ABOUT A POSSIBILITY OF CREATION OF A DETERMINISTIC UNIFIED MECHANICS

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The possibility of creation of a unified deterministic scheme of classical and quantum mechanics, allowing to preserve their achievements, is discussed. It is shown that the canonical system of ordinary differential equation of Hamilton classical mechanics can be added with the vector system of ordinary differential equation for the variables of quantization. The interpretational problems of quantum mechanics are considered.

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1. INTRODUCTION

It is possible without exaggeration to term the creation of quantum mechanics as the greatest reaching of physical thought of the 20-th century. Two formalisms of quantum mechanics - the Bohr - Sommerfeld formalism (BSF, that is "the old quantum mechanics") and the Heisenberg-Schrödinger formalism (HSF, that is the conventional quantum mechanics) are most known. Both of them with great success were applied to description of the motion of microparticles.

BSF uses the classical mechanics equations to describe dynamics of particles and additional rules to select real orbits of all the plurality of orbits supposed by classical mechanics.

This statement of quantum mechanics has allowed considerably to progress in understanding of atomic spectrums, since has given a common method for calculation of spectral terms of major number of atomic and molecular systems. The results obtained for atom of hydrogen are extended on a case of hydrogen-like systems and atoms of alkaline metals. The theory is used also to vibrational and rotational spectrums of molecules, X-ray spectrums of atoms, the normal Zeeman effect.

BSF gives exact selection rules and probabilities of possible quantum transitions, if to add the semiclassical theory of interaction of substance and radiation to this scheme. Almost in all these cases theory is in perfect consent with experiments. The separate discrepancies are observed for small quantum numbers. These discrepancies can be eliminated, if to add some empirical allowances (for example, the zero value exclusion of the azimuth quantum number, l=0) to quantization rules. However, theory meets major difficulties at description of multielectronic systems, scatterings etc. cases [1]. It is necessary to underline, that in BSF the motion of particles is described by deterministic fashion in the Newton space-time.

HSF [2] gives the perfect coincidence of theory and experiment for all those cases, when such coincidence is present in BSF. In addition it allows describe a scattering, multielectronic systems and other cases, when BSF does not work. However reference feature of HSF is that knowledge of the state vector (SV is a state vector) of the quantum mechanical system puts only statistical restrictions on results of measurements. Accordingly, the predictions of HSF with inevitability have the probability character.

Within the framework of the Copenhagen interpretation of HSF the SV is only "mathematical representation of knowledge" about system. Thus the laws of HSF superimpose certain restrictions on possible simultaneously observed probability distributions of different observations. Besides they give the differential equations, integrating which it is possible to receive change of these time distributions.

Thus, in frameworks of HSF the detailed description of the separate particle motion in the Newtonian space - time is not achieved. However such description of appearances is typical for the remainder base physical theories (classical mechanics and electrodynamics). Therefore in this connection with inevitability of probability interpretation of HSF many physicists consider that this theory is improved variant of statistical mechanics [3].

Within the framework of this improved statistical mechanics (i.e. HSF) at study of change in time of probability measures any more it is not supposed, that the motion of these measures are generated by the motion of points in the phase space. At such sight naturally there is a guess, that, probably, the statistical devices should arise in HSF approximately in the same way, as well as in a classical statistical mechanics. In last one the individual motions of particles is masked by the law of major numbers. Therefore, probably, the quantum mechanical SV is some average on better particular states [4].

These hypothetical quantum-mechanical dispersion-free states should be defined not only the quantum mechanical state vector of HSF, but also additional variables. If the states with given quantities these additional variable can be really prepared and measured, HSF becomes observant inadequate. At least individual results both theoretically and observationally will be completely reliable.

For these dispersion-free states all physical quantities bound with them also will be free from

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dispersion. However even in the event that these dispersion-free states will appear observationally not implemented, the searches of these additional variables represent considerable interest. It is necessary only that these variables introduction was justified enough by physical reasons.

The supporters of this point of view consider that the Einstein-Podolsky-Rosen (EPR) definition "of physical reality of observabling quantity" is approaching for description of the individual motion.

Before beginning of quantum mechanics the physics history supported the point of view of EPR. However such base theory as HSF is elimination because of its probability interpretation.

In spite of the fact that such searching is represented rather interesting, it conjugates to major difficulty: necessity of maintenance of huge number of remarkable achievements of HSF.

According to the usual tradition the models of quantum mechanics are constructed which or cannot contain additional parameters (since a von Neumann), or contain them explicitly (since de Broglie, Bohm). The model, offered in the given paper, concerns to the second direction and explicitly contains additional variable, at an means on which is gained HSF. The uniform deterministic scheme includes classical and quantum mechanics and maintains "transparence" of description of phenomena in the Newton space - time and the prediction force of HSF.

2. THE BASIC PRINCIPLES

In the offered formalism the particle dynamics is described within the framework of classical mechanics. As we shall compare the equations of classical mechanics to the Schrödinger equation, from all schemes of classical mechanics it is most convenient to take advantage of the Hamilton-Jacobi equation in partial derivates [5]. In this part we will analyze the states where H is independent of time. Therefore

$$S(\mathbf{r},\alpha,t) = W(\mathbf{r},\alpha,E) - E\cdot(t-t_0),$$

where S is the action, E is the energy, W is the characteristic Hamilton function. For the characteristic Hamilton function W we have an equation

$$H(\mathbf{p},\nabla W) = E, \tag{2.1}$$

In this case momentum

$$\mathbf{p} = \nabla W \tag{2.2}$$

For dynamic equations (2.1) and (2.2) we proposed the quantum conditions with the assistance of the equations set

$$(\hbar^2\nabla\nabla + \mathbf{pp})\cdot Z = 0, \tag{2.3}$$

where **pp** is the dyad, which done using (2.2) , $\hbar$ is the Planck constant .

The operators in the left part of (2.3) are symmetric affine orthogonal tensors, and the Z-function is a scalar. Therefore the tensor equation (2.3) has necessary transform properties, which are dictated by such properties of the Euclidean space, as homogeneity and isotropy.

The important property of (2.3) is its unique compatibility with the stationary Schrödinger equation. Really, in the one-dimensional case the equation (2.3) is an ordinary one-dimensional Schrödinger equation. In the three-dimensional case Sp (Tr) of the tensor equation (2.3) gives for one particle case

$$-\hbar^2\Delta Z = (p_x^2 + p_y^2 + p_z^2)\cdot Z = \mathbf{p}^2\cdot Z$$

where Δ is Laplacian.

In our case where E = $\mathbf{p}^2/2m+V$ we obtain the equatin that is the Schroedinger equation for Z:

$$(-\hbar^2\frac{\Delta}{2m} + V)\cdot Z = E\cdot Z\,. \tag{2.4}$$

Therefore choice of the base equation as (2.3), for which the Schrödinger equation is an affine invariant (Sp), allows save the Schrödinger eigenvalues of energy.

For the multiparticle system $\mathbf{p_i}$ is the momentum of i-particle and $\mathbf{r_i}$ is its radius-vector. Z($\mathbf{r_i}$,$\boldsymbol{\gamma_i}$) depends on $\mathbf{r_i}$ and $\boldsymbol{\gamma_i}$ of all particles, where $\boldsymbol{\gamma_i}$ – additional state parameters. If we take the sum by i (i=1..N) for Tr_i of equations (3) and take into consideration $T=\Sigma\mathbf{p_i}^2/2m_i$ then we receive the Schrödinger equation for N particles [2].

By virtue of equality to zero of right parts o the equations for Z (2.3) and of the Schrödinger stationary equation

$$\Psi(\mathbf{r}) = \int\rho(\gamma)Z(\mathbf{r},\gamma)d\gamma \tag{2.5}$$

is the solution of the Schrödinger equation where $\rho(\gamma)$ is an average function for additional variables. Therefore, in proposed formulation, the ψ-function describes an ensemble of the particles.

Therefore it is naturaly to require, that Z must sutisfy as conditions, as ψ in HSF.

3. DESCRIPTION OF MOVEMENT ON TRAECTORIES

The set of equations (2.1), (2.2) and (2.3) is one of partialy differential equations set. However, this set, in contrary to the Schröedinger equation., has appropriate ordinary differential equation set.

Eqs. (2.1), (2.2) in a well-known way are connected with the ordinary differential Hamilton equation set. For the one particle case

$$\frac{d\mathbf{p}}{dt} = -\frac{\partial H}{\partial\mathbf{r}};\ \ \frac{d\mathbf{r}}{dt} = \frac{\partial H}{\partial\mathbf{p}}\,. \tag{3.1}$$

To obtain the quantification equation set, in accordance with (2.3), let's enter quantizing vector–variable

$$\mathbf{P} = -i\hbar\nabla Z/Z, \tag{3.2}$$

In virtue of Eq. (2.3)

$$\frac{d\mathbf{P}}{d\mathbf{r}} = \frac{i}{\hbar}(\mathbf{pp} - \mathbf{PP}). \tag{3.3}$$

Really, taking a j-component of **P** (3.2), differentiating it with respect to x_i components of **r** and excluding Z, then obtain (3.3). In time of particle movement on trajectory, vector change by

$$\frac{d\mathrm{P}}{dt} = \frac{i}{\hbar}(\mathrm{pp} - \mathrm{PP})\cdot \mathrm{v}, \tag{3.4}$$

where **v**=d**r**/dt.

In case H=T+V, where T=$\mathbf{p}^2$/2m we have **p**=m**v**. In addition to **pp*v**=**p***(**p**,**v**) and **PP*v**=**P***(**P**,**v**). Therefore

$$\frac{d\mathbf{P}}{dt} = \frac{i}{\hbar \cdot m}[\mathbf{p}^2 \cdot \mathbf{p} - \mathbf{P} \cdot (\mathbf{P},\mathbf{p})].$$

Thus the particle movement in time is described by (3.1) and (3.4). The vector **P** has the same rights as dynamical variables, that is of **r** and **p**. The quantum conditions are laid on **P**. This is a selection of the real orbit.

Eq. (3.4) is addition to the equations of classical mechanics, which allows entering quantization. The proposed method of quantization is somewhat similar to a method of quantization of Bohr-Sommerfeld, but it is joint with HSF. The vector **P** allows to introduce additional connections for selection of real trajectories from all population of trajectories supposed by classical mechanics. It is naturally to call a quantum momentum, because is transformable as like as.

The extension of (3.1), (3.4) in multiparticle case is made in the way of adding indexes to **r**,**p**,**P** and m.

The physical solutions of Eq. (3.1), (3.4) are often the singular functions. For example, in the one-dimensional case with E<0 solutions have poles in the points where Z=0.

To receive the equations for the smooth functions (2.3) may be divided on two equations:

$$\begin{gathered} -\hbar^2 \frac{d\mathbf{R}}{d\mathbf{r}} = \mathbf{pp} \cdot Z, \quad \text{where } \mathbf{R} = \nabla Z \text{ or} \\ -m \cdot \hbar^2 \frac{d\mathbf{R}}{dt} = \mathbf{p}^2 \cdot \mathbf{p} \cdot Z, \; m\frac{dZ}{dt} = (\mathbf{R},\mathbf{p}). \end{gathered} \tag{3.5}$$

4. THE STATIONARY STATES

The set of Eqs. (3.1,3.4) is the ordinary differential equations set. Its solutions are the continuous set which is generated by initial values of variables. Physically, the solutions which correspond to the stationary states are of sense. In BSF these solutions are selected by the following principle [1]

$$\oint p \cdot dq = n \cdot \hbar.$$

In our case the stationary states may be defined by the next conditions:

a) variables **P, R,** Z have the same period, as that of classical variables **r,p**;

b) the analog of the Bohr-Sommefeld rule is

$$\oint (\mathbf{P},\mathbf{p}) \cdot dt = n \cdot \hbar \cdot m,$$

where *n* is the main quantum number;

c) E is real.

5. TO INTERPRETATION OF THE SCHRODINGER WAVE FUNCTION

The HSF is well supported by the experimental proofs and enters as a constituent part in the proposed formalism. Therefore the basic principles of interpretation of the proposed mathematical formalism should be close to principles of the Copenhagen interpretation of HSF.

The Copenhagen interpretation was observed in detail in the book of A. Sudbery[6], Chapter 5.

The interpretational problems of the Copenhagen HSF interpretation observed in detail in paper [7] p. 659. Its author considers, that more "ideal" interpretation should have the following plurality of properties:

(1) it should permit the operation of the microcosm to be isolated from the macrocosm and particularly from intrinsically complicated macroscopic concepts, e.g., knowledge, intelligent observers, consciousness, irreversibility, and measurement;

(2) it should account for the nonlocal correlations of the Bell inequality tests in a way consistent with relativity and causality;

(3) it should explain the collapse of the state vector without subjective "collapse triggers" (e.g., consciousness); and

(4) it should give additional knowledge about the state vector and provide insight into the problems of complexity, completeness, and predictability.

In the proposed formalism (Eqs. 3.1, 3.4) and (2.5):

(1) Microcosm motion and macrocosm motion are integrated. Motion is reversible in time and agrees with causality of microcosm. If the ensemble of microsystems is subjected to measuring by a macroscopic device, the processes are interpreted as well as in HSF. The key concepts of the Copenhagen interpretation for ensemble (2.5) are kept without changes. The offered formalism does not deal with such composite concepts as knowledge, intellectual observers and consciousness.

(2) In the proposed formalism non-local correlations in experiments on checkout of the Bell inequalities and the inconsistencies in interpretation of HSF with relativity and causality are explained that SSV is the performance of the ensemble (2.5), instead of the performance of microparticle [8]. The physical performances of motion of an individual microparticle are **r**, **p**, **P** (or **r, p, R**, Z). The equations of motion of microparticles are compatible to relativity (see [9]) and causality.

(3) Collapse SSV is an instantaneous change of the state vector with jump in time according to the laws of probability, when (and only, when) the measuring is made on a system [10]. If under a system to understand an ensemble (2.5), then as a result of measuring the ensemble of an initial state is sorted (breaks up) to

plurality of distinctive in experiment ensembles of terminating states. Thus, however, the course of individual process is not traced in the space - time.

There is no necessity in any "a trigger mechanism of a collapse" in the proposed formalism with deterministic description of motion.

(4) Additional knowledge about the state vector consists in possibility of representation SSV by the special mean (2.5) on better determined particular states Z (**r**,γ), where γ are additional parameters fixing dispersion-free states. The quantity ρ(γ) is description of population of these states. By nature the mean (2.5) is close to concept of the mean in classical statistical mechanics. At such guess the Heisenberg "knowledge of system" is rather "knowledge of ensemble ". Imposed on a vector **P** requirements answer for belonging the solution (3.1, 3.4) to the class of physically possible solutions. The predictivity of the proposed formalism corresponds to the predictivity of classical mechanics and HSF.

6. CONCLUSION

With fundamental point of view, this theory is interesting, because it is exactly overlapping for classical mechanics and quantum mechanics simultaneously. It should be emphasized that the proposed formalism is a development and extension of the conventional quantum-mechanical one; therefore it may be have the extent regions of the description of the experimental facts.

The proposed theory has also shown that a synthesis is possible between the ideas of Heisenberg and Bohr, who emphasized the intrinsic uncertainty and complementarity of quantum processes, and the ideas of Einstein, who emphasized the need to view the reality behind the formalism with an interpretation that is compatible with our understanding in other areas of physics.

From the applied point of view this theory may be useful for calculation of multiparticle systems, for estimation of semiclassical approximation precision, for discussion of measurement mechanism in Heisenberg-Schrödinger mechanics and so.

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REFERENCES

1. A. Sommerfeld. *Atombau und Spektrallinien.* I band. Braunschweig. 1951, 694 p. (in Russian).
2. A. Messia. *Quantum Mechanics.* V. 1. M.: “Nauka”, 1978, 480 p. (in Russian).
3. G.W. Mackey. *The Mathematical Foundations of Quantum Mechanics.* New York Amsterdam. 1963, 220 p. (in Russian).
4. J. Bell. On the Problem of Hidden Variables in Quantum Mechanics *// Rev. Mod. Phys.* 1966, v. 38, №3, p. 637-657.
5. H. Goldstein, *Classical Mechanics.* Addison-Wersley Press, Inc.Cambridge, 1950, 416p, (in Russian)
6. A. Sudbery. *Quantum mechanics and the particles of nature.* Cambridge University Press, 1986, 488 p. (in Russian).
7. J. Cramer. The transactional interpretation of quantum mechanics // *Rev. Mod. Phis.* 1986, v. 58, №3, p. 647-687.
8. G. Adenier. *A Refutation of Bell's Theorem*. quant-ph/0006014, 2001, 8 p.
9. G.K. Khomyakov *The noncanonical formulation of quantum mechanics (zero-spin relativistic particle) // Nauchnie Vedomosti BelSU,* 2000, №1(10), p. 130-134.
10. J. Von Neumann, *Mathematische Groundlagen der Quantenmechanik.* 1932, Berlin, Springer-Verlag (in Russian).

О ВОЗМОЖНОСТИ ПОСТРОЕНИЯ ОБЪЕДИНЕННОЙ МЕХАНИКИ

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Обсуждается возможность построения единой детерминистской механики, сохраняющей достижения классической и квантовой механик. Показано, что в нерелятивистском случае для достижения этой цели каноническая система обыкновенных дифференциальных уравнений Гамильтона может быть дополнена подходящей системой обыкновенных дифференциальных уравнений для переменных квантования.

ПРО МОЖЛИВІСТЬ ПОБУДОВИ ДЕТЕРМІНІСТСЬКОЇ ОБ’ЄДНАНОЇ МЕХАНІКИ

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Обговорюється можливість побудови єдиної детерміністської механіки, яка зберігає досягнення класичної і квантової механік. Показано, що в нерелятивістському випадку для досягнення цієї мети канонічна система звичайних диференціальних рівнянь Гамільтонової механіки може бути доповнена відповідною системою звичайних диференціальних рівнянь для змінних квантування.