MULTI-BEAM ACCELERATING MODULE CALCULATION

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INTRODUCTION

As has been pointed in [1], a multi-beam accelerating cavity with drift tubes is suggested to be used in a linac for the purpose of overcoming limitations on the mean current.

The multi-beam accelerating cavity containing a cylindrical screen 1 with end flanges 2 is shown in Fig. 1. Current conducting rings 3 are set inside the screen perpendicularly to its axis. Drift tubes 4 with integral constant magnet focussing quadrupoles are uniformly distributed along the ring circumference. Each ring is attached to the screen inner surface by means of two diametrically placed supports 5 in such a way that supports of neighbouring rings are mutually perpendicular. At the operational mode the resonant elements (ring 3 with supports 5) are exited so that the oscillation phase is opposite.

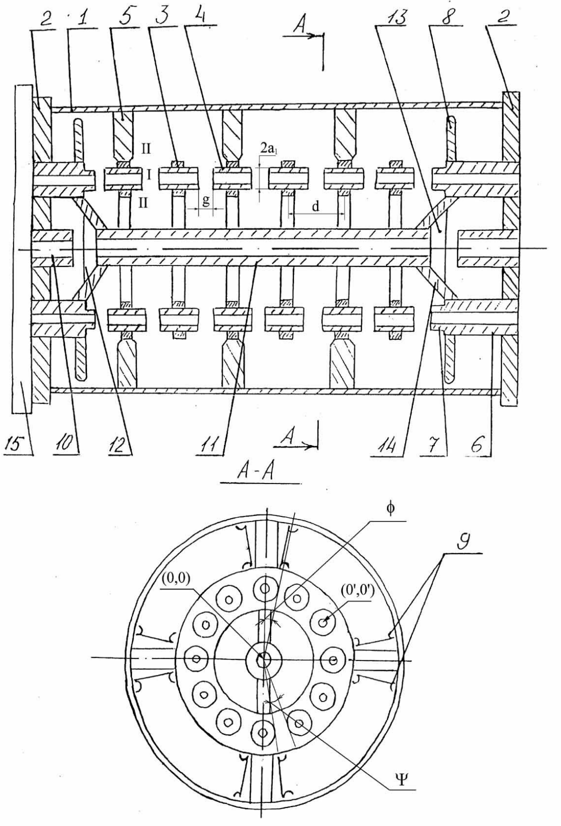

Fig.1

In this paper the dispersion equation for such a structure was deduced and analyzed.

DISPERSION EQUATION

The calculations have been carried out under the following assumptions. The structure geometry is approximated as shown in Fig. 2.

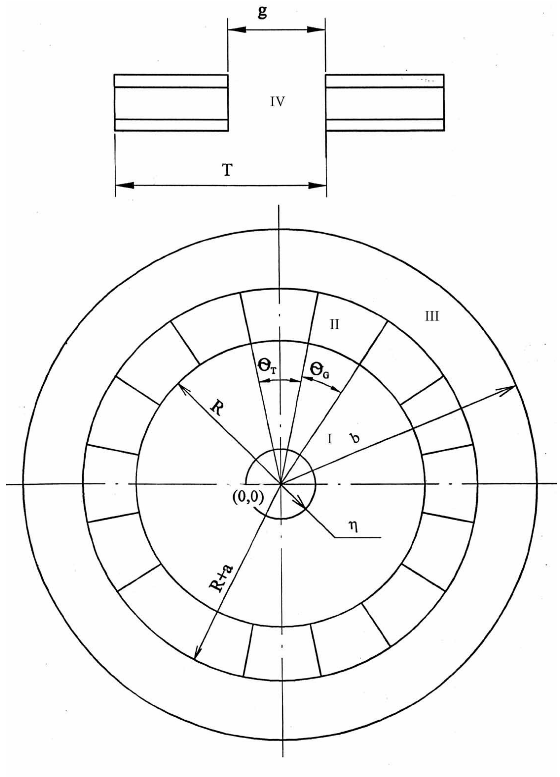

Fig.2

The drift tubes and the cavity body are assumed to be ideally conducting. The structure length is infinitely long. Only E and H modes can be exited in the structure that follows from the module geometry. Let us divide the structure into several regions. The complete mathematical modeling of the electromagnetic wave propagation requires some factors to be specifically accounted for. Among them there are the boundary conditions on the conductor surface (its geometrical configuration), the conditions on the boundaries dividing the regions, the two-dimensional periodicity. Taking into account these requirements Helmholtz equations were solved for each region with corresponding boundary conditions. Thus, for the region 1 we have:

$$\Delta E_z^I + k^2 E_z^I = 0, \qquad (1)$$

$$E_z^I(\eta, \varphi, z) = 0, \qquad (2)$$

$$E_z^I(R, \varphi, z) = f(\varphi, z), \qquad (3)$$

where the function f(φ,z) is given by

$$f(\varphi, z) = \begin{cases} 0, \varphi \in \{\nu\theta; (\nu+1)\theta - \theta_m\} \\ E_z^{II}(R, \varphi, z); \varphi \in \{\nu\theta - \theta_m; \nu\theta\} \\ E_z^{IV}(R, \varphi, z); z \in \{\nu(T-g), \nu T\}, \nu = 0,1,2,... \end{cases} \qquad (4)$$

It should be noted that E-modes are considered. Here Laplacians are expressed in a cylindrical coordinate system, its center being at the module axis. The solution of Eq. (1) is represented as a series of own functions. The required dispersion equation is obtained by means of using expressions of magnetic field strength components and determination of series coefficients A_{ik}.

For the purpose of analysis it seems to be convenient to consider the harmonic (0,0) for which the dispersion equation can be written as

$$\frac{1}{T\Theta}\left\{\int_0^L\int_{\theta-\theta_r}^{\theta}\left[\left(\frac{1}{\Theta_T L}\int_{\theta_G}^{\theta}\int_0^L K\xi'_{00}(R+a,\varphi,z)\bar{u}_{00}(R+a,\varphi,z)d\varphi\,dz\right)\bar{u}_{00}(R,\varphi,z)+\right.\right.$$

$$+\left(\frac{1}{\Theta_T L}\int_{\theta_G}^{\theta}\int_0^L K_{ml}\xi'_{ml}(R+a,\varphi,z)\bar{\bar{u}}_{00}(R+a,\varphi,z)d\varphi\,dz\right)\bar{\bar{u}}_{00}(R,\varphi,z)\Bigg]\nu_{00}(R,\varphi,z)d\varphi\,dz+$$

$$+\left[\left(\frac{1}{\Theta g}\int_0^{\theta}\int_L^T K\xi_{00}(R+a,\varphi,z)\bar{\Gamma}_{00}(R+a,\varphi,z)d\varphi\,dz\right)\bar{\Gamma}_{00}(R,\varphi,z)+\right.$$

$$\left.\left.+\left(\frac{1}{\Theta g}\int_0^{\theta}\int_L^T K\xi_{00l}(R+a,\varphi,z)\bar{\bar{\Gamma}}_{00}(R+a,\varphi,z)d\varphi\,dz\right)\bar{\bar{\Gamma}}_{00}(R,\varphi,z)\right]\nu_{00}(R,\varphi,z)d\varphi\,dz\right\}=$$

$$=\frac{1}{T\Theta}\left\{\int_0^L\int_{\theta_r}^{\theta}\left[\left(\frac{1}{\Theta_T L}\int_{\theta_G}^{\theta}\int_0^L K\xi'_{00}(R+a,\varphi,z)\bar{u}'_{00}(R+a,\varphi,z)d\varphi\,dz\right)\bar{u}'_{00}(R,\varphi,z)+\right.\right.$$

$$+\left(\frac{1}{\Theta_T L}\int_{\theta_G}^{\theta}\int_0^L K\xi'_{00}(R+a,\varphi,z)\bar{\bar{u}}'_{00}(R+a,\varphi,z)d\varphi\,dz\right)\bar{\bar{u}}'_{00}(R,\varphi,z)\Bigg]\nu'_{00}(R,\varphi,z)d\varphi\,dz+$$

$$+\left[\left(\frac{1}{\Theta g}\int_0^{\theta}\int_L^T K\xi_{00}(R+a,\varphi,z)\bar{\Gamma}'_{00}(R+a,\varphi,z)d\varphi\,dz\right)\bar{\Gamma}'_{00}(R,\varphi,z)+\right.$$

$$\left.\left.+\left(\frac{1}{\Theta g}\int_0^{\theta}\int_L^T K\xi'_{00}(R+a,\varphi,z)\bar{\bar{\Gamma}}'_{00}(R+a,\varphi,z)d\varphi\,dz\right)\bar{\bar{\Gamma}}'_{00}(R,\varphi,z)\right]\nu'_{00}(R,\varphi,z)d\varphi\,dz\right\} \quad (5)$$

where the functions ξ, u, Γ, v – are taken to be Bessel and Neumann functions, such as (6).

$$\left\{K_{ij}(x)-\frac{K_{ij}(c)}{I_{ij}(c)}I_{ij}(x)\right\}e^{-if}, \quad (6)$$

That equation was solved graphically (Fig 3, Table 1) for λ=10 cm. For any module geometry chosen in advance (Q, Q_T, L, g) such a graph makes it possible to determine the required diameter of drift tubes and to analyze the dynamics of its variation with changing β:

$$a=\frac{const}{\sqrt{k^2-\left(\frac{2\pi}{\beta\lambda_0}\right)^2}}, \quad (7)$$

where the *const* is defined by graph (Fig. 3).

Table 1.

νa	$\theta_m g/\theta\ L$		
	1	2	3
0,5	5,513	1,508	0,811
1,0	4,742	2,112	0,813
1,5	6,803	1,834	0,605
2,0	9,610	2,122	0,523

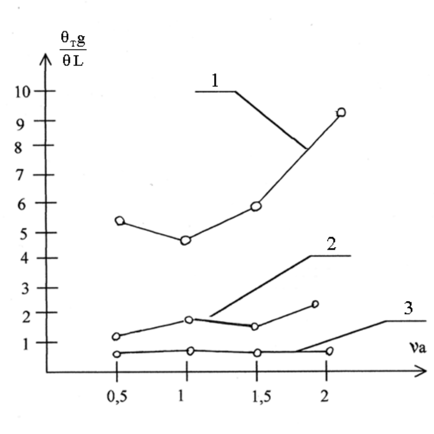

Fig. 3. The parameters are: ***1*** – *R/a*=1.25, *a/η*=1, *b/a*=4: ***2*** — *R/a*=1.5, *a/η*=1.25, *b/a*=5: ***3*** — *R/a*=2.0, *a/η*=1.5, *b/a*=6.

It is of interest to note that in the limiting case when $Q/Q_m \to 1$ Eq. (5) can be written as

$$\xi_2\frac{K_1[\nu(R+a)]\ \{K_1[\nu(R+a)]K_1(\nu R)-I_1[\nu(R+a)]I_1(\nu R)\}}{K_0[\nu(R+a)]\ \{I_0[\nu(R+a)]I_0(\nu R)-K_0[\nu(R+a)]K_0(\nu R)\}}=1 \quad (8)$$

Eq. (8) is similar to that obtained by P. Alliot for such a structure [2].

CONCLUSION

The correlation of geometrical parameters of the structure was demonstrated in this work. In addition, recommendations are given for the determination of the drift tube diameter for cases of given geometry and dynamics. The limiting case $Q/Q_m \to 1$ was investigated. The obtained results could be used in the development of accelerators.

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