\frac{c}{\omega}\frac{\partial A_z}{\partial z}\right\} = \frac{\omega_p^2}{c\omega_c}B_z + i\frac{4\pi}{\omega}\nabla_\perp \mathbf{j}_{b\perp} \quad (6)$$

$$\nabla^2 B_z = \frac{\omega_p^2}{c\omega_c}\nabla^2\varphi - \frac{4\pi}{\omega}[\nabla, \mathbf{j}_{b\perp}]_z$$

where $\mathbf{j_b}$ (j_{bz}, $j_{b\perp}$) is the beam current. If the beam modulation is changing slowly, that is $\partial n_b/\partial z << k_z n_b$, one can separate in the potential fast phase and slow amplitude variations. Finally, one obtains explicit expressions for potentials in terms of Green functions and summation on macro-particles, which are used for numerical simulation of wave-beam interaction for any resonance and beam structure. Calculations of Green functions are rather complex and will be presented elsewhere. For a thin beam and Cherenkov resonance, properties of whistler dispersion allow the beam to be in resonance simultaneously with several waves having the same phase velocity but different frequencies $\omega_m << \omega_c$ and wave numbers $k_{\perp m} >> k_{zm}$. This situation is realistic as the spectral analysis of the modulated current of beams injected from guns on board satellites or in vacuum chambers typically exhibits not only the modulation frequency but also higher harmonics; moreover, modulation at different frequencies can also be applied simultaneously to the beam density.

At a given frequency ω_m, there are two resonant whistlers with the same k_{zm} (verifying the Cherenkov resonance condition $k_{zm}v_{bz} = \omega_m$) and with two different perpendicular wave numbers k_{1m} and k_{2m}. In the case of a thin beam and for any wave in a packet of M sheared whistlers, all fields and potentials inside and outside the beam can be described in terms of potential amplitudes Ψ_m at the beam center. Using Maxwell equations, matching conditions at the beam boundary as well as conditions of free wave propagation to infinity, one can find the M equations describing the self-consistent nonlinear evolution of whistlers' amplitudes along the beam, including the slow modulation of the parallel beam current j_{bz} as a source term

$$\frac{\partial \Psi_m}{k_{zm}\partial z} + \kappa_m \Psi_m = -i\frac{2\pi v_{bz}}{\omega_p^2}\left\langle j_{bz,m}\right\rangle = i\frac{2\pi e n_b v^2{}_{bz}}{\omega_p^2}\frac{1}{N}\sum_{j=1}^{N} e^{-i\theta_{j,m}} \tag{7}$$

$\theta_{j,m} = k_{zm}z_j - \omega_m t \equiv m\theta_j$, where $\theta_j = k_{z1}z_j - \omega_1 t$ is the phase of the particle j in the fields of the wave harmonic m. The complex factor κ_m depending on r_b and $k_{\perp m}$ describes new effects of energy loss by wave emission out of the beam to infinity : whereas $Re(\kappa_m)$ represents the rate of emission, $Im(\kappa_m)$ controls the reversible exchange of the wave field energy inside the beam with that of the outside waves. The right hand side of (7) describes the nonlinear interaction of the harmonic m with resonant electrons; it results from the averaging of the beam current over the beam cross section. The slow evolution of the current results from variations of particles' phases θ_j due to the parallel motion of electrons in the M wave fields

$$\frac{d^2\theta_j}{dz^2} = -\frac{ek_{z1}}{m_e v^2{}_{bz}}\left(E_z + \frac{B_\theta E_\theta + B_r E_r}{B_0}\right) = \sum_{m=1}^{M} a_m(r)\Psi_m e^{im\theta_j} + c.c. \tag{8}$$

where $E_{r,\theta,z}$ and $B_{r,\theta,z}$ are respectively the total electric and magnetic fields at the particle position (r,θ,z); $a_m(r)$ depends on the parameters of the beam-plasma system.

3. Numerical simulation results

As a full analytical study is impossible, equations have been solved by a numerical code for different physical parameters and resonance conditions $k_z v_{bz} = \omega - n\omega_c$, that is, Cherenkov resonance ($n=0$) as well as anomalous ($n=-1$) and normal ($n=1$) Doppler shifted cyclotron resonances.

3.1 Cherenkov resonance

For Cherenkov resonance, numerical results obtained in the case of a spiral beam are qualitatively similar to those obtained for a cylindrical beam interacting with a whistler wave [7] or a lower hybrid wave. At a small distance from the injector, the wave amplitude is growing and, at the same time, the formation of a vortex corresponding to resonant particle trapping by the wave starts in the phase space. Then, in contradiction with the well known case of a conservative system (no dissipation), the energy given by the beam to the waves cannot be returned to them due to the irreversible loss of wave energy and momentum: no quasi-periodic exchange of energy between waves and trapped particles is observed. A part of resonant particles is gathered in a continuously decelerated bunch. At the same time, the phase velocity of waves is self-adjusted owing to a nonlinear shift of their parallel wave number so that Cherenkov resonance with bunched particles is held. Thus, dynamically stable nonlinear bunches are formed. Beam energy is continuously transferred to the wave which damps due to electrostatic wave dissipation out of the bounded beam volume.

Figure 1 shows, as a function of the normalized time τ (proportional to the distance z from the injector $\tau \propto z/v_{bz}$), (a) the variation of the normalized wave energy E_w integrated on the total beam cross-section, (b) the total normalized longitudinal beam momentum P_z, and (c)-(d) phase spaces η_p-v_z at different τ showing an electron bunch continuously decelerated along z.

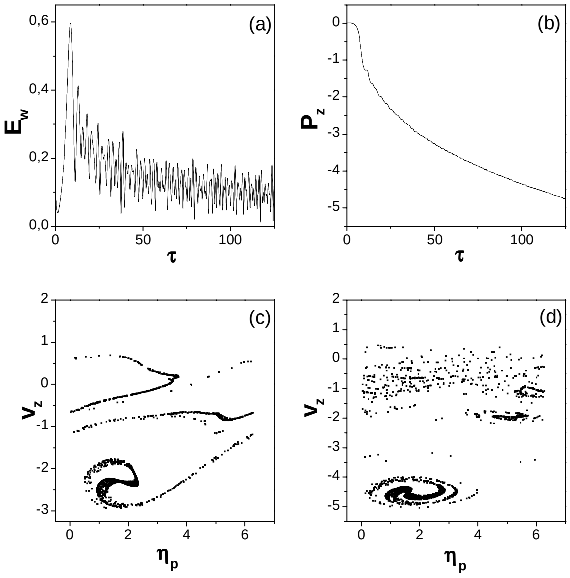

Fig. 1. Nonlinear interaction of a modulated electron beam with a whistler wave (Cherenkov resonance n=0).

The bulk of nonresonant particles is heated, what is confirmed by the growth of parallel temperature (not shown here). A small decrease of the electrons' perpendicular velocity occurs, corresponding to the decreasing of their rotation radius. Indeed, resonant particles are subjected to strong interaction with the wave to which they give parallel energy, while their loss of perpendicular energy is much weaker. Let us also notice the formation of secondary vortices in phase space (see Fig. 1d), as observed in our previous works.

In the case of a single quasi-monochromatic whistler, continuous bunch deceleration and Cherenkov resonance tuning processes can be explained using a simple model describing bunches as nonlinear resonant structures. At the asymptotic stage of the interaction, a well-formed bunch can be considered as a single particle with a weight $n_{tr}=N_{tr}/N$ proportional to the number N_{tr} of particles it contains; current modulation is only due to the bunched particles. As confirmed by the numerical solution, wave and bunch remain in phase. Then, bunch interaction with the dissipating wave results from the normalized form of (7)-(8)

$$\left|A\right| \approx -\frac{n_{tr}}{v}\sin\phi, \quad \frac{dv^2}{d\tau} \approx -2n_{tr}\sin 2\phi \qquad (9)$$

where $A=\left|A\right|e^{i\phi}$ is the normalized wave amplitude and $v \approx -d\phi/d\tau$ the nonlinear shift of phase velocity. Even in the single wave case, not only one but several bunches with different velocities can exist; they are resonant with waves present in the Fourier spectrum: indeed, slow changes in the main wave characteristics can be considered as a result of the superposition of several waves whose wave numbers differ slightly. Similar considerations are valid for lower hybrid waves.

The case of a beam-wave packet interaction is more complex, as the different waves can trap successively electrons and as result form a more wide variety of bunches; those can be accelerated or decelerated according to their phase matching with waves. As bunches' velocity decreases continuously, the bunched electrons can start interactions with waves of smaller phase velocities than the initial trapping wave. Each formed bunch makes a finite contribution to the amplitude A_m of each harmonic m according to the phenomenological estimation (derived from (8), with δ_m standing as normalized form of κ_m)

$$A_m \approx i\sum_{l=1}^{M_b}\frac{mn_l e^{-im\theta_b}}{\left(i\partial\theta_b/\partial\tau + m\delta_m\right)} \qquad (10)$$

where n_l is the relative number of particles inside the bunch l and M_b the total number of bunches; θ_b is the phase of the bunch l (all particles it contains have roughly the same phase). When the formed bunches have similar characteristics, their behavior with respect to the waves is roughly equivalent and all can be resonant with waves. When one bunch contains much more electrons than others, it dominates the system's dynamics as only it interacts strongly with waves. Other bunches participate weakly to the radiation process, even though they are in resonance with waves. But, as each A_m results from the sum of components resonant with each bunch (see (10)), a non vanishing (on average) resonant deceleration force $F_l \approx d^2\theta_b/d^2\tau$ acts on particles in bunch l; since the phases of all waves are well correlated, this force is proportional to the relative number of particles in the bunch, $F_l \propto \mathrm{Re}\left[\sum_m iA_m e^{im\theta_b}\right] \propto n_l$.

Finally, beam particles are definitively separated in two distinct groups: the dynamically stable bunches continuously decelerated in resonance with the waves and a bulk of non resonant electrons, presenting more or less diffusion to lower velocities. The number of particles in each bunch, as well as in the bulk, is established during the trapping process by waves and the subsequent bunches' formation; it depends on beam parameters and initial conditions. This picture is different from plateau formation in velocity distribution, as expected from quasilinear theory; additional nonlinear stable structures are present in the velocity distribution which allow the beam to radiate energy out of its volume at a significant distance from the injection point. Thus, strong effective dissipation can prevent the stochastic phase mixing required for the validity of quasilinear theory and keep the phases of all waves well correlated. Sequences of bunches propagating together with forced electric field perturbations (modulated wave packet) can be considered as dissipative nonlinear Van-Kampen modes.

Figure 2 presents the variation as a function of τ of (a) the amplitudes' evolution of M=12 waves, (b) the corresponding average parallel momentum P_z and, (c)-(f) phase spaces η_p-v_z at four τ ($\tau_{(c)} < \tau_{(d)} < \tau_{(e)} < \tau_{(f)}$).

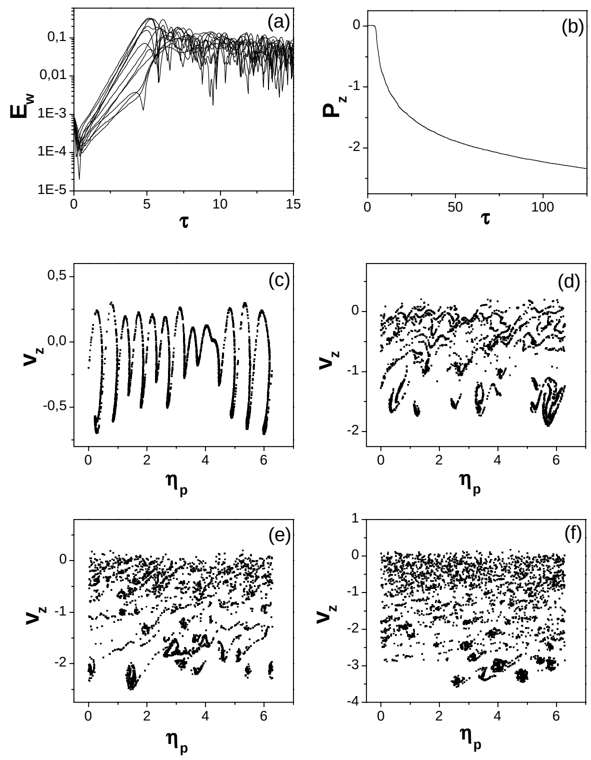

Fig. 2. Cherenkov resonant interaction of a thin modulated electron beam with a packet of whistlers in presence of dissipative effects.

3.2 Beam-wave interaction at cyclotron resonances

For cyclotron resonances, simulations of beam-lower hybrid wave interaction show results which are partly analogous to the Cherenkov case: the similar partial trapping process occurs as well as the subsequent bunch formation, which happen now in the total phase $\eta = k_z z + n\theta - \omega t$ ($n\neq 0$). Let us point out a remarkable conclusion that follows from equation (2). It shows that the rate of energy exchange between wave and beam particles is a fraction $\propto \omega/(n\omega_c - \omega)$ of the rate of change of parallel beam energy; the interaction of electrons with wave can roughly be considered as their scattering on the static potential. So, at cyclotron resonances, contrary to the Cherenkov case, (i) resonant electrons only redistribute their perpendicular and parallel energies and only a small part of it, proportional to ω/ω_c, is transferred to the wave; (ii) because wave gains energy from the beam, parallel energy of electrons is decreasing (resp. increasing) for $n=1$ (resp. for $n=-1$). This is confirmed by the numerical solution and the equation of electron motion.

The formation of electron bunches and their deceleration along z with respect to the heated bulk, together with wave excitation and damping, are illustrated on Fig. 3 which shows the variation with τ of (a) E_w, (b) P_z , $\Delta E_\perp$ (average perpendicular energy change) and ΔE (total beam energy change), and (c)-(d) phase spaces η_p-v_z and η_p-$v_\perp$ at fixed τ.

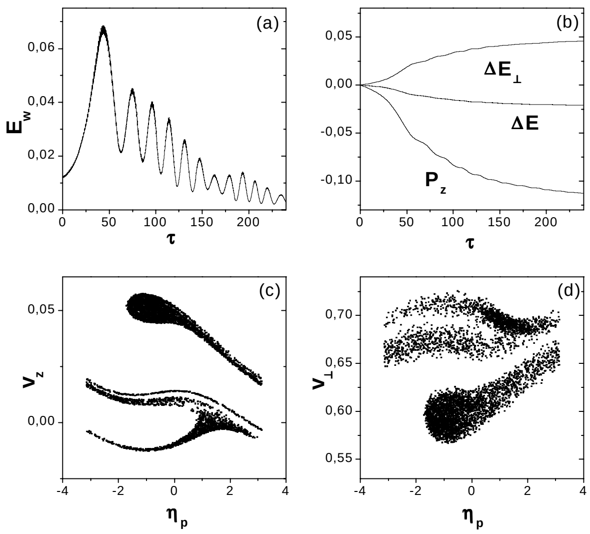

Fig. 3. Interaction of a modulated electron beam with a lower hybrid wave (Doppler resonance n=1).

4. Conclusion

Numerical simulations of beam interaction with a finite number of waves in presence of effective dissipation have shown that, independently of the dissipation type and the nature of the considered waves, the nonlinear evolution of the particles' distribution has a tendency to self-organization, leading to the formation of highly concentrated electron structures. These bunches of resonant particles are decelerated continuously by friction on waves and their dynamics shows noticeable stability in a range of time exceeding several characteristic times of their formation. When the number of waves in the packet is large and the wave spectrum is continuous, quasilinear diffusion of particles in velocity space and plateau formation in the velocity distribution are usually expected to occur during the beam relaxation stage. In the strongly dissipative case however, our calculations show the coexistence in the velocity distribution of a wide and very low plateau together with small peaks (at lower velocities) corresponding to stable electron bunches, which typically contain around 1-10% of the total number of particles. On the other hand, the plateau itself exhibits a fine structure consisting in a large set of small and almost indistinguishable bunches. At the asymptotic stage, the deceleration rate of bunches and, correspondingly, their whistler emission rate is proportional to the number of particles they contain. If the total number of particles gathered in bunches is not very small, the whistler energy emitted during the long asymptotic stage of beam relaxation can exceed the whistler emission in the initial stage of the interaction (i.e., near the injector) although the latter is much more intense. Our calculations show that one possibility to control the amount of particles organized into bunches in order to increase the emission efficiency is to premodulate the beam at one or several given frequencies.

5. References

1. J.R. Winckler The applications of artificial electron beams to magnetospheric research // Rev. Geophys. Space Phys. 1980. vol. 18. p. 659.

2 Z.. Nemecek et al. Artificial electron and ion beam effects : Active Plasma Experiment // J. Geophys. Res. 1997. vol. 102. p. 2201.

3 C.. Krafft et al. Whistler emission by a modulated electron beam // Phys. Rev. Lett. 1994. vol. 72. p. 649.

4. M. Starodubtsev, C. Krafft Resonant cyclotron emission of whistler waves by a modulated electron beam // Phys. Rev. Lett. 1999. vol. 83. p. 1335.

5. C. Krafft, A. Volokitin Nonlinear interaction of whistler waves with a modulated thin electron beam // Phys. Plasmas. 1998. V. 5. P. 4243.

6. T.M. O'Neil et al. Nonlinear interaction of a small cold bam and a plasma // Phys. Fluids. 1971. V. 14. P. 1204.

7. A. Volokitin et al. Nonlinear interaction of a thin modulated electron beam with a quasi-resonant whistler wave // Phys. Plasmas. 1997. V. 4. P. 1.

8. C. Krafft et al. Nonlinear dynamics of electron bunches in presence of dissipative effects // J. Plasma Phys. 1999. V. 61. P. 275.

9. A.A. Ivanov et al. Nonlinear theory of the interaction between a monoenergetic beam and a dense plasma // Sov. Phys. JETP. 1973. V. 36. P. 887.

10. K.I. Nishikawa et al. Three-dimensional simulation of whistler mode excited by the Spacelab 2 electron beam // J. Geophys. Res. 1989. V. 94. P. 6855.

11. K.L. Harker, P.M. Banks Near fields in the vicinity of pulsed electron beams in space // Planet. Space Sci. 1987. V. 35. P. 11.

12. G. Golubyatnikov, R.L. Stenzel Cyclotron harmonic lines in magnetic fluctuations of spiraling electrons in plasmas // Phys. Rev. Lett. 1993. V. 70. P. 940.

13. P.M. Banks, W.J. Raitt Observation of electron beam structure in space experiments // J. Geophys. Res. 1988. V. 93. P. 5811.