UDK 533.9

LONG-WAVELENGTH ION-SOUND PARAMETRIC INSTABILITY OF PLASMA IN MAGNETIC FIELD

Mikhailenko V.S., Ol'shansky V.V. **, Stepanov K, N. ***
** V.N.Karazin Kharkov National University, Kharkov, Ukraine*
*** Institute of Plasma Physics, NSC «Kharkov Institute of Physics and Technology», Kharkov, Ukraine, e-mail olshansky@ipp.kharkov.ua*

We have studied numerically the dependence of the frequency and growth rate of the ion sound wave on the wave vector in plasma in the electric field of a helicon for long-wavelength oscillations when the wavelength of ion sound wave is equal to the order of magnitude to or exceeds the electron Larmor radius. Such instability may be separated if the pumping wave frequency (helicon) is equal to the order of magnitude to or less than the lower hybrid frequency. The growth rate approaches some tenth parts of the lower hybrid frequency if the electron drift velocity in the field of the helicon exceeds the ion sound velocity by some times. As for short wavelength oscillations, one should expect the anomalous damping of the helicon, turbulent heating of electrons and sustainment of gas discharge at the nonlinear stage of such instability.

1. Introduction

The papers [1,2] have shown that high efficiency of helicon plasma sources, in which heating is performed by a helicon, may be due to the short wavelength kinetic ion-sound parametric instability. For this instability the wavelength of unstable ion-sound oscillations $1/k$ is considerably less than the electron Larmor radius, $k\rho_e >>1$. As these oscillations are excited at the pumping frequency $\omega_0 > \omega/m$ (m is an integer) and $a_E = ku/\omega_0 > 1$, then the condition $k\rho_e >>1$ holds for $\omega_0 >> \sqrt{\omega_{ce}\omega_{ci}}$, $m \sim 1$ (here k is the wavenumber, $v_s = \sqrt{T_e/m_i}$ is the velocity and $\omega \sim kv_s$ is the frequency of the ion-sound wave, $u = c(E_{\perp 0}/B_0)$ is the velocity of electron oscillations in the field of the helicon with the frequency ω_0, wave vector $\mathbf{k}_0$ and the strength of the transverse electric field of the wave $E_{\perp 0}$). It is assumed that $u \gtrsim v_s$. Thus the separation of unstable short wavelength oscillations is possible only for «weak» magnetic fields. Such condition holds in a number of experiments by F.Chen et al [3], for which the correspondence between theoretical data [1,2] and measured ones is obtained. However there are a number of experiments with helicon sources in which this condition does not hold and the magnetic field is «strong», $\omega_0 \gtrsim \sqrt{\omega_{ce}\omega_{ci}}$. In this case it is necessary to consider the long wavelength, $k\rho_e \lesssim 1$, ion-sound instability. The present paper is devoted to the theoretical treatment of this problem.

1. Dispersion equation

If the plasma is immersed in the electric field of the pumping wave (helicon) with the frequency ω_0, considerably below the electron cyclotron frequency ω_{ce}, but considerably more than the ion cyclotron frequency ω_{ci}, then with $u \gtrsim v_s$ in the plasma the ion-sound oscillations may be excited for which one can neglect the plasma and pumping field nonuniformity in the direction perpendicular to the magnetic field. As the oscillations may be excited that propagate almost across the magnetic field, then the longitudinal wavenumbers of the unstable oscillation $k_{\parallel}$ and the helicon $k_{\parallel 0}$ may be the same to the order of magnitude, generally speaking. Therefore we will take into account the nonuniformity of the helicon electric field along the magnetic field.

We describe the linear stage of the ion-sound parametric instability with the following infinite set of difference equations for the Fourier components of the electric potential of the ion-sound wave determined in the reference frame oscillating together with ions in the pumping wave field (see. [4,5]):

$$\left(1+\delta\varepsilon_i(\omega,\mathbf{k})\right)\varphi(\omega,\mathbf{k}) + \sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} J_p(a_E)J_{p+n}(a_E)e^{in\delta}\cdot \\ \cdot\delta\varepsilon_e(\omega - p\omega_0, \mathbf{k} - p\mathbf{k}_0)\varphi(\omega + n\omega_0, \mathbf{k} + n\mathbf{k}_0) = 0 \qquad (1)$$

where $\delta\varepsilon_\alpha$ is the contribution to the longitudinal dielectric permittivity made by α-species particles. For the oscillations considered the growth rate exceeds the ion cyclotron frequency substantially and the wavelength is much less than the ion Larmor radius. In this case we can neglect the action of the magnetic field on ion motion, so that

$$\delta\varepsilon_i(\omega,\mathbf{k}) = \frac{\omega_{pi}^2}{k^2 v_{Ti}^2}\left[1 + i\sqrt{\pi}z_i W(z_i)\right]. \qquad (2)$$

For electrons, neglecting the small terms proportional to ω^2/ω_{ce}^2 and $\left(k_{\parallel}^2 v_{Te}^2\right)/\omega_{ce}^2$, we obtain

$$\delta\varepsilon_e(\omega,\mathbf{k}) = \frac{\omega_{pe}^2}{k^2 v_{Te}^2}\left[1 + i\sqrt{\pi}A_0\left(k^2\rho_e^2\right)z_e W(z_e)\right]. \qquad (3)$$

Here the following notation is introduced

$\omega_{p\alpha} = \sqrt{4\pi n_0 e_\alpha^2/m_\alpha}$ is the Langmuir frequency of α -species particles, $v_{T\alpha} = \sqrt{T_\alpha/m_\alpha}$ is their thermal velocity, $z_i = \omega/\left(\sqrt{2}|k|v_{Ti}\right)$,

$$z_e = \omega/\left(\sqrt{2}\left|k_\parallel\right|v_{Te}\right),\ W(z) = \left(1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}dt\right).$$

Making in eq.(1) the substitution $\omega \to \omega - m\omega_0$, $k_\parallel \to k_\parallel - mk_{\parallel 0}$, we obtain the set of difference equations, whose solubility criterion is the vanishing of the determinant

$$D = det\left\|a_{mn}\right\|_{m,n=-\infty}^{\infty} = 0\,, \qquad (4)$$

where

$$a_{mn} = \delta_{mn} + \frac{\exp[-i(n-m)(\delta+\pi)]}{1+\delta\varepsilon_i\left(\omega - m\omega_o,\mathbf{k}\right)}\sum_{p=-\infty}^{\infty} J_{p+m}(a_e)\cdot$$
$$\cdot J_{p+n}(a_E)\delta\varepsilon_e\left(\omega + p\omega_o, \mathbf{k} + p\mathbf{k}_{\parallel 0}\right).$$

For short wavelength oscillations with $k\rho_e >> 1$ there immediately follows from eq. (4) the vanishing of any diagonal element, i.e.,

$$D \sim 1 + \frac{1}{1+\delta\varepsilon_i\left(\omega - m\omega_0,\mathbf{k}\right)}\sum_{p=-\infty}^{\infty} J_{p+m}^2(a_E)\cdot$$
$$\cdot\,\delta\varepsilon_e\left(\omega + p\omega_0, \mathbf{k} + p\mathbf{k}_{\parallel 0}\right) = 0 \qquad (5)$$

This equation was studied in paper [4] at $k_{\parallel 0} = 0$ and in paper [2] at $k_{\parallel 0} \neq 0$. Our problem consists in studying equation (4) for $k\rho_e \lesssim 1$ and pumping frequencies $\omega_0 \gtrsim \sqrt{\omega_{ce}\omega_{ci}}$.

2. Numerical solution of the dispersion equation for long wavelength oscillations

On solving eq. (4) numerically we assumed $\omega_0/\sqrt{\omega_{ce}\omega_{ci}} = 0{,}6$, $\omega_{pe}^2/\omega_{ce}^2 = 25$ (helicon branch of oscillations is usually treated under the condition $\omega_{pe}^2 >> \omega_{ce}^2$), $T_e/T_i = 20$, $u/v_s = 5$, the operating gas is hydrogen. Conventionally one uses argon in helicon sources as an operating gas. It is easy to prove that under the condition $kr_{De} << 1$ the result of calculation does not depend on the mass of the operating gas ions if the quantities $\omega_0/\sqrt{\omega_{ce}\omega_{ci}}$ and u/v_s are regarded as fixed.

Figure 1 shows the dependence of the oscillation frequency on the wave vector. For comparison the same figure shows the dependence of the ion-sound frequency $\omega = kv_s$. It is seen that the frequency depends actually linearly on the wave vector. The difference of the frequency of unstable oscillations on the frequency of ion-sound oscillations of the unmagnetized plasma is associated with the effects of finite electron Larmor radius and the presence of the pumping wave.

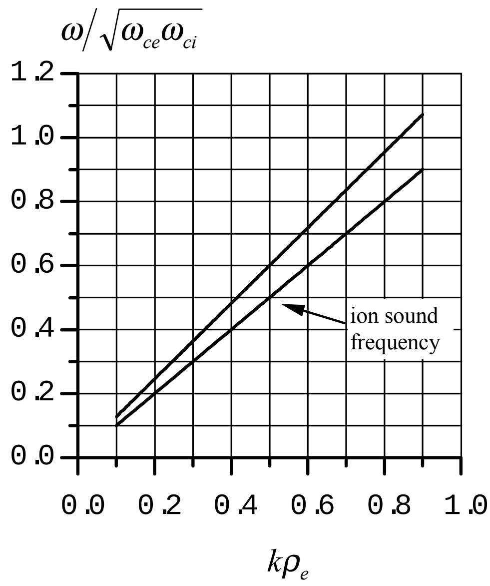

Fig.1. Frequency of parametrically unstable ion-sound oscillations in the helicon field against the wavenumber ($\omega_{pe}^2/\omega_{ce}^2 = 25$, $u/v_s = 5$, $T_e/T_i = 20$,. $\omega_0/\sqrt{\omega_{ce}\omega_{ci}} = 0{,}6$, $k_{\parallel 0} = 0$)

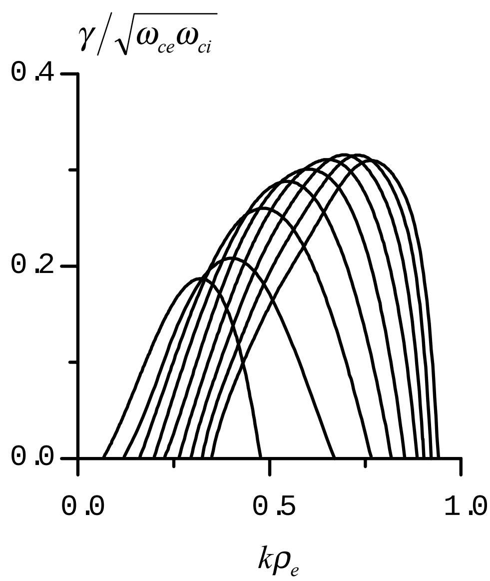

Fig. 2. Growth rate of unstable ion-sound oscillations in the helicon field against the wavenumber for different $\cos\theta$ values (the parameters are the same as in Fig.1). The curves correspond to various $\cos\theta$ values from 0,01to 0,05 wth the step 0,005

Figure 2 depicts the growth rate against different values of the angle θ between the magnetic field and the wave vector. We see that on increasing $cos\theta$ the maximum value of the growth rate increases up to the value $cos\theta = 0.045$, at which the maximum value of the growth rate, $\gamma_{max} \approx 0{,}3\sqrt{\omega_{ce}\omega_{ci}}$,appears to be the biggest. At larger $cos\theta$ values the maximum value of the growth rate decreases with the quantity $cos\theta$ increasing. This biggest value is achieved at $k\rho_e \approx 0{,}75$. In this case the frequency is $\omega \approx 0{,}9\sqrt{\omega_{ce}\omega_{ci}}$ and the excitation of oscillations is due to the beats with p=2 ($p\omega_0 > \omega$). The maximum value of the growth rate in Fig. 2 is related with the value $|z_{ep}| \sim 1$. In this case the interaction of resonant electrons possessing the velocity of the order of the thermal velocity with the p-th beats appears to be important.

Conclusion

The study performed shows that in the case of «strong» magnetic field the ion-sound parametric instability in the helicon field also sets in. And it is even stronger than in the case of «weak» magnetic field for which $\gamma_{max} \sim 0{,}05\sqrt{\omega_{ce}\omega_{ci}}$. Therefore one can also expect in the «strong» magnetic field case the appearance of a strong ion-sound turbulence leading to electron heating and discharge sustainment in the alternating field of the helicon.

Note that recently there is actively discussed (see review [6]) the mechanism of strong absorption of the pumping wave in the helicon source due to the excitation of electrostatic oscillations called Gould-Trivelpiece waves in the surface layer of plasma and the coupling of these modes with helicons due to plasma nonuniformity. In the presence of such mechanisms the parametric excitation of ion-sound oscillations is also possible for the strong as well for the weak magnetic field. In this case the absorption of the Gould-Trivelpiece mode is determined by the effective collision frequency depending on the level of the ion-sound turbulence.

References

1. O.I.Akhiezer, V.S.Mikhailenko, K.M. Stepanov Ion-Sound Plasma Turbulence with Transverse Current in Magnetic Field //Ukrainian Journal of Physics, 1997 vol. 42, No. 8, p. 990.
2. A.I. Akhiezer, V.S. Mikhailenko, K.N. Stepanov Ion-sound parametric turbulence and anomalous electron heating with application to helicon plasma sources //Physics Letters A, 1998, vol. 245, No. 1-2, p. 117.
3. F.F. Chen Physics of helicon discharges //Physics of Plasmas, 1996, vol. 3, p. 1783.
4. A.B. Kitsenko, V.I. Panchenko, K.N. Stepanov, F.F. Tarasenko Parametric instabilities and turbulent heating of a plasma in the field of a fast magneto-acoustic wave //Nuclear Fusion, 1973, vol. 23, p. 527.
5. A.F. Korzh, V.S. Mikhailenko, K.N. Stepanov Parametric Ion Cyclotron Instability of Plasma in the Electric and Magnetic Fields of the Nonuniform MHD wave //Sov. Plasma Physics, 1989, vol. 15, p. 413.
6. K.P. Shamrai Collective mechanisms of absorbing RF power in helicon plasma sources //Fizika plazmy, 1999, vol. 25, p. 934.