SURFACE ELECTROMAGNETIC WAVES IN LEFT-HANDED MATERIAL SLAB EMBEDDED IN PLASMALIKE MEDIA

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This study is devoted to the dispersion properties of the electromagnetic surface eigen waves that propagate along the left-handed planar slab that is bounded by the air (vacuum) and the medium with positive plasma-like permittivity. Both the left-handed slab and the media are assumed to be isotropic and have zero losses. We present the results of the studying the phase and group velocities of the considered waves.

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INTRODUCTION

At last decades both theoretical and experimental studies of new composed materials which are often called as metamaterials are carried out in the whole world. Such metamaterials possess many unique physical properties which are not found among the all usual substances [1].

The basic feature of these materials is the fact that the directions of the group and phase velocities of electromagnetic eigen waves in such materials are opposite directed, that's why such materials are called left-handed materials (LHM). These unique properties give the opportunity to design innovative devices with previously unavailable characteristics [2]. Possible technological applications include many areas, such as superlensing, optical cloaking, image processing and so on. In addition it should be mentioned the research carried out at the Argonne Wakefield Accelerator Facility; specifically, the investigation of the LHM application to control the dispersion relation in a loaded waveguide [3, 4].

Designing such new devices is impossible without detail theoretical investigation of the electromagnetic properties of the waveguide structures with metamaterials. At present planar waveguide structures with metamaterials are widely used in the investigations. In different leading scientific laboratories of the world it was considered various planar waveguide structures which contains LHM embedded either by vacuum or by different ordinary dielectrics with constant permittivity [5-7]. But using ordinary dielectrics which bound the metamaterial slab leads to the narrowing of possible wavenumbers range for the eigen surface electromagnetic waves of such structures as compared with vacuum bounds.

In the present work we study the planar waveguide structure that consists of the left-handed metameterial slab immersed in the non-magnetic plasma-like media which permittivity depends on the wave frequency. Plasma, metals, semi-metals and metamaterials with appropriate parameters [8] possess such electric and magnetic properties in some frequency ranges.

1. STATEMENT OF THE PROBLEM

The considered electromagnetic waves propagate along the planar waveguide structure that is made of isotropic LHM slab of thickness Δ. This material is characterized by effective permittivity $\varepsilon(\omega)$ and permeability $\mu(\omega)$ which depend on the wave frequency and are commonly expressed with the help of experimentally obtained expressions [2]:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \tag{1}$$

$$\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2}, \tag{2}$$

here ω_p is effective plasma frequency, ω_0 is the characteristic frequency of LHM. In further study we consider the LHM with $\omega_p / 2\pi = 10$ *GHz*, $\omega_0 / 2\pi = 4$ *GHz* and parameter $F = 0,56$ [5].

This LHM slab is bounded on one side by the artificial isotropic nonmagnetic ($\mu_2 = 1$) material with no losses and the effective permittivity

$$\varepsilon_2(\omega) = 1 - \frac{\omega_{p2}^2}{\omega^2}, \tag{3}$$

here ω_{p2} is effective plasma frequency for neighbourhood medium, we choose $\omega_{p2} / 2\pi = 3,2$ *GHz*. Such parameters ratio leads to the existence of the frequency range within which $\varepsilon(\omega) < 0$ and $0 < \varepsilon_2(\omega) < 1$ simultaneously. The semi-bounded conventional dielectric with constant dielectric permittivity $\varepsilon_1 = 1$ and permittivity $\mu_1 = 1$ is located on the other side of LHM slab.

Let's consider electromagnetic wave that propagates along this structure. It is assumed that wave disturbance tends to zero far away from LHM and the dependence of the wave components on time t and coordinate z is chosen in the following form:

$$E, H \propto E(x), H(x) \exp[i(k_3 z - \omega t)], \tag{4}$$

here x is the coordinate rectangular to the wave propagation direction and to the LHM slab.

In this case it is possible to split the system of Maxwell equations into two subsystems. One of them describes the waves of H-type and the other – wave of *E*-type.

The wave of *E*-type possesses the dispersion relation in the following form:

$$\varepsilon(\omega)\,\kappa\left[h_2\varepsilon_1 + h_1\varepsilon_2(\omega)\right] + [h_1h_2\varepsilon(\omega)^2 + \varepsilon_1\varepsilon_2(\omega)\kappa^2] * \tanh(\kappa\,\Delta) = 0 , \quad (5)$$

here $h_1 = \sqrt{k_3^2 - \varepsilon_1\mu_1 k^2}$, $h_2 = \sqrt{k_3^2 - \varepsilon_2(\omega)\mu_2 k^2}$, $\kappa = \sqrt{k_3^2 - \varepsilon(\omega)\mu(\omega)k^2}$, $k = \omega/c$, c is the speed of light in vacuum.

In the region of LHM slab the wave field components, normalized by the $H_y(0)$, can be written as:

$$H_y(x) = C_1 e^{\kappa x} + C_2 e^{-\kappa x},$$
$$E_x(x) = k_3\left(C_1 e^{\kappa x} + C_2 e^{-\kappa x}\right)/(k\,\varepsilon(\omega)), \quad (6)$$
$$E_z(x) = i\kappa\left(C_1 e^{\kappa x} - C_2 e^{-\kappa x}\right)/(k\,\varepsilon(\omega)),$$

here C_1 and C_2 are wave field constants.

In the air (vacuum) region the wave field components, normalized on the $H_y(0)$, possess the form:

$$H_y(x) = e^{h_1 x}$$
$$E_x(x) = k_3 e^{h_1 x}/(k\,\varepsilon_1) \quad (7)$$
$$E_z(x) = i h_1 e^{h_1 x}/(k\,\varepsilon_1)$$

In the plasma-like half-space the wave field components, normalized by the $H_y(0)$, can be written as:

$$H_y(x) = A e^{-h_2 x},$$
$$E_x(x) = A k_3 e^{-h_2 x}/\left(k\,\varepsilon_2(\omega)\right), \quad (8)$$
$$E_z(x) = -i\,A\,h_2 e^{-h_2 x}/\left(k\,\varepsilon_2(\omega)\right),$$

here A is wave field constant. These constants are of the following form:

$$A = -2\,h_1\,\varepsilon_2(\omega)\,\varepsilon(\omega) e^{(h_2+\kappa)\Delta}/\left(\varepsilon_1\Psi_E\right),$$
$$C_1 = h_1\,\varepsilon(\omega)\left[h_2\varepsilon(\omega) - \varepsilon_2(\omega)\kappa\right]/\left(\varepsilon_1\,\kappa\,\Psi_E\right), \quad (9)$$
$$C_2 = -h_1\,\varepsilon(\omega)\left[h_2\varepsilon(\omega) + \varepsilon_2(\omega)\kappa\right]e^{2\kappa\Delta}/\left(\varepsilon_1\,\kappa\,\Psi_E\right),$$
$$\Psi_E = (e^{2\kappa\Delta}+1)\,\varepsilon(\omega)\,h_2 + (e^{2\kappa\Delta}-1)\,\varepsilon_2(\omega)\,\kappa .$$

Similarly wave of H-type possesses the dispersion relation in the following form:

$$\mu(\omega)\,\kappa\left(h_2\mu_1 + h_1\mu_2\right) + [h_1h_2\mu(\omega)^2 + \mu_1\mu_2\kappa^2]\tanh(\kappa\,\Delta) = 0 . \quad (10)$$

In the region of LHM slab the wave field components, normalized by the $E_y(0)$, can be written as:

$$E_y(x) = D_1 e^{\kappa x} + D_2 e^{-\kappa x},$$
$$H_x(x) = -k_3\left(D_1 e^{\kappa x} + D_2 e^{-\kappa x}\right)/\left(k\,\mu(\omega)\right), \quad (11)$$
$$H_z(x) = -i\kappa\left(D_1 e^{\kappa x} - D_2 e^{-\kappa x}\right)/\left(k\,\mu(\omega)\right),$$

here D_1 and D_2 are wave field constants.

Wave field components, normalized on the $E_y(0)$, in the air (vacuum) region:

$$E_y(x) = e^{h_1 x},$$
$$H_x(x) = -k_3 e^{h_1 x}/(k\,\mu_1), \quad (12)$$
$$H_z(x) = -i h_1 e^{h_1 x}/(k\,\mu_1).$$

In the plasma-like half-space the wave field components, normalized by the $E_y(0)$, can be written as:

$$E_y(x) = B e^{-h_2 x},$$
$$H_x(x) = -B k_3 e^{-h_2 x}/(k\,\mu_2), \quad (13)$$
$$H_z(x) = i\,B\,h_2 e^{-h_2 x}/(k\,\mu_2),$$

here B is wave field constant. These constants are of the following form:

$$B = -2\,h_1\,\mu_2\,\mu(\omega) e^{(h_2+\kappa)\Delta}/\left(\mu_1\Psi_H\right),$$
$$D_1 = h_1\,\mu(\omega)\,(h_2\mu(\omega) - \mu_2\kappa)/\left(\mu_1\,\kappa\,\Psi_H\right), \quad (14)$$
$$D_2 = -h_1\,\mu(\omega)\,(h_2\mu(\omega) + \mu_2\kappa)e^{2\kappa\Delta}/\left(\mu_1\,\kappa\,\Psi_H\right),$$
$$\Psi_H = (e^{2\kappa\Delta}+1)\,\mu(\omega)\,h_2 + (e^{2\kappa\Delta}-1)\,\mu_2\,\kappa .$$

2. MAIN RESULTS

The results of numerical calculation of dispersion relations for E and H-waves for the selected set of parameters are shown at Fig. 1. There are four solutions of two dispersion equations (5) and (9).

Curves marked by the numbers 1, 2 correspond to the waves of E-type and curves marked by the numbers 3, 4 correspond to the waves of H-type. For the chosen set of parameters the condition that the central metamaterial demonstrates left-handed properties ($\varepsilon(\omega) < 0$ and $\mu(\omega) < 0$ simultaneously) is valid for the normalised frequency $1 < \Omega < 1.5$. The lines (a) and (b) represent the curves $\xi = \Omega\sqrt{\varepsilon_2}$ and $\xi = \Omega\sqrt{\varepsilon_1}$, and the line(c) – $\xi = \Omega\sqrt{\varepsilon(\omega)\mu(\omega)}$.

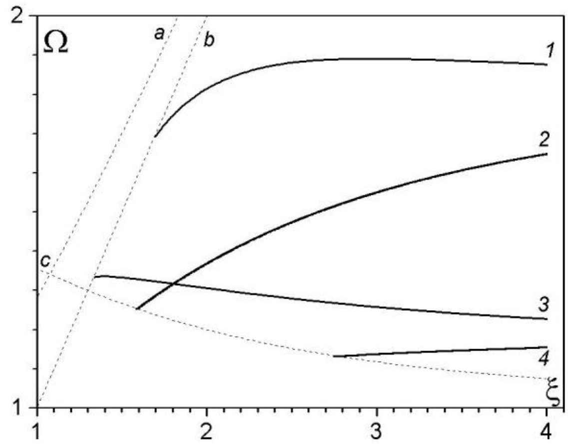

Fig. 1. The dependence of the normalised frequency $\Omega = \omega/\omega_0$ on dimensionless wave number $\xi = k_3 c/\omega_0$ for LHM slab thickness $\omega_0\Delta/c = 0{,}5$, $\varepsilon_1 = 1$, $\mu_1 = \mu_2 = 1$

Let's consider the phase $V_{ph} = \omega/k_3$ and group velocity $V_{gr} = d\omega/dk_3$ for E- and H-waves which correspond to lines 2, 3, 4 on Fig. 1.The dependence of the normalized phase velocity on normalized frequency for fixed different values of slab thickness Δ is shown on Figs. 2, 3.

For the one of H-waves (its dispersion is represented by the line 3 on Fig. 1), dependence of the phase velocity on the frequency varies substantially with decreasing of the slab thickness (see Fig. 2). Number 1 on Fig. 2 corresponds to the slab thickness $\omega_0\Delta/c$=1, number 2–$\omega_0\Delta/c$=1.5, number 3 – $\omega_0\Delta/c$=2.

For the other *H*-mode (its dispersion is given by the line 4 on see Fig. 1), dependence of the phase velocity on the frequency does not change with decreasing of the slab thickness (see Fig. 3).

It is shown that the surface waves are slow and strong frequency dependent in that frequency range where central slab material demonstrates the left-handedness.

Phase velocity of the considered waves at fixed frequency changes with changing the value of slab

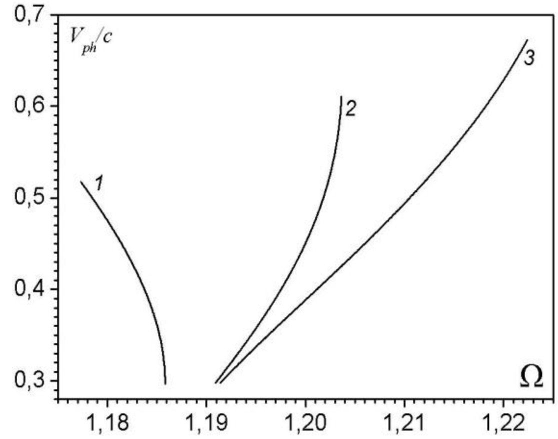

Fig. 2. The dependence of the phase velocity V_{ph}/c of the one H-wave which dispersion is shown by the line 3 on Fig. 1 on the frequency $\Omega = \omega/\omega_0$ for different left-handed material slab thickness $\omega_0\Delta/c = 1; 1.5; 2$

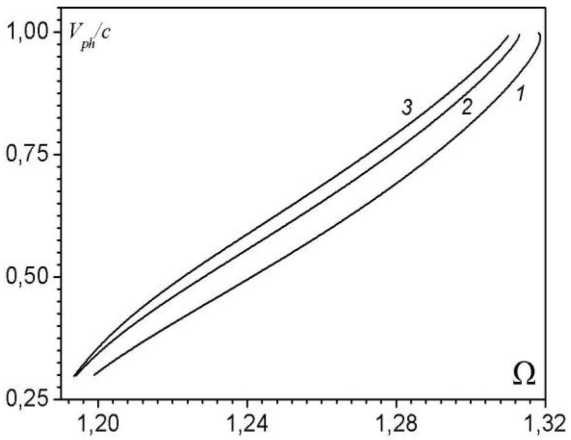

Fig. 3. The dependence of the phase velocity V_{ph}/c of the other H-wave which dispersion is presented by the line 4 on Fig. 1 on the frequency $\Omega = \omega/\omega_0$ for different left-handed material slab thickness $\omega_0\Delta/c = 1; 1.5; 2$

thickness Δ. This dependence is called as geometric dispersion – it is shown on Fig. 4.

Dependence of V_{ph} and V_{gr} on the slab thickness is most strong for those values of metamaterial thickness, which, as we know, are not succeeded to be produced up to now. Practical possibility of managing the waves velocities will be run in such structures when rather thin metamaterials will be produced.

On Fig. 5 the line 1 corresponds to *E* –wave which dispersion is represented by the line 2 on see Fig. 1. The lines 2 and 3 correspond to *H* –waves which dispersions are given by the lines 3 and 4 on see Fig. 1. As we can see the modes considered are both forward and backward. Especially it should be noted the presence of *H*-mode with zero group velocity.

The modes on see Fig. 5 show strong group velocity dependence on frequency, including linear dependence. These features can be very useful for practical applications.

CONCLUSIONS

The possibility of existence of surface electromagnetic eigen waves which propagate along the left-handed planar slab bounded by the vacuum and the medium with positive plasma-like permittivity is shown. It is demonstrated that the surface *E*-wave is a direct

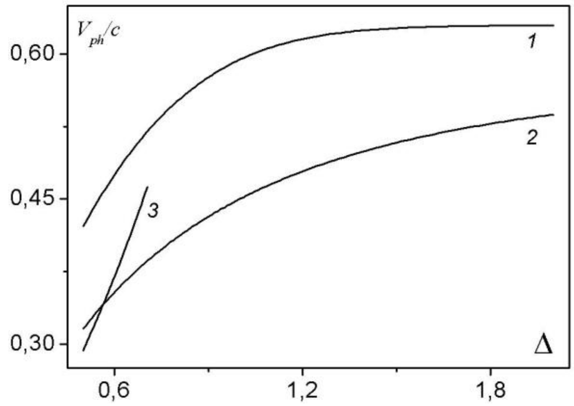

Fig. 4. The dependence of the phase velocity V_{ph}/c on left-handed material slab thickness. 1: E-wave (with dispersion shown by the line 2 on Fig. 1, $\Omega = 1,64$); 2: H-wave (with dispersion shown by the line 3 on Fig. 1, $\Omega = 1,23$); 3: H-wave (with dispersion shown by the line 4 on Fig. 1 , $\Omega = 1,154$)

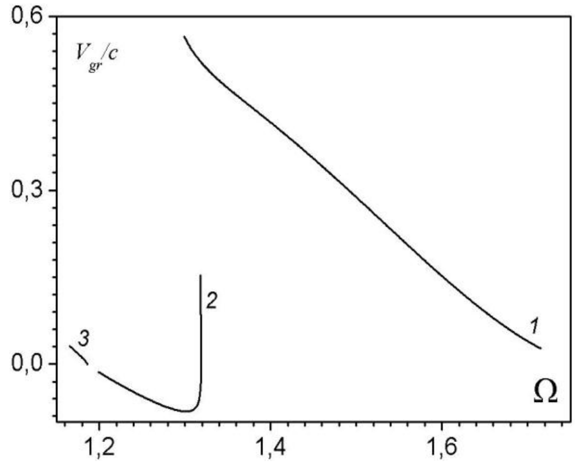

Fig. 5. The dependence of the group velocity V_{gr}/c on of the frequency $\Omega = \omega/\omega_0$ for left-handed material slab thickness $\omega_0\Delta/c = 1$. 1: E-wave (with dispersion shown by the line 2 on Fig. 1); 2: H-wave (with dispersion shown by the line 3 on Fig. 1); 3: H-wave (with dispersion shown by the line 4 on Fig. 1)

one (group and phase velocities coincide in the direction) and *H*-polarized wave is backward one (phase and group velocities are directed oppositely).

It is shown that the phase velocity of one of the *H*-waves is weakly dependent on the thickness of the metamaterial slab, that is important for technological applications. The absolute value of the group velocity of *H*-waves can be quite low, down to zero. The linear frequency dependence of group velocity of *E*-waves in a combination with a relatively broad range of its existence makes this mode very perspective one.

In particular it should be noted that the length of the surface waves is much less than the length of bulk electromagnetic waves, which significantly reduces the size of technological devices.

The results obtained in this work can be useful for the modelling and designing of modern nanodevices based on metamaterials.

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ПОВЕРХНОСТНЫЕ ЭЛЕКТРОМАГНИТНЫЕ ВОЛНЫ В СЛОЕ ЛЕВОСТОРОННЕГО МАТЕРИАЛА, ПОГРУЖЕННОГО В ПЛАЗМОПОДОБНУЮ СРЕДУ

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Изучены свойства собственных поверхностных электромагнитных волн, распространяющихся вдоль плоской волноводной структуры, состоящей из слоя левостороннего материала, ограниченного вакуумом, и плазмоподобной средой с диэлектрической проницаемостью, зависящей от частоты. Слой левостороннего материала и окружающие среды предполагаются изотропными и бездиссипативными. Представлены результаты изучения фазовой и групповой скоростей рассматриваемых волн.

ПОВЕРХНЕВІ ЕЛЕКТРОМАГНІТНІ ХВИЛІ В ШАРІ ЛІВОСТОРОННЬОГО МАТЕРІАЛУ, ЗАНУРЕНОГО В ПЛАЗМОПОДІБНЕ СЕРЕДОВИЩЕ

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Вивчено дисперсійні властивості власних поверхневих електромагнітних хвиль, які поширюються в пласкій хвилеводній структурі, яка складається з шару лівостороннього середовища, що межує з вакуумом та плазмоподібним середовищем з діелектричною проникливістю, яка залежить від частоти. Шар лівостороннього матеріалу та оточуючі середовища вважаються ізотропними та бездисипативними. Представлено результати дослідження фазової та групової швидкостей хвиль, що вивчаються.