# THE RELATIVISTIC E-BEAM INTERACTION WITH PLASMA-FILLED COAXIAL CORRUGATED WAVEGUIDE

K.A. Lukin, I.N. Onishchenko*
Institute of Radio-Electronics, NASU, *NSC KIPT, Kharkov, Ukraine

## 1. INTRODUCTION

The conventional FEL-devices are based on relativistic electron beams and magnetic undulators. For wavelength shortening and beam current arising the plasma undulator has been proposed. Taking into account the recent achievements in high power generation (e.g. above $10^{12}$ W for the corrugated systems) it is alluring to use these intense HF waves as an undulator. Let the additional REB of energy $mc^2\gamma_1$ generates the eigenmode of the slow wave structure (SWS) with frequency $\omega_1$ and wave number $k_1 = 2\pi/D$ which propagates oppositely to the main REB direction. The main REB of energy $mc^2\gamma_2$ being undulated in the fields of excited wave (by other words in the fields of induced charges) irradiates the other eigenmode with frequency $\omega_2$ and wave number $k_2$. Using laws of energy and momentum conservation

$$\omega_2 = \omega_1 + \omega_b\ ,\ k_2 = k_b - k_1$$

we can obtain the following relation for the frequency of irradiated wave caused by the FEL mechanism:

$$\omega_2 = k_1\ (v_1 + v_2)\ (1 + v_2/c)\ \gamma_2^{\,2}$$

where $v_1$ and $v_2$ are the velocities of the additional and the main REB, respectively. It is evident that for the ultrarelativistic case, i.e. $v_1 = v_2 = c$ it gives a known formula: $\omega_2 = 2\pi c/D\ 4\ \gamma_2^{\,2}$.

From the other side, the novel kind of SWS with plasma assistance is being developed now. The vacuum SWS, which are used in Cherenkov microwave generators and amplifiers, have an essential shortcoming due to the surface character of a slow wave. The decreasing of longitudinal field component from periphery to system axis causes the fall of the coupling coefficient of the near-axis beam with a slow wave. It leads to decreasing the instability in growth rates. This shortcoming is especially strong in the high frequency range. In the other kind of SWS - homogeneous plasma waveguides [1] - the slow waves are volumetric and have the maximal longitudinal electrical field at the axis where charged particles move. However, in the non-relativistic region of phase velocities the plasma waves are quasi-longitudinal, with small transversal components of fields, that complicates the microwave energy input and output. Hybrid systems, which are promizing in the non-relativistic region of phase velocities, were offered for the first time in KIPT. They combine the advantages of vacuum and plasma systems and have no shortcomings marked above. The hybrid structure uses a plasma waveguide as the beam transition channel of vacuum SWS [2,3]. In such a structure the beam-plasma interaction plays the determining role in excitation of oscillations, and periodic waveguide system is used for power output. The coaxial systems have an additional advantage due to the presence of a cable mode providing a wide frequency band.

In this work we investigate the first stage of the problem considered, namely the interaction of the electron beam with the plasma-filled coaxial corrugated waveguide. The dispersion equation, which describes interaction of REB with SWS, is obtained. The non-linear stage and excitation efficiency is considered.

## 2. THE DISPERSION EQUATION

The axially-symmetric waveguide, formed by two coaxial ideally conducting cylinders is considered. The inner cylinder is smooth, the external one is corrugated sinusoidally with a period D. In the cylindrical system of coordinates $(r,\varphi,z)$ waveguide surfaces are given as:

$R_s(z)=R_{0s}=const$, $R_g(z)=R_{0g}(1+\delta\cos(k_0 z))$, $k_0=2\pi/D$,

where $R_{0s} < R_{0g}(1-\delta)$, $-\infty < z < \infty$, $0 \le \varphi \le 2\pi$, $0 < \delta << 1$.

It is supposed, that waveguide is filled with homogeneous plasma and is placed in a strong magnetic field $\omega_c >> \omega_p$, where $\omega_c$ is the cyclotron frequency and $\omega_p$ is the electron plasma frequency. Waves of E-type $(E_z, E_r, H_\varphi)$ are considered. We solve the Maxwell equations system together with boundary conditions for tangential component of electrical field on a surface of waveguide and we find the dispersion of electromagnetic waves $\omega(k_3)$ proceeding from a condition of existence of the non-trivial solution of this system, as it was made in [4].

### 2.1. THE DISPERSION EQUATION OF PLASMA-FILLED COAXIAL WAVEGUIDE

As the corrugated waveguide is periodical along Z-axis, components of electromagnetic field, according to the Flocke theorem, can be presented as a seriesof spatial harmonics: $F(\vec{r}_\perp, z) = \sum_{n=-\infty}^{\infty} A_n f_n(\vec{r}_\perp) e^{ik_{||n} z}$, where $k_{||n} = k_3 + k_0 n$, $k_3$ is the longitudinal wave number. For axially-symmetric E-wave fields in the region between internal and external cylinders are:

$$\begin{aligned}
E_z(r,z,t) &= \sum_{n=-\infty}^{\infty} E_{zn} = \sum_{n=-\infty}^{\infty} A_n F_0(k_{\perp n} r)\ e^{i(k_{||n} z - \omega t)},\\
E_r(r,z,t) &= \sum_{n=-\infty}^{\infty} E_{rn} = -\sum_{n=-\infty}^{\infty} \frac{i\varepsilon_{||} k_{||n}}{k_{\perp n}} A_n F_1(k_{\perp n} r)\ e^{i(k_{||n} z - \omega t)},\\
H_\varphi(r,z,t) &= \sum_{n=-\infty}^{\infty} H_{\varphi n} = -\sum_{n=-\infty}^{\infty} \frac{i\varepsilon_{||}\, \omega/c}{k_{\perp n}} A_n F_1(k_{\perp n} r)\ e^{i(k_{||n} z - \omega t)}.
\end{aligned} \tag{1}$$

where $k_{\perp n}^2 \equiv \varepsilon_{||}\left(\omega^2/c^2 - k_{||n}^2\right)$, $\varepsilon_{||} = 1 - \omega_p^2/\omega^2$, $\omega_p^2 = 4\pi n_p e^2/m_e$ is the electron plasma frequency, $n_p$ is the plasma density, $-e$ and $m_e$ are the charge and mass of electron accordingly,
$F_0(k_{\perp n}r) = N_0(k_{\perp n}R_{0s})J_0(k_{\perp n}r) - J_0(k_{\perp n}R_{0s})N_0(k_{\perp n}r)$,
$F_1(k_{\perp n}r) = N_0(k_{\perp n}R_{0s}) \times J_1(k_{\perp n}r) - J_0(k_{\perp n}R_{0s})N_1(k_{\perp n}r)$; $J_0, J_1, N_0, N_1$ are cylindrical Bessel and Nejmann functions.

The boundary condition on corrugated surface of waveguide can be written in components of electrical field $E_z$ and $E_r$ in the following form:

$$E_z(R_g(z)) + E_r(R_g(z)) \cdot tg(\theta(z)) = 0, \quad (2)$$

where $tg\theta = \frac{d}{dz}R_g = -\delta k_0 R_{0g}\sin(k_0 z)$. Substituting fields (1) into (2) after appropriate transformations we received the following infinite system of algebraic equations for $A_n$:

$$\sum_{n=-\infty}^{\infty} A_n C_{mn}(\omega, k_3) = 0, -\infty < m < \infty, \quad (3)$$

$$C_{mn}(\omega, k_3) = \frac{1}{D}\int_{-D/2}^{D/2} F_0[f_n(z)]\, e^{ik_0(n-m)z} dz \left[1 + \frac{\varepsilon_{||}k_{||n}k_0(n-m)}{k_{\perp n}^2}\right] \quad (4)$$

where $f_n(z) \equiv k_{\perp n}R_{0g}(1 + \delta \cdot \cos(k_0 z))$. The system (3) has the non-trivial solution, only if its determinant is equal to zero. It defines the required dispersion equation:

$$\det \| C_{mn} \| = 0. \quad (5)$$

### 2.2. THE DISPERSION EQUATION FOR STRUCTURE WITH A BEAM

Let the axially-symmetric tubular infinitely thin monoenergetic electronic beam propagates along the system axis. Beam density $n_b(r) = [I/2\pi e v_b R_b]\delta(r - R_b)$, where $v_b$, I, $R_b$ are velocity of particles, current and radius of beam, respectively. We consider fields in two regions: I - between an internal cylinder surface and beam, II - between beam and external corrugated cylinder surface. The boundary conditions on beam for fields $E_z$ and $E_r$ are:

$$[rE_r] = \frac{2iek_3 IE_z}{m_e v_b \gamma_b^3(\omega - k_3 v_b)^2}, \qquad [E_z] = 0, \quad (6)$$

where $[f] \equiv f(R_b + 0) - f(R_b - 0)$, $\gamma_b$ is the relativistic factor of beam. In the region 1 field can be presented in the form (1). In the region 2 we have:

$$E_z^2(r,z,t) = \sum_{n=-\infty}^{\infty} A_n[x_n J_0(k_{\perp n}r) + y_n N_0(k_{\perp n}r)]\, e^{i(k_{||n}z - \omega t)},$$
$$E_r^2(r,z,t) = -\sum_{n=-\infty}^{\infty} A_n \frac{i\varepsilon_{||}k_{||n}}{k_{\perp n}}[x_n J_1(k_{\perp n}r) + y_n N_1(k_{\perp n}r)]\, e^{i(k_{||n}z - \omega t)}. \quad (7)$$

We matched fields from regions I and II on the boundary $r = R_b$ taking into account boundary conditions (6) and satisfying boundary conditions (2) on corrugated waveguide surface. Similarly to the case with no beam we obtained the dispersion equation in the form (5) with the only difference, that $F_0[f_n(z)]$ in (4) should be replaced by:

$$T_0(z, k_3, \omega, n) \equiv G_0(k_{\perp n}, R_{0s}, R_g(z)) - \frac{2ek_{\perp n}IG_0(k_{\perp n}, R_{0s}, R_b)}{m_e v_b \gamma_b^3 R_b \varepsilon_{||}(\omega - k_{||n}v_b)^2}\frac{G_0(k_{\perp n}, R_b, R_g(z))}{G_1(k_{\perp n}, R_b, R_b)} \quad (8)$$

where
$G_i(k_{\perp n}, R_1, R_2) \equiv N_0(k_{\perp n}R_1)J_i(k_{\perp n}R_2) - J_0(k_{\perp n}R_1)N_i(k_{\perp n}R_2)$.

It is easy to notice, that when $I \to 0$ function $T_0(z, k_3, \omega, n)$ turns into function $F_0[f_n(z)]$. The choice of a thin beam allowed us to "extract" beam component from arguments of cylindrical functions, where it would enter in the case of REB with final thickness [4]. This eliminates set of beam harmonics appearing in case of beam with final thickness when $\omega \to k_{||n}v_b$, and simplifies the analysis of results.

## 3. EXCITATION EFFICIENCY

The linear stage of interaction of electronic beam with a synchronous wave of corrugated coaxial line continues until non-linear processes of multi-mode interactions appear - disintegrations, modulation instability etc. They redistribute microwave power of a growing synchronous wave across a spectrum and that stabilizes level of excited microwave oscillations. The essential mechanism of instability stabilization of microwave oscillations build-up, which is dominating, is the beam trapping in a field of the main synchronous wave (non-linearity of "wave - particle" type) [7]. Growth rate of this wave was determined in the linear theory. In this case [7] the saturation takes place when growth rate $\mathrm{Im}(\omega)$ is comparable with frequency $\Omega$ of trapped oscillations of beam particles in the wave field:

$$\mathrm{Im}(\omega) \approx \Omega, \quad (9)$$

where $\Omega = \sqrt{\frac{eE_{\max z}k_3}{m\gamma_b^3}}$, $E_{\max z}$ is the saturation amplitude of the longitudinal component of electrical field intensity. From (9) it follows:

$$E_{\max z} \cong \frac{[\mathrm{Im}(\omega)]^2 m\gamma_b^3}{ek_3}. \quad (10)$$

The received expressions allow to estimate efficiency of excitation of microwave fields in the structure under consideration. The efficiency of generation was defined as the ratio of a microwave power flow to a flow of electronic beam particles kinetic energy through the waveguide cross section. To calculate the microwave power flow the eigenwaves of coaxial corrugated waveguide without beam were taken as:

$$E_z = E_{zs} = E_z^0 = A^0 \cdot F_0(k_{\perp 0}r) \cdot e^{i(k_3 z - \omega t)},$$
$$H_\varphi = -i\varepsilon_{||}\frac{\omega}{c}A^0 \cdot \left[\frac{F_1(k_{\perp 0}r)}{k_{\perp 0}} - \frac{C_{00}}{C_{0-1}}\frac{F_1(k_{\perp -1}r)}{k_{\perp -1}}e^{-ik_0 z}\right] \cdot e^{i(k_3 z - \omega t)}, \quad (11)$$

i.e. it was supposed, that the basic contribution to a longitudinal electrical field gives the harmonics with number 0, and the power flow is determined by fields of harmonics with numbers 0 and -1. As a result the following expression for efficiency was obtained:

$$\eta = \frac{v_g}{D}\frac{m}{e}\frac{\gamma_b^6}{\gamma_b - 1}\frac{\omega^2\varepsilon_{||}^2[\mathrm{Im}(\omega)]^4}{k_3^2 c^4 F_0^2(k_{\perp 0}R_b)} \times$$

$$\times \int_0^D dz \int_{R_{0s}}^{R_{0g}(1+\delta\cos(k_0 z))} dr \cdot r \left| \frac{F_1(k_{\perp 0}r)}{k_{\perp 0}} - \frac{C_{00}}{C_{0-1}}\frac{F_1(k_{\perp -1}r)}{k_{\perp -1}} e^{-ik_0 z} \right|^2, \quad (12)$$

where $v_g$ is the group velocity of wave.

Results of investigations of influence of various parameters of electron beam - SWS system on the efficiency value are presented below. For calculation of ω, $k_3$ and Im(ω) the dispersion equation, taking into account only harmonics with numbers 0 and -1, was taken. The beam current was equal to 400 A, $D = 10cm$, $R_{0s} = 2cm$, $R_{0g} = 4cm$, $R_b = 3cm$, $\delta = 0{,}1$. The frequency ω, appropriate to the wave number $k_3$ of a maximum of growth rate of the instability Im(ω) in case with no beam, was substituted into expression (30). The calculations were carried out for a case of vacuum structure, and also for the following values of plasma density: $n_{p1}$=1,3683·$10^{10}$ $cm^{-3}$, $n_{p2}$=2.2619·$10^{10}$ $cm^{-3}$, $n_{p3}$=2,8·$10^{10}$ $cm^{-3}$, $n_{p4}$=1,3·$10^{10}$ $cm^{-3}$.

Table

| $\beta_b$ | $\gamma_b$ | $S_b$ (MW) | η (%) for plasma densities: | | | | |
|---|---|---|---|---|---|---|---|
| | | | $n_p$=0 | $n_{p1}$ | $n_{p2}$ | $n_{p3}$ | $n_{p4}$ |
| 0,6 | 1,25 | 51,2 | 1,08 | 1,17 | 1,23 | 1,27 | 2,84 |
| 0,7 | 1,4 | 82 | 1,69 | 1,74 | 1,79 | 1,81 | 2,44 |
| 0,8 | 1,67 | 136,5 | 2,69 | 2,71 | 2,72 | 2,73 | 2,98 |
| 0,9 | 2,29 | 265 | 2,55 | 2,62 | 2,60 | 2,63 | 2,75 |

In the Table the values of η for various densities of plasma and for various values of $\beta_b = v_b/c$, and the appropriate values of beam energy flow $S_b$ are presented. Let us note the following: firstly, η increases with the growth of plasma density, and with $\gamma_b = 1{,}15$ for $n_{p4} = 1{,}3\cdot 10^{11} cm^{-3}$ η becomes more than two and a half times as much as the vacuum efficiency. Secondly, with growth of $\gamma_b$ for vacuum case and for all plasma cases except $n_{p4}$, η increases reaching the maximal value with $\gamma_b = 1{,}67$ and then decreases with $\gamma_b = 2{,}29$.

This work was partly supported by STCU Project No. 365.

## REFERENCES

1. Fainberg Ya.B., Gorbatenko M.F. ZhTF (in Russ.), 1959.-V.29, N 5.-P.549-562.
2. Berezin A.K., Buts V.A., Kovalchuk I.K. et al. Preprint KhFTI 91-52 (in Russ.).
3. Fainberg Ya.B., Bezjazychnyj I.A., Berezin A.K. et al. Plasma Physics and Controlled Nuclear Fusion Research.-IAEA, Vienna, 1969.-V.2.-P.723-732.
4. Ostrovsky A.O., Ognivenko V.V. R. & E. (in Russ.), 1979.-V.24, N 12.-P.2470-2477.
5. Carmel Y., Minami K., Kehs R.A. et al. Phys. Rev. Lett., 1989.-v.62.-p.2389-2392.
6. Fainberg Ya.B., Bliokh Yu.P., Kornilov Ye.A. et al. Dokl. AN UcrSSR. Ser. À. Fiz.-mat. & techn. nauki (in Russ.), 1990.-N11.-P.55-58.
7. Onishchenko I.N., Linetsky A.R., Matsiborko N.G. et al. Pisma v ZhETF (in Russ.), 1970.- V.12, N 8.-P.407-411.