PLASMA FOCUSING DEVICES IN EXTERNAL PROGRAMMED MAGNETIC FIELD

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1. INTRODUCTION

At present there is a significant requirement for the development of devices for focusing high-energy intense ion beams for the solution of actual scientific and technological problems (inertial thermonuclear fusion on heavy and light ions, radiotherapy, high energy investigations, research of radiation resistance of materials, implantation metallurgy, etc. [1,2]). For this purpose the quadrupole magnetic lenses are widely used. Last time plasma lenses with essentially larger focusing force are used. It is essential that the charge of focused beam is compensated in these lenses. With energy and current growth of accelerated beams plasma lenses should replace the conventional ones.

Plasma lens investigation is performed in many scientific centers such as Lawrence Berkeley Laboratory, CERN, GSI (Darmstadt) and others (see, e.g., one of the last reviews [3]). Nowadays three types of lenses are usually used: Gabor electron (electrostatic) lens, current (magnetic) plasma lens, Morozov electrostatic plasma lens. As for historical approach, the self-focusing of charged particles beams by own magnetic field was considered in thirties by H.Alfven and W.Bennet. In 1947 D.Gabor proposed the space charge lens for electrostatic focusing of ion beams [4]. Gabor lens consists of cylindrical column of electron plasma confined with magnetic field. Further W.Panofsky and W.Baker performed experiments on high-energy ion beams focusing by magnetic plasma lens which represents itself the discharge in plasma [5]. In this case focusing is provided by the azimuthal magnetic field produced by longitudinal current in plasma column. Last years lenses of such type are successfully elaborated and used in CERN and GSI. In sixties A.Morozov proposed a plasma electrostatic lens in which magnetic surfaces are the equipotentials of electric field [6]. Further this direction was successfully developed in experiments of A.Goncharov, i.e., for ion beam focusing with the energy of tens kV [7, 8]. In eighties for focusing ultra-high energy electron beams it was proposed «passive» plasma lenses [9, 10] based on conception of magnetic self-focusing. Later the conception of «passive» plasma lenses was expanded on more worth-while adiabatic plasma lenses [11].

For focusing of intense ion beams of high energy the longitudinally homogeneous «active» magnetic plasma lenses is applied with the greatest success (e.g., see [12, 13]). With the help of such lens the ions of gold with energy 2 GeV were focused on the distance of about 30 cm [12, 13]. For such focusing the discharge of a capacity unit with current of about 10 kA was used at time duration of beam focusing about 1 (sec. The efficiency of such lens was (10^{-5} in the first work [12]. In the second work it was enhanced to (10^{-3} [13] but was rather small yet. The beam was focused to possible minimum radius 125 μm conditioned by initial emittance. In the work [14] it was also proposed to use adiabatic plasma lens for slow compression of ion beam on the length of several betatron oscillations. These data show good prospects of the focusing method in question and the necessity of efficiency enhancement investigation.

This work continues the investigations of plasma lenses performed at the NSC KIPT in the frame of the STCU Project No.298 [15,16]. In this work for the enhancement of focusing efficiency by current (magnetic) plasma focusing device (plasma focuser) it is proposed to use the external magnetic field that changes along the device length so that the focusing current channel radius be close to the focused beam radius and be decreasing with the beam radius decrease. (In this consideration the fact is used that the current channel radius is inversely proportional to the square root of magnetic field strength). It will allow to reduce the focusing current, to increase the efficiency, entrance radius and pulse duration of the focused beam.

In this work the studies of non-uniform adiabatic plasma lenses of different types (magnetic lens, Gabor and Morozov lenses, charge-current lens) are performed. It should be noted, that in [11, 14] the criterium of adiabatic property is small changing of plasma lens parameters over focusing length (slight non-uniformity). In this work the another case will be studied when that criterium is small changing of focusing device parameters on much smaller «cyclotron length» which is equal to the product of electron cyclotron frequency by longitudinal velocity of plasma electrons (the condition of drift approximation). In this case the plasma parameters change mainly on the focusing length (strong nonuniformity).

CALCULATIONS OF ION FOCUSING IN NON-UNIFORM PLASMA LENSES

At first we study in detail the current (magnetic) plasma lens, and then discuss the usefulness of these results to Gabor and Morozov lenses of charge (electrostatic) type. In conclusion, the charge-current lens will be studied. Under the term «lens», the long focusing channel will be understand in this paper (i.e., «plasma focuser» [11]).

2.1. Plasma current (magnetic) lens

Let us consider the problem of ion beam focusing by an azimuth magnetic field of longitudinal current in plasma. We investigate the case when the current radius is determined by the external non-uniform longitudinal magnetic field. The problem is being solved at the paraxial approximation. In this case the equation of the magnetic surfaces is as follows:

$$a^2(z) = \frac{a^2(0)B_z(0)}{B_z(z)}, \qquad (1)$$

where $a(z)$ is the variable radius of the magnetic surface, $B_z(z)$ is the longitudinal magnetic field on the axis, $B_z(0)$ and $a(0)$ are determined by the boundary conditions at are determined by the boundary conditions at $z = 0$. We assume that in the case of the strong magnetic field the electrons which transport the current in plasma are moving along cylindrical magnetic surfaces enclosed one into another. The boundary conditions are defined as follows: at z=0, $a(0) = b$, where b is the radius of an electrode that supply the current in the plasma (e.g., it is the inner electrode of the plasma gun). From Eq.(1) it follows: if the equidistantness of the magnetic surfaces is set in some cross-section, then it is conserved in any other one. As a result, if the current density is homogeneous in the electron emitter region, then it is homogeneous in any other current channel cross-section. It is necessary for focusing without spherical aberration because the Lorenz force focusing an ion toward the axis is proportional to the distance of the ion from the axis:

$$F_m = -\frac{e}{c} v B_\varphi = -\frac{ev}{c}\frac{2I}{cr} = -\frac{2evI}{c^2 a^2(z)} r . \quad (2)$$

As a result, the equation for the focusing ions trajectories will take the form:

$$r'' + k^2 \frac{B_z(z)}{B_z(0)} r = 0, \; k^2 = \frac{2I e}{Mc^2 v b^2} . \quad (3)$$

In (2), (3), I is the current in plasma, e and M are the charge and mass of the ion (i.e., the proton), c is the light velocity, v is the ion velocity, $B_z(0)$ is the magnetic field induction in the region of the plasma gun output, B_φ is the azimuthal magnetic field of the current. Under condition $B_z(z)$ = const from Eq.(3) we have: $r = r_0 \cos kz$, and the focusing distance in the plasma: $L_f = \pi / 2k$. For a lens of length $l < L_f$ we have:

$$L_f = l + k^{-1} ctg(kl) ,$$

where at $kl <<1$ it is easy to obtain the well-known expression for focusing distance of a thin lens: $L_f = (k^2 l)^{-1}$.

In general case, the trajectories of focused particles are calculated with the help of a computer [16]. For some cases the Eq.(3) has an analytic solution, e.g., for the «bell-liked» distribution of the magnetic field

$$B_z(z) = B_z(0)\left[1 + (z/d)^2\right]^{-2} . \quad (4)$$

In this case Eq.(3) takes the form:

$$r'' + k^2 r\left[1 + (z/d)^2\right]^{-2} = 0 . \quad (5)$$

The solution of Eq.(5) that is known from the electron optics [18] can be written as it follows:

$$r = \frac{r_0}{\sqrt{1+k^2}} \frac{\sin\left(\sqrt{1+k^2}\,\mathbf{arcctg}\, z/d\right)}{\sin\left(\mathbf{arcctg}\, z/d\right)} . \quad (6)$$

The coordinate of the ion beam focus corresponds to the condition r=0, and is defined by the expression:

$$z_f = d\,\mathbf{ctg}\frac{\pi}{\sqrt{1+k^2}} . \quad (7)$$

The calculations based on the above formulae show: due to compression of the current channel by the external magnetic field, the one -order decrease of the focusing current can be reached. In this case the focusing of intense proton beams (of MeV range energy) in steady state regime can be realized.

During the ion focusing and compression of the current channel by the magnetic field of a solenoid, some ions (with large injection radius) can move partly out of the current channel. They also deflected to the axis but not get to the common focus. The moving equation for them has the form:

$$r'' + \frac{\kappa}{r} = 0, \text{ where } \kappa = \frac{2ZeI}{c^2 Mv} . \quad (8)$$

To put together all ions in the focus, it is necessary to optimization the external magnetic field distribution. For this aim we can determine the form of the magnetic surface that limits the current channel. Then we can calculate the parameters of the solenoid (for producing such magnetic surface) and determine the focusing ion trajectories (see [16,17]). The calculation can be carried out for paraxial ion trajectories and paraxial magnetic surfaces where particles and magnetic force lines go through input and output butt-endes (faces) of a cylindrical lens.

The limit magnetic surface is determined from the condition that its radius (R) coincides with the current channel radius (a) and the radius of the focused beam. The functions $R(z)$ and $B_z(z)$ are determined from the equation similar to (8):

$$R'' + \frac{\kappa}{R} = 0, \; \kappa = \frac{2ZeI}{c^2 Mv} . \quad (9)$$

The solution of the Eq.(9) (with initial conditions: $R = R_0, R' = R_0'$ at z=0) has the form:

$$z = \pm \int_{R_0}^{R} \frac{dr}{\sqrt{R_0'^2 - 2\kappa \ln R/R_0}} . \quad (10)$$

Using the substitution:

$$t^2 = \frac{R_0'^2}{\kappa} - 2\ln\frac{R}{R_0} \quad (11)$$

and the definition of the tabulated function (the probability integral):

$$\Phi(R) = \sqrt{\frac{2}{\pi}} \int_0^R e^{-\frac{1}{2}t^2} dt , \quad (12)$$

we reduce the solution (10) to the form:

$$z = \pm\sqrt{\frac{\pi}{2\kappa}} R_0 \exp\left(\frac{R_0'^2}{2\kappa}\right) \times \left[\Phi\left(\sqrt{\frac{R_0'^2}{\kappa} - 2\ln\frac{R}{R_0}}\right) - \Phi\left(\frac{R_0'}{\kappa}\right)\right] . \quad (13)$$

In the case of the parallel ion beam injection, at z=0 we have $R_0' = 0$, besides, in the focusing region $z > 0$. As a result, the Eq.(13) takes the form:

$$z = \sqrt{\frac{\pi}{2\kappa}} R_0 \Phi_0\left(\sqrt{2\ln\frac{R_0}{R}}\right) . \quad (14)$$

In the real experiment the current channel compression leads to the certain value R_g (not equal to zero) that corresponds to the coordinate z_g . At this place the current channel is finished (by a wire mesh or metallic foil). Later on the inertial ion focusing in the focal spot takes the place. This point's coordinate is defined as follows:

$$z_f = \sqrt{\frac{\pi}{2\kappa}} R_0 \Phi_0 \left(\sqrt{2\ln\frac{R_0}{R_g}} \right) + \frac{R_g}{\sqrt{2\kappa \ln\frac{R_0}{R_g}}} . \qquad (15)$$

The numerical calculations of the Eq.(8) by PC (see [16], Fig.2) give the result for z_f coinciding with the (15).

2.2. The long Gabor lens

For the uniform Gabor lens the expression for the focusing electrostatic force have the form:

$F_e = -2\pi ne^2 r$, where n is the electron density.

The equation for ions' movement has the form:

$$r'' + k_G^2 r = 0,\; k_G^2 = \frac{2\pi ne^2}{Mv^2}, \qquad (16)$$

the equation of ions trajectories and focusing distance are defined by the expressions:

$$r = r_0 \cos k_G z,\; L_f = \frac{\pi}{2k_G} = \frac{\pi v}{2e}\sqrt{\frac{M}{2\pi n}} . \qquad (17).$$

For a lens of length l: $L_f = l + k_G^{-1} ctg(k_G l)$,

where at $k_G l << 1$ it is easy to obtain the known expression for focusing distance of a thin lens [3, 4].

2.3. The long Morozov lens

In the Morozov lens the electric potentials are inserted into plasma by the concentric ring electrodes. In this case, the system of the «charged» magnetic surfaces are created in the plasma. It is supposed that in the applied strong magnetic field the transverse current is absent. For the long Morozov lens, it is worth-while to place the concentric ring electrodes on the lens faces (or near by them at lateral surface), i.e. at the input and output of the focused beam. In these conditions, the expression for the focusing force of electric field has the form:

$$F_e = -eE_r = e\frac{\partial \varphi(r,z)}{\partial r} . \qquad (18)$$

With the given distribution of potential along the radius: $\varphi = \varphi_0 r^2 / a_0^2$, (were a_0 is the radius of the external magnetic surface at $z=0$, and φ_0 is its potential), one can have:

$$F_e(r,z) = e\frac{2\varphi_0}{a^2(z)} r . \qquad (19)$$

With taking into account Eq.(1), the equation for movement of focused ions is:

$$r'' + \frac{B_z(z)}{B_z(0)} k_M^2 r = 0, \text{ where } k_M^2 = \frac{2e\varphi_0}{Mv^2 a_0^2} \qquad (20)$$

In the uniform case the focusing distance is equal to:

$$L_f = \pi (2k_M)^{-1} = \frac{\pi v a_0}{2}\sqrt{\frac{M}{2e\varphi_0}} . \qquad (21)$$

In the non-uniform case, the external magnetic field can increase from the input to the output of the lens in such a manner, that the limiting magnetic surface can coincide with the focusing beam radius. Simultaneously the focusing electric field is increasing that results in the enhancement of the lens efficiency and focusing distance decreasing. The solution of this problem is similar to the case of the non-uniform magnetic lens (see above).

2.4. The charge-current lens

In conclusion, we study briefly the case of ion beam focusing by the counter-stream intense electron beam. As it is known, the electron concentration in such beams can reach to $10^{12} - 10^{13}$ cm^{-3}, therefore its using for ion beam focusing can have good prospects.

The expression for the focusing force has the form:

$$F_r = F_e + F_m = -2\pi ne^2 r - 2\pi ejvc^{-2} r , \qquad (22)$$

where n is the electron concentration, j is the current density. Let us express n and j by the current I, radius a and velocity v_e of the electron beam:

$$n(z) = \frac{I}{ev_e \pi a^2(z)},\; j(z) = \frac{I}{\pi a^2(z)} . \qquad (23)$$

The expression for the focusing force takes the form:

$$F_r = -\frac{2eI}{a^2(z) v_e}\left(1 + \frac{vv_e}{c^2}\right) . \qquad (24)$$

The equation for focused ion trajectories (with taking into account Eq.(1)):

$$r'' + \frac{B_z(z)}{B_z(0)} k_e^2 r = 0,\; k_e^2 = \frac{2eI(1 + vv_e c^{-2})}{Mv^2 v_e a_0^2} . \qquad (25)$$

In the uniform or non-uniform cases one can use similar methods and formulae (with the own k_e) as in the above-mentioned case of magnetic lens.

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