# STUDY OF WAKEFIELDS IN LONGITUDINALLY AND TRANSVERSELY INHOMOGENEOUS RECTANGULAR DIELECTRIC RESONATORS

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At Institute of Plasma Electronics and New Methods of the Acceleration of NSC KIPT the experimental inspection of the principles of creation of multi-bunch and the multimode wakefield dielectric accelerator is carried out. For clearing up of the physical processes arising in case of excitation the wakefield and dynamics of electron bunches in such accelerator we executed a series of analytical and numerical calculations, and also full three-dimensional simulation by means of the developed 3D code is carried out. Wakefields were excited in case of injection of sequence of electron bunches in a rectangular vacuum wave guide of R32 type (72.14 ×34.04 mm) or R26 type (86.36 ×43.18 mm), filled with dielectric with the relative dielectric permittivity $\varepsilon = 3.8$, covering two opposite wide walls of a waveguide. The bunch repetition frequency was 2,805 GHz, energy of electrons was 4.5 MeV, average current was 0.73 A and a charge of one bunch was 0.26 nC. The electron bunches arriving through an open input end of a dielectric waveguide were deviated by a cross magnetic field of permanent magnets to a metal wall of waveguide free of dielectric. Depending on the location of magnets along a structure axis the length of interaction of electron bunches with a dielectric waveguide varied. Experimentally the linear dependence of the electric field value excited by bunches, near an output end of a semi-limited dielectric waveguide versus the interaction length is found. The 3D modeling executed by us researched possibility of the growing dependence of amplitude of a longitudinal electric field found in experiment at an output end of a waveguide from interaction length. The chronology of longitudinal electric field near output end on axis of system was probed and it spectral characteristic were analyzed too. The carried out researches allowed to better understanding the physical processes happening in the wakefield dielectric accelerator.

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## INTRODUCTION

Wakefields excited when relativistic electron bunches are injecting in dielectric waveguide or plasma filled waveguide (or their combination) may be used for electrons acceleration in perspective two-beam accelerator methods, or for obtaining short-wave radiation. That is why research of wakefield or in other words estimation of it amplitude and spectral composition has a great interest.

For a given type of waveguide, dielectric permittivity and electron beam injected in it a wakefield amplitude will be higher if the interaction length of electrons with dielectric waveguide will be greater. One way of varying of interaction length lies in changing the length of a dielectric waveguide. This method gives nonlinear dependence of wakefield amplitude on a dielectric length [1]. Another way that gives a rectangular dielectric waveguide for varying of interaction length consists in deviation of electrons on a side wall of waveguide free of dielectric at different distances from input edge by permanent magnets. Wakefield amplitude on interaction length in this method has rather linear dependence [1].

Wakefield dielectric structure can operate in monochromatic regime and multimode regime [2, 3]. Operation in multimode regime allows increasing amplitude of wakefields.

In this work we describe the results of 3D-PIC simulation of wakefields excitation and particles dynamics when train of relativistic electron bunches are injected in rectangular dielectric waveguide or resonator boxed in magnetic deflection field.

## STATEMENT OF THE PROBLEM

Rectangular dielectric waveguide represents the metal waveguide having the cross sizes $a \times b$ with two dielectric slabs (dielectric permittivity is equal to $\varepsilon$), covering opposite wide walls of a waveguide. Dielectric slabs are shifted from the input end of waveguide for a distance $z_{sh}$. Waveguide is boxed in magnetic deflection field formed by permanent magnets placed around wide walls (Fig. 1). Electron bunches passing through a slow-wave structure excite the wakefield which we examined.

Parameters of bunches in numerical calculations were chosen corresponding to the experimental installation "Almaz-2".

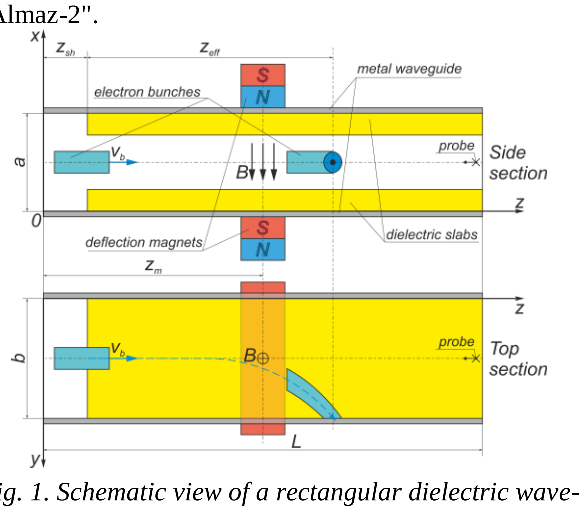


*Fig. 1. Schematic view of a rectangular dielectric waveguide. Yellow bricks show dielectric slabs, blue cylinder shows electron bunch*

For numerical simulation dielectric slab dimensions for given bunches and waveguide were calculated using theory of excitation of multizone dielectric waveguides [4, 5] and consideration expounded in [6]. In table parameters used in calculation are given.

*Parameters used in calculation*

| Waveguide | R32 | R26 |
|---|---|---|
| Dimensions ($a \times b$), mm | 34.04×72.14 | 45.00×90.00 |
| Operating frequency, GHz | 5.594 | 2.802 |
| Slab dimensions (are located along wide wall of a waveguide), mm | 5.7×72.14 | 15.3×90.00 |
| Full waveguide length $L$ | 49.18 cm | |
| Slab length $L_d$ | 42.54 cm ($3\lambda_d$) | |
| Shift of dielectric plates from an input end $z_{sh}$ | 6.64 cm ($\lambda_{vac}/2$) | |
| Relative dielectric constant ε | 3.8 (quartz) | |
| Bunch energy $E_0$ | 4.5 MeV | |
| Total bunch charge | 0.26nC | |
| Number of bunches | 40 | |
| Bunch axial RMS dimension 2σ (Gaussian charge distribution) | 17.0 mm | |
| Full bunch length used in PIC simulation | 34.0 mm | |
| Bunch diameter | 10.0 mm | |
| Bunch repetition period $T_r$ | 0.35654 ns ($f_r$=2.805 GHz) | |

## GENERALITIES

For numerical simulation we use dependence of magnetic induction on longitudinal coordinate obtained from experimental measurements (Fig. 2).

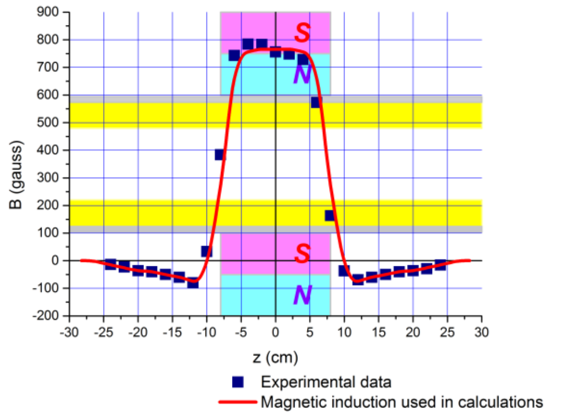


*Fig. 2. Dependence of magnetic induction on longitudinal coordinate*

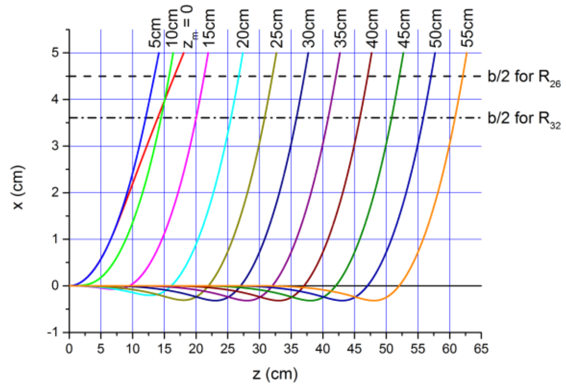


*Fig. 3. Assemblage of trajectories of 4.5 MeV electron injected along longitudinal coordinate z at the waveguide axis for different magnet position $z_m$ (given above). Dotted line show the R26 and R32 waveguide wall position*

For electrons with energy $E_0 = 4.5\,\text{MeV}$ injected along the longitudinal coordinate at the waveguide axis for different magnet position $z_m$ we obtain assemblage of trajectories shown in Fig 3.

Intersection of electron trajectories with R26 and R32 waveguide wall (that is equal to sum $z_{sh} + z_{eff}$) on a magnet position $z_m$ is given in Fig 4. For $z_m \geq 8\,\text{cm}$ curves in Fig. 4 has nearly linear dependence. So at that the interaction length of bunches with rectangular dielectric waveguide $z_{eff}$ will be proportionally to $z_m$ also.

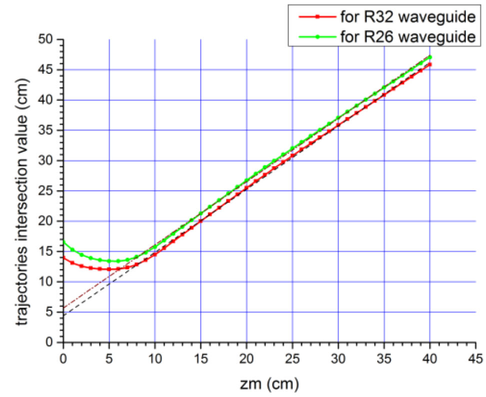


*Fig. 4. Dependence of intersection of electron trajectories with R26 and R32 waveguide wall $\left(z_{sh} + z_{eff}\right)$ on a magnet position $z_m$*

## RESULTS OF 3D-PIC CODE SIMULATION

During numerical simulation using our 3D-PIC code we obtain nearly linear dependence of wakefield amplitude $E_z$ on $z_{eff}$ both for dielectric resonator and waveguide based on R26 (Fig. 5). It is in good compliance with experiment [1]. The wakefield for the resonator approximately by 3 times exceeds a field for a waveguide for train of 40 bunches.

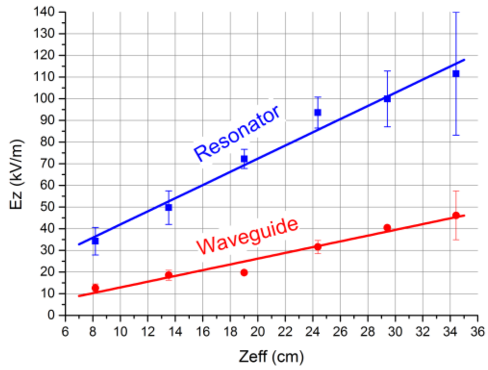


*Fig. 5. Dependence of wakefield amplitude $E_z$ on interaction length $z_{eff}$ of bunches with rectangular dielectric waveguide (red) and resonator (blue) based on R26*

For modal analysis of multimode structure we offer advanced procedure of Fourier transformation [7]. In Fig. 6 are presented the results of applying of improved procedure for determination of spectrum of the wakefield energized by the long bunch train in resonator based on R26 at $z_m = 9\,\text{cm}$. It is seen that spectrum of total wakefield consist of frequency of main mode, frequencies multiple them and also frequency of 2-nd LSE mode of oscillations that is close to 3-rd harmonic of main mode.

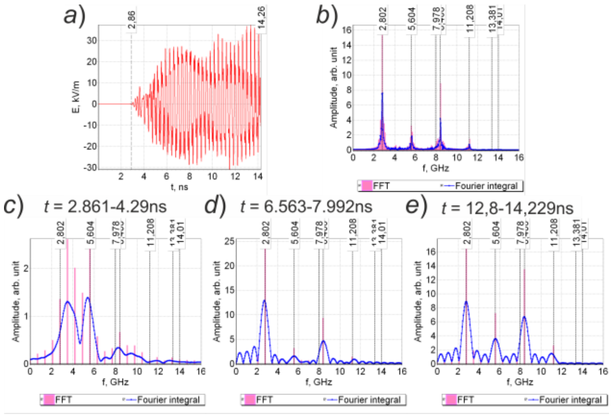


*Fig. 6. Spectra $E_z$(t,x=2.25 cm, y=4.5 cm, z=48.64 cm) of resonator based on R26 at $z_m$=9 cm for the train of 40 bunches with using of sampling interval: b) t=2.861…14.261ns; c) t=2.861…4.29ns; d) t=6.563…7.992ns; e) t=12.8…14.229ns* (*see Fig. 6,a*). *Blue lines show spectra computed by decomposition in Fourier's integral, and magenta bars show the spectra with using of standard FFT*

In Fig. 7 show the spectrum of wakefield excited in resonator based on R32 at $z_{sh}=0$, $L=L_d$ and $z_m=24\text{cm}$. It is seen that in this case spectrum of total wakefield consist of doubled repetition frequency i.e. $2f_r$ that is equal to frequency of main mode and frequencies multiple $f_r$: $3f_r$ and $4f_r$.

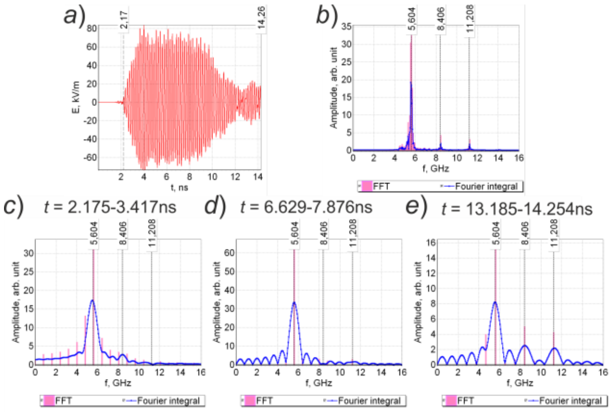


*Fig. 7. Spectra $E_z$(t,x=1.702 cm, y=36.07 cm, z=42 cm) of resonator based on R32 at $z_{sh}$=0 and $z_m$=24 cm for the train of 40 bunches with using of sampling interval: b) t=2.175…14.26 ns; c) t=2.175…3.417 ns; d) t=6.629…7.876 ns; e) t=13.185…14.254 ns* (*see Fig. 7,a*). *Blue lines show spectra computed by decomposition in Fourier's integral, and magenta bars show the spectra with using of standard FFT*

Spectral composition of dielectric resonator based on R32 is more poorly than of resonator based on R26. That is why resonator based on R32 may be considering as a single-mode resonator, while resonator based on R26 is a multi-mode one.

Fig 8,a shows the phase plane "energy-longitudinal coordinate" for resonator based on R26 at $z_m=24\text{cm}$ for last three bunches of the train. Fig 8,b demonstrate energies distribution function of electrons on a resonator output for last bunch of the train.

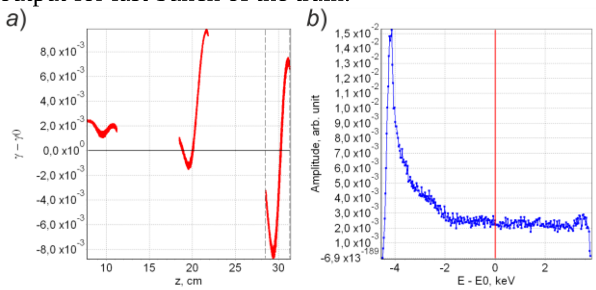


*Fig. 8. a) phase plane "energy-longitudinal coordinate", b) energies distribution function of electrons on a resonator output. $E_0$=4.5 MeV for* ***resonator*** *based on R26 at $z_m$=24 cm for last three bunches of the train* (*a*) *and last bunch* (*b*)

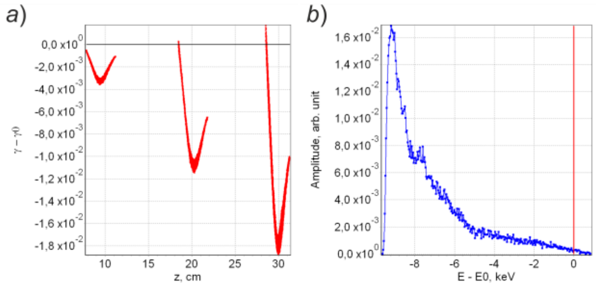


*Fig. 9. Same contents as in Fig. 8, but for the waveguide based on R26*

Fig. 9 present the same contents as see Fig. 8, but for the waveguide based on R26. Comparing see Figs. 8 and 9 one can see that in a waveguide case electrons better give up energy to wakefield than for resonator. In addition in a dielectric waveguide an accelerated electrons are practically absent whereas in resonator a number of accelerated and decelerated particles are comparable.

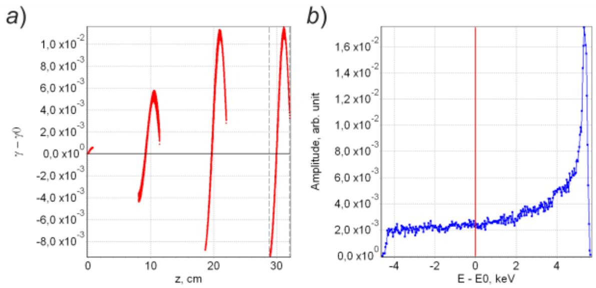


*Fig. 10. Same contents as in Fig. 8, but for the resonator based on R32 at $z_{sh}$=0 and $z_m$=24 cm*

Even greater accelerated electrons we obtain in case of resonator based on R32 at $z_{sh}=0$, $L=L_d$ and $z_m=24\text{cm}$ (Fig. 10). This fact and wakefield behaviour shown in Fig. 7,a may follow from frequency detuning between doubled repetition frequency $2f_r$ and frequency of main mode of dielectric resonator based on R32.

## CONCLUSIONS

The 3D modeling executed by us shows the growing dependence of amplitude of a longitudinal electric field found in experiment at an output end of a waveguide from the interaction length of electrons with dielectric waveguide.

Analysis of spectrums of a longitudinal electric field $E_z(t)$ showed existence of multimode structure of oscillations in R26 dielectric waveguide or resonator and single-mode structure in R32 resonator.

While studying of energies distribution function of electrons on a resonator output we find accelerated electrons along with decelerated electrons.

At the same time in R26 waveguide structures an accelerated electrons practically does not appear.

Existence of accelerated electrons in R32 dielectric resonator may be caused by frequency detuning between doubled repetition frequency $2f_r$ and frequency of main mode.

## ACKNOWLEDGEMENTS

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## ИССЛЕДОВАНИЕ КИЛЬВАТЕРНЫХ ПОЛЕЙ В ПРОДОЛЬНО И ПОПЕРЕЧНО НЕОДНОРОДНЫХ ПРЯМОУГОЛЬНЫХ ДИЭЛЕКТРИЧЕСКИХ РЕЗОНАТОРАХ

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В ХФТИ выполнена экспериментальная проверка принципов создания многосгусткового и многомодового кильватерного диэлектрического ускорителя. Кильватерные поля возбуждались при инжекции последовательности электронных сгустков в прямоугольном вакуумном волноводе типа R32 (72,14×34,04 мм) или типа R26 (86,36×43,18 мм), заполненном диэлектриком с относительной диэлектрической проницаемостью $\varepsilon = 3,8$, покрывающем две противоположные широкие стенки волновода. Частота повторения сгустков составляла 2,805 ГГц, энергия электронов была 4,5 МэВ, заряд одного сгустка составлял 0,26 нКл. Электронные сгустки, поступающие через открытый входной торец диэлектрического волновода, отклонялись поперечным магнитным полем постоянных магнитов к металлической стене волновода, свободной от диэлектрика. В зависимости от расположения магнитов вдоль оси структуры изменялась длина взаимодействия электронных сгустков с прямоугольным диэлектрическим волноводом. Выполненное нами 3D-моделирование показало линейный рост амплитуды продольного электрического поля на выходном торце волновода с длиной взаимодействия, что находится в хорошем соответствии с экспериментально полученной зависимостью. Исследовались хронологические зависимости продольного электрического поля вблизи выходного торца волновода на оси системы, получены и проанализированы спектральные характеристики колебаний поля. Проведенные исследования приводят к лучшему пониманию физических процессов, происходящих в кильватерном диэлектрическом ускорителе.

## ДОСЛІДЖЕННЯ КІЛЬВАТЕРНИХ ПОЛІВ У ПОЗДОВЖНЬО Й ПОПЕРЕЧНО НЕОДНОРІДНИХ ПРЯМОКУТНИХ ДІЕЛЕКТРИЧНИХ РЕЗОНАТОРАХ

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У ХФТІ виконана експериментальна перевірка принципів створення багатозгусткового й багатомодового кільватерного діелектричного прискорювача. Кільватерні поля збуджувалися при інжекції послідовності електронних згустків у прямокутному вакуумному хвилеводі типу R32 (72,14×34,04мм) або типу R26 (86,36×43,18 мм), заповненому діелектриком з відносною діелектричною проникністю $\varepsilon = 3.8$, що покриває дві протилежні широкі стінки хвилеводу. Частота повторення згустків становила 2,805 ГГц, енергія електронів була 4,5 МеВ, заряд одного згустка становив 0,26 нКл. Електронні згустки, що надходили через відкритий вхідний торець діелектричного хвилеводу, відхилялися поперечним магнітним полем постійних магнітів до металевої стіни хвилеводу, вільної від діелектрика. Залежно від розташування магнітів уздовж осі структури змінювалася довжина взаємодії електронних згустків із прямокутним діелектричним хвилеводом. Виконане нами 3D-моделювання показало лінійне зростання амплітуди поздовжнього електричного поля на вихідному торці хвилеводу з довжиною взаємодії, що перебуває в гарній відповідності до експериментально отриманої залежності. Досліджувалися хронологічні залежності поздовжнього електричного поля поблизу вихідного торця хвилеводу на осі системи, отримані й проаналізовані спектральні характеристики коливань поля. Проведені дослідження призводять до кращого розуміння фізичних процесів, що відбуваються в кільватерному діелектричному прискорювачі.