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ESTIMATION OF ULTRASHORT LASER PULSE PENETRATION DEPTH INTO PLASMA AT MULTIPHOTON COLLISIONAL ABSORPTION

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The interaction of ultrashort laser pulses with plasma is studied, where the pulse is considered as a photon flux and the plasma is presented by an ensemble of free electrons absorbing photons due to backstopping. The penetration depth is estimated for powerful ultrashort laser pulses at multiphoton collisional absorption.

Inroduction

One of the important aspects in the problems of laser thermonuclear fission is estimating the powerful laser penetration depth. In [1], the infinitely penetration depth of ultrashort laser pulses into conducting media is substantiated, even when the field spectrum is below the plasma frequency. In [1], the field dynamic in metals to be described by the sine-Gordon equation having undamped soliton solutions. Here due to pulse duration, ultrashort as compared to the electron relaxation time in conductors, the beam absorption due to backstopping is neglected. Meanwhile, it is important to take into account the collisional absorption by electrons, otherwise one can overestimate penetration of the pulse of arbitrary duration, when considering it as a sequence of ultrashort pulses. In [2], we show, that account the collisional absorption by electrons in process of ultra-short pulses interaction with a conductors lead to finite penetration depth. But in [2] we neglect the processes of multiphoton absorption. In the present work, we estimate the penetration depth of ultrashort laser pulses into plasma at multiphoton collisional absorption.

The theory multiphoton absorption on the basis of the quantum-mechanical theory is developed (for example, [1] and reference there). An estimation of *n* quantum absorption probability in one act of interaction is known

$$P_n=\alpha_n I^n, \tag{1}$$

here I - intensity of radiation; α_n - factor of proportionality.

However, from the practical point of view, more important estimation of mathematical expectation and variation of quantum quantity, absorbed by electron in one act of interaction, then estimation of probability of absorption of quantum quantity. Besides the existing theory multiphoton absorption is developed for monochromatic field, in this connection the analysis of interaction of ultrashort pulses with plasma requires attraction of the new approaches.

In the present work the research of multiphoton collisional absorption by electron is spent with use of methods of the Markov processes theory with discrete states and continuous time. In order to demonstrate it we shall carry out a substantiation of the offered approach, that is we shall show compatibility of Markov processes theory with quantum mechanics**.** The principles of this approach was reported in [3].

1. Compatibility of markov processes theory with quantum mechanics

In work [4-7] is shown, that effects compelled stimulated Brillouin scattering (SBS), the amplification and generation of laser radiation, chemical reaction rate can be correctly described on the basis of the Markov processes theory with discrete states and continuous time. In a basis of the given approach the description of physical effects with the help the graph (Fig.1) is necessary, each state S_i answers a certain condition of examined process (system). Dynamics of system is described with the help of the Kolmogorov equations for probabilities P_i of each state S_i.

$$dP_i/dt= -\nu_2 P_i + \nu_1 P_{i-1}. \tag{1.1}$$

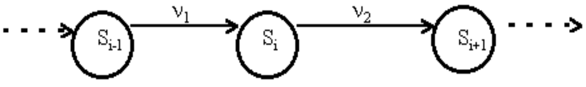

Fig.1. Graph of a some quantum system

Here ν_1, ν_1 - random flux intensities transportation's between states.

As is known, considered in work [4-7] the effects are described within the framework of the quantum mechanics on the basis of the Shrodinger equation

$$i\hbar\frac{\partial\Psi}{\partial t}=H\Psi, \tag{1.2}$$

We shall note formal similarity of expressions (1.1) and (1.2) - the left parts of the equations contain first *derivative* on time. Besides $|\Psi(r,t)|^2$ - probability of a finding of system in a vicinity of a point r at the moment of time t, that is, actually, P_i. There is the hypothesis that there is the deep interrelation the Kolmogorov equations system with the Shrödinger equation not only for the same problems [4-7], but also in a general case. We shall show, that the system of the Kolmogorov equations can be received from the Shrodinger equation.

We shall substitute in the Shrödinger equation (1.2) wave function $\Psi=ae^{iS/\hbar}$ (a - slowly varying function; S - action) [8], we shall receive by making differentiation

$$a\frac{\partial S}{\partial t}-i\hbar\frac{\partial a}{\partial t}+\frac{a}{2m}(\nabla S)^2-\frac{i\hbar}{2m}a\Delta S-\frac{i\hbar}{m}\nabla S\nabla a-\frac{\hbar^2}{2m}\Delta a+Ua=0$$

In the last equation are available real and imaging terms. Equating that and other separately to zero, we shall receive two equations:

$$\frac{\partial S}{\partial t}+\frac{1}{2m}(\nabla S)^2+U-\frac{\hbar^2}{2ma}\Delta a=0\,, \tag{1.3}$$

$$\frac{\partial a}{\partial t}+\frac{a}{2m}\Delta S+\frac{1}{m}\nabla S\nabla a=0\,. \qquad (1.4)$$

The first equation without the account last term, containing $\hbar^2$, represents the classical Hamilton-Jacob equation for action S.

The second of the received equations after multiplication on $2a$ can be rewritten as (with the account $a^2=P_i$ and $\nabla S/m=\mathrm{v}$ - velocity)

$$dP_i/dt=-\,div\,\mathrm{v}\cdot P_i-\mathrm{v}\cdot grad\,P_i. \qquad (1.5)$$

We shall consider for simplicity one dimension movement. By choosing $dx_1=dx_2\approx\Delta x$, (here Δx - size of quantum system) we shall receive, that flux intensities transportation's between states depend on life-time of quantum system state $\nu_1=1/dt_1$ and $\nu_2=1/dt_2$ accordingly

$$div\,v=\partial v/\partial x\approx(\nu_2-\nu_1)/\Delta x=(dx_2/dt_2)/\Delta x-(dx_1/dt_1)/\Delta x= \\ =\nu_2-\nu_1,$$

$$v\cdot grad\,P_i=v\cdot\partial P_i/\partial x\approx v\,(P_i-P_{i-1})/\Delta x=(P_i-P_{i-1}) \\ (dx_1/dt_1)/\Delta x=\nu_1(P_i-P_{i-1}).$$

By substituting the last expressions in (1.5), we shall receive

$$dP_i/dt=-(\nu_2-\nu_1)P_i-\nu_1(P_i-P_{i-1})=-\nu_2P_i+\nu_1P_{i-1},$$

That is, Kolmogorov equation (1.1) for probability P_i of a finding of considered quantum system in a state S_i.

Thus, is shown equivalence of the description of quantum systems on the basis of the Shrodinger equation and on the basis of the Markov processes theory together with the Hamilton-Jacob equation.

2. Investigation of multiphoton collisional absorption on the basis of the Markov processes theory

Investigation of multiphoton collisional absorption on the basis of the Markov processes theory we shall begin from construction the graph (Fig.2).

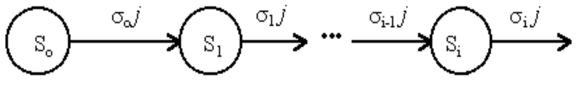

Fig.2. Graph of multiphoton collisional absorption process

On Fig.2. are designated:

S_i - states of an electron in adequate an absorption i photons;

σ_i - cross section of an absorption of i-th a photon;

j - photon flux of radiation interacting with plasma.

For each state S_i the Kolmogorov equation for probability of each state [9] can be written as

$$dP_i/dt=-\sigma_i j\,P_i+\sigma_{i-1}j\,P_{i-1} \qquad (2.1)$$

with initial conditions

$$P_o=1,\ P_i=0,\ i=1,\infty. \qquad (2.2)$$

Us will be interest the probability P_i in an instant t, when the electron already has absorbed i-1 a photons.

Then $P_i\approx 0$ and the equation (2.1) can be rewritten as

$$dP_i/dt=\sigma_{i-1}j\,P_{i-1}. \qquad (2.3)$$

By dividing the last expression on intensity of photon flux j, we shall receive (by definition [10] P.439) cross section of an absorption i photons

$$\sigma_i=\sigma_{i-1}P_{i-1}. \qquad (2.4)$$

The system of the equations (2.1) with the initial conditions (2.2) allows the analytical solutions

$$P_k=\sum_{i=0}^{k}\frac{\prod_{l=0}^{k-1}\sigma_l}{\prod_{\substack{m=0,\\ m\neq i}}^{k}(\sigma_i-\sigma_m)}exp(-\sigma_i jt)\,. \qquad (2.5)$$

Expression (2.5) allows to carry out estimations of mathematical expectation of quantity absorbed photons by electron at its collision with an ion in a field of a powerful pulse of laser radiation. However for this purpose an estimation of section σ_i is necessary. In work [11] with use of methods of quantum electrodynamics we estimate cross section of collision absorption photon by electron for unrelativistic case

$$\sigma^{(ac)}=\frac{1}{\pi^2}Z^2\alpha\,r_e^2\,\lambda^3 n_i\frac{c}{v}F(\frac{v}{v'},\theta), \qquad (2.6)$$

Where Z - the charge number of an ion;

$\alpha=1/137$ - constant of a thin structure;

r_e - classical radius of an electron;

λ - wavelength of radiation;

n_i - concentration of ions in plasma;

c - light velocity;

$v,\ v'$ - velocities of a colliding and scattered electron accordingly;

$F(\theta)$ - factor of the order 1;

θ - angel between photon and colliding electron.

The estimation σ_i on the basis (2.6) turns out by substitution of velocities of a colliding and scattered electron $v\ v'$ before and after absorption i-th a photon.

The further analysis the most simple to carry out for a case of that answers case of interaction of photons with wavelength up to a soft X-radiation with low- and high-temperature plasma with $T<10^8\ K$. The consideration of relativistic case does not cause basic difficulties. For unrelativistic case it is possible to consider that $\sigma_i\approx$const. Then from (2.1) instead of (2.5) easily to receive the following analytical solutions for each probability P_k

$$P_i=\frac{(\sigma j\tau)^i}{i!}exp(-\sigma j\tau), \qquad (2.7)$$

here $\tau=min\{\tau_c,\tau_u\}$ - minimum time from time of impact of an electron with an ion and ultrashort laser pulse duration. As the intensity of photon flux j is proportional to an radiation intensity I ($j=I/\hbar\omega$ in a laboratory system, where the ion is stopped), probability of an absorption of n photons, as follows from (2.7), is proportional I^n pursuant to (1), that testifies to adequacy of the offered approach to the analysis effect of a multiphoton absorption. The evaluation of probabilities (2.7) allow us to receive an analytical evaluation of expectation of number of absorbed photons in one act of interaction in case of a powerful ultrashort laser pulse

$$k=\Sigma iP_i=\sigma j\tau. \qquad (2.8)$$

The obvious result - average of absorbed photons number equally to number photons witch interacted with electron in this case received, that is a corollary of approximation σ_i = *const*. In more general case $\sigma_i\neq const$

the given approach also allows to receive an analytical evaluation of an average of absorbed photons number.

3. Estimation of ultrashort laser pulse penetration depth into plasma at multiphoton collisional absorption

To evaluate the penetration depth for ultrashort laser pulses, let us consider the light-matter interaction in the framework of a next concept. We consider the interaction of laser photons flux with plasma electrons which periodically colliding with the ions ones within the average time

$$<t>=1/\nu_{ei}, \tag{3.1}$$

where ν_{ei} is the frequency of those collisions. Note that such an approach naturally relieves the problem of time representation for the field of laser pulse with a duration of few field oscillations or even shorter than one period.

To simplify further analysis, we assume the medium parameters (ρ, T, P, n_e etc.) to be unchanged during irradiation, which is not correct in the case of long (nanosecond and longer) laser pulses. However, this assumption is valid for the pulse duration comparable or shorter than the average time of collision between quasifree electrons with ions. On this basis, the "ultrashort" laser pulse is defined as a pulse during which the medium has no time to change its parameters. These parameters vary after passing the ultrashort laser pulse through the medium. Since $<t>=1/\nu_{ei}\approx 10^{-13}s$, this definition embraces femtosecond and shorter pulses.

The flux intensity j after passing the thin layer plasma dx is additionally reduced by $\nu_{ei}n_e k\,dx$

$$dj=-\nu_{ei}n_e k\,dx, \tag{3.2}$$

where k is average number of absorbed photons in one act of interaction in case of a powerful ultrashort laser pulse. After substituting k from (2.8) easily to receive the following equation

$$dj=-\nu_{ei}\,n_e\sigma\,j\tau\,dx, \tag{3.3}$$

which, in turn, is integrated to yield

$$j=j_0\,exp(-\sigma\,n_e\nu_{ei}\,\tau\,x) \tag{3.4}$$

from here the penetration depth for powerful ultrashort laser pulses is estimated as

$$l=\frac{1}{\sigma n_e}\frac{1}{\nu_{ei}\tau} \tag{3.5}$$

For case $\tau_c<\tau_u$ the penetration depth is not depended from pulse duration $l=\frac{1}{\sigma n_e}\frac{1}{\nu_{ei}\tau_c}$. The pulse of arbitrary duration, when considering it as a sequence of ultrashort pulses, would have the same penetration depth. However pulses longer than the ultrashort ones heat the medium, thus changing the beam penetration.

Thus, in the present work on the basis the Markov processes theory with discrete states and continuous time receives expectation of number of absorbed photons and receives estimation of ultrashort laser pulse penetration depth into plasma at multiphoton collisional absorption. The given results can be of interest for research of heating dynamics of plasma by powerful laser radiation.

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