INTENSITY OF CASCADES OF TRANSITIONS AT DECAY OF RADIONUCLEI

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We present a method of the calculation for intensity of cascades of transitions at decay of radioactive nuclei. The decay scheme of radioactive nucleus and branch coefficient of transitions are used.

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1. INTRODUCTION

Intensities of cascades of transitions at decay of radioactive nuclei are the nuclear-physical constants describing as process of transformation of one nucleus in another, and, basically, structure of a nucleus-product.

Intensity of cascades of transitions can be used, for example, for simulation spectra γ-γ-coincidences, necessary at detection radioactive nuclei with the help of a multidetector spectrometer, and also for testing of their contents in complex mixes on structure. Such nuclear-physical constants are of interest for studying of structure of states of atomic nucleus with the help so-called "a spectroscopic method".

As the cascade of transitions we shall understand group of consistently let out γ-quanta (α- and β-particles). We shall identify steps of the cascade as consistently let out γ-quanta.

It is obvious, that besides emission of γ-quanta at transition of a nucleus from an initial state in another can be let out internal-conversion electrons. However, the probability of their emission, as a rule, is much less than probability of emission of γ-quanta. Therefore terms both "intensity of γ-transition", and "intensity of transition" can to be used. Intensities (probability) of transitions are the characteristic of a nucleus, while intensities of γ-transitions (electrons) are the characteristic of external emission in relation to a nucleus. Thus, radioactive decay can be characterized as a set of intensities of cascades.

The method of calculation for intensity of cascades is based on balance of population of levels of a nucleus and the sum of transitions intensity at their decay. The nuclear-physical constants from last version of evaluated nuclear data [1] and also results of monographs [2] are used at the calculations of intensity of cascades.

The decay scheme of a radioactive nucleus is its basic physical characteristic. It includes the energy scheme of levels with their quantum characteristics and intensity of transitions between these states.

The basis of construction of the scheme of γ-transitions of a nucleus is Ryts's method, i. e. a method

of coincidence of the sums of values of energy of cascade γ-transitions between the states. Demerit of Ryts's method is the opportunity of casual coincidence of the sums of values of energy of γ-transitions. It leads to false levels and wrong order for the decay of γ-transitions in some cases. A method γ-γ-coincidence as well as a method of summation of amplitudes of coincident pulses from γ-quanta are used for reliable order of the γ-transitions taking place in the cascade.

Obviously, that the strict balance between energy of levels and the sum of energy γ-transitions and balance of intensity of transitions for each level is observed for correctly made scheme. It allows drawing up the table of intensity of transitions cascades accompanying decay of a radioactive nucleus based on the scheme of transitions.

As it is told above, as the cascade we shall understand the γ-quanta emitting consistently at transition of a nucleus from higher state in lower or basic "simultaneously", i. e. within the limits of resolution of the circuit of gate of a spectrometer. The table of intensity of cascades includes a consecutive set of γ-quanta' energy and their frequency of emanation per one decay of a parent nucleus.

Intensities of cascades are interested in, first of all, at the analysis of the contents radioactive nuclei in samples of complex composition by γ-γ-coincidence spectrometers.

2. DESIGNATIONS OF THE PHYSICAL QUANTITY USED AT THE ANALYSIS DECAY SHEME

We shall enter definitions of physical quantity and give their formulation before to pass to the equations to analyze decay scheme. Let q mean number of a level of a daughter nucleus. Numbers of the excited levels change from 1 up to Q; E_q is the energy of a level with number q; P_q is the probability of formation of a condition with energy E_q at decay of a parent nucleus; W_q is the probability of decay of a state with energy E_q per one decay of a parent nucleus; Z_q is the part of α-, β-, ε-decay of a parent nucleus falling a level q of a daughter nucleus; T_{qq}' is the probability of transition of a nucleus from a state q' in a state q. Population of level P_q make up from α-, β-, ε-transitions of parent nucleus

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Z_q and the sum of probabilities of transitions $T_{qq'}$ with more high-energy state q' on a level q

$$P_q = Z_q + \sum_{q=q'+1}^{Q} T_{qq'+1}. \qquad (1)$$

$T_{q'q}$ is the intensity of transition from a level q' on a q.

3. MODEL DECAY SHEME OF A NUCLEUS

We shall accept the following nucleus decay schemes according to the file ENSDF. Decay from one level (basic or metastable one) of a parent nucleus is considered in each scheme only. Thus decay can yield two daughter nucleus. The factor of decay in each of daughter nucleus is given so that the sum is equal to unit. The sum of transitions from one level of a parent nucleus on two daughter ones is equal to unit. β-decay of a nucleus does not take into account in comparison with γ-transition to lower level. Decay of metastable level is considered as disintegration of a separate nucleus at considerable probability of β-decay in comparison with γ-transition.

Emission of internal-conversion electrons is competing process in relation to γ-transitions. Process of internal conversion is taken into account by a factor of internal conversion a_e

$$a_e = {f_e}/{f_\gamma}. \qquad (2)$$

Here f_e and f_γ is the numbers of electrons and γ-quanta emitted at transition of a nucleus from one state in another one.

From balance of population q level and its decay

$$P_q = W_q \qquad (3)$$

it is possible to receive an expression containing intensity of transitions

$$Z_q + \sum_{q'=q+1}^{Q} T_{qq'} = \sum_{q'=q+1}^{q-1} T_{qq'} \qquad (4)$$

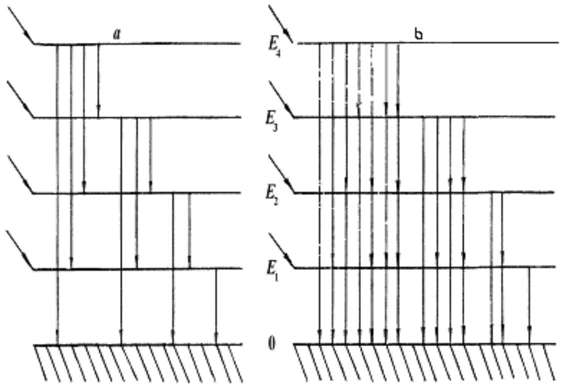
The probability of transition $T_{q'q}$ consists of intensity of γ-transition and transition with emission internal-conversion electron

$$T_{q'q} = I_{q'q}\left(1 + a_{q'q}\right). \qquad (5)$$

4. PROBABILITIES OF CASCADES OF TRANSITIONS CALCULATION

We shall designate probability of the cascade of transitions from a level q containing k steps multiplicity and having number n as X_{qkn}.

To get the required information – intensity γ-cascades - we shall transform the usual scheme containing total probabilities of transitions between levels (Fig. *a*) to the cascades scheme (Fig. *b*). The scheme from Fig. *a* shows that all levels of an daughter nucleus (q=1,2,…, Q) are occupied by transitions at decay of parent nucleus for completeness of the analysis. It also shown that every possible transitions between levels can be carried out. As a rule it is not realized and is only some abstraction allowing simplifying the decision of the problem.



The schemes of transitions of a nucleus:
a - in a usual state; b - as cascades

The analysis of the decay scheme (Fig. *a*) shows that the level with number q decays up by cascades of transitions with different multiplicity from k=1 (transition in the basic condition). The number of cascades of identical multiplicity rises with k achiving a maximum and decreasing up to unit (transitions with participation of all excited levels) q. The numbers of cascades given in Table 1 are binomial factors. The sum of cascades for a level q is determined by the formula

$$N_q = 2^{q-1}. \qquad (6)$$

The sum of all cascades of a nucleus for the scheme with Q of levels calculates as the sum of members of a geometrical progression

$$N(Q) = \sum_{q=1}^{Q} N_q = 2^{Q-1}. \qquad (7)$$

Let's note that number of transitions in the circuit submitted on Fig. 1*a* is equal

$$M(Q) = \sum_{q=1}^{Q} M_q \frac{(1+Q)Q}{2}. \qquad (8)$$

Table 1. *Number of cascades of identical frequency rate for level k (given in upper row), their sum N_q for level q (left column) and the common number of cascades for level Q (N_Q)*

k / *q*	1	2	3	4	5	6	$N_q = \sum_{k=1}^{q} N_{qk}$	$N_Q = \sum_{q=1}^{Q} N_q$
1							1	1
2	1	1					2	2
3	1	2	1				4	7
4	1	3	3	1			8	15
5	1	4	6	4	1		16	31
6	1	5	10	10	5	1	32	63

Table 1 shows the dependence of number of cascades of identical frequency rate on number of a level q.

Probability of transition between levels (see Fig. *a*) we shall designate as $T_{q'q''}$, i. e. T_m

$$T_{qq'} \equiv T_m . \quad (9)$$

The probability of decay of a level q the cascade of radiations of multiplicity k having number n X_{qkn} is equal to probability of occurrence of one of steps of this cascade (for example, transition between levels $q' \rightarrow q''$) . We shall redefine this probability as

$$X_{ljn} \equiv X_n . \quad (10)$$

It is possible to write down the following equality compared the decay scheme from Figs. 1*a* and 1*b*,:

$$X_{qkn} = T_m , \quad m = 1,2,...,M . \quad (11)$$

Values N and M can be calculated by formulas (7) and (8) (see Table 1). Expression (11) represents system of the linear algebraic equations with M unknowns.

Since Q=3 the number of cascades becomes big numbers of transitions and system (11) – not predetermined. The missing equations can be received using values of branching factor of the top level. So for Q=3 (M=6, N=7) the system looks like

$$\begin{cases} T_1 = X_1 + 0 + X_3 + 0 + X_5 + 0 + X_7 \\ T_2 = 0 + X_2 + 0 + 0 + 0 + X_6 + 0 \\ T_3 = 0 + 0 + X_3 + 0 + X_5 + 0 + X_7 \\ T_4 = 0 + 0 + 0 + 0 + X_5 + 0 + X_7 \\ T_5 = 0 + 0 + 0 + 0 + X_5 + 0 + 0 \\ T_6 = 0 + 0 + 0 + 0 + 0 + X_6 + X_7 . \end{cases} \quad (12)$$

We shall write down missing 7-th equation from a condition of proportionality of the relation of probability of 6-th cascade to 7-th one to the relation of probabilities (branching) of transitions from lower levels

$$\frac{X_6}{X_7} = \frac{T_2}{T_3} . \quad (13)$$

Whence

$$T_3 X_6 - T_2 X_7 = 0. \quad (14)$$

In case Q=4 (N=15; M=10) the missing 5 equations we shall receive by factors of branching of lower levels:

$$\begin{cases} \frac{X_6}{X_7} = \frac{T_2}{T_3}, \quad \frac{X_{10}}{X_{11}} = \frac{T_2}{T_3}, \quad \frac{X_{14}}{X_{15}} = \frac{T_2}{T_3}, \\ \frac{X_{12}}{X_{13}} = \frac{T_4}{T_5}, \quad \frac{X_{12}}{X_{14} + X_{15}} = \frac{T_4}{T_6} . \end{cases} \quad (15)$$

Feature of a technique of drawing up of the missing equations is usage of proportionality of known factors of branching (T_i/T_k) of lower levels to the relation of probabilities of cascades for more high levels.

Probabilities of cascades of transitions X_n (n=1,2,… ,N), which design procedure is given, are the decision of system of linear algebraic equations of N-th order.

As an example of calculation of probabilities of cascades are chosen radioactive nuclei, formed in reactions to thermal neutrons on nucleus of the most widespread in an earth's crust in present work. Corresponding probability calculation of cascades for transitions ^{42}K are presented in Table 2.

***Table 2.** Probabilities of cascades of transitions ^{42}K*

$^{42}_{19}\text{K} \xrightarrow{\beta^-} {}^{42}_{20}\text{Ca}, \quad T_{1/2} = 12{,}36$ h		
k	$E_{\beta^-} \rightarrow E_{\gamma 1} \rightarrow E_{\gamma 2}$, keV	p, r.u.
1	1564	0,821
2	510→2424 882,0→1525	0,000211 0,175
3	700,8→312,7→1525 510→899→1525 67→1922→1525	0,0032 0,000516 0,000412

Probabilities of cascades for transitions of some natural and the most widespread in technique radioactive nuclei are calculated also.

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ИНТЕНСИВНОСТИ КАСКАДОВ ПЕРЕХОДОВ ПРИ РАСПАДЕ РАДИОНУКЛИДОВ

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Представлен метод расчета интенсивностей каскадов переходов радионуклидов. Для этого используется схема распада радионуклидов, а также коэффициенты ветвления переходов.

ІНТЕНСИВНОСТІ КАСКАДІВ ПЕРЕХОДІВ ПРИ РОЗПАДІ РАДІОНУКЛІДІВ

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Представлено метод розрахунку інтенсивностей каскадів переходів радіонуклідів. Для цього використано схему розпаду радіонуклідів, а також коефіцієнти розгалуження переходів.