THE DISTRIBUTION FUNCTION OF PLASMA PARTICLES IN LONGITUDINAL MAGNETIC AND RADIAL ELECTRIC FIELDS

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The problem of determining the distribution function of particles in plasma that is produced in crossed longitudinal magnetic and radial electric fields is solved. It is assumed that the velocity probability distribution functions for the particles of the residual gas before its ionization or initial plasma before it has appeared in crossed fields are Maxwellian with non-zero thermal velocities. Then produced plasma particles are moving in crossed fields without collisions. The obtained distribution function is written in the coordinates of the guiding center. The expressions for the distribution function in the limiting cases of strong and weak electric field are also obtained.

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INTRODUCTION

In the problem of plasma stability in crossed longitudinal magnetic B and radial electric E_r fields, it is important to know the distribution function on the particle velocity and coordinates. For example, in [1-3], the ion cyclotron instabilities of plasma in crossed fields were studied. In [1], the ion cyclotron instability of rotating plasma was considered in the case of a weak radial electric field, when the energy of a particle motion in the electric field was much less than their thermal motion energy. In [2, 3], the ion cyclotron instability of the plasma was studied in the opposite case of strong radial electric field. In each of these limiting cases the particle distribution function was defined in different way based on the specific features of the problem. Accordingly, in each of these cases the excitation of the ion cyclotron instability is determined by different mechanisms. In some cases, an intermediate version when the energy of the particles thermal motion is comparable to the energy transferred to the particle from the electric field is also interesting.

In cylindrically symmetric plasma it is convenient to use independent variables which define the energy of the transverse particle motion $\varepsilon_\perp$ and its generalized angular momentum μ_φ instead of the coordinates and velocities. These new coordinates are the integrals of motion. Distribution function was chosen in [2, 3] as follows

$$F(\varepsilon_\perp, \mu_\phi, v_z) \sim Y(e\Phi(a) - \varepsilon_\perp)\delta(\varepsilon_\perp - \omega_e \mu_\phi)\delta(v_z). \quad (1)$$

In (1), Y is the Heaviside step function, δ is the Dirac delta function, $\Phi(a)$ is the potential on the external electrode (anode), $\omega_e = -cE_r / Br$. The radial profile of the electric field potential was assumed to be parabolic, $\Phi(r) = \Phi(a)(r^2/a^2)$, which ensured the independence of the angular rotation rate of the particle on the radius (rigid rotor model). One of the conditions of the limit of strong electric field is that the initial energy of ions produced in the plasma as a result of the ionization should be considerably less than the ion energy obtained in the radial electric field. Second condition is the inequality $\Omega_r > \omega_{ci}$, where Ω_r is the frequency of the radial ion oscillations (modified cyclotron frequency in crossed fields), ω_{ci} is the ion cyclotron frequency. In this paper, the unperturbed distribution function is determined for an arbitrary ratio of the initial ion energy and the energy of motion in crossed fields. Furthermore, the ratio of frequencies Ω_r and ω_{ci} is assumed to be arbitrary one, which corresponds to an intermediate case in which the energy of the particles thermal motion is comparable to the energy obtained by the particle from the electric field.

1. GENERAL EXPRESSION FOR THE DISTRIBUTION FUNCTION

We consider the same model of plasma as in the papers [2, 3]. Cylindrically symmetric collisionless plasma is placed in crossed longitudinal magnetic and radial electric fields. In the radial direction, plasma is limited by metal electrode (anode) with the radius r_a. The electrode has a positive potential relative to the axis, so that the electric field is directed inside the plasma. The potential in plasma is assumed to have a quadratic dependence on the radius $\Phi(r) = \Phi_0 (r/r_a)^2$ so that the rotational speed of the particles does not depend on the radius. This potential distribution arises in negatively charged plasma with uniform radial distribution of the electron density. The plasma is considered to be unlimited along the magnetic field. The equilibrium distribution function of plasma particles of a specie α (ion or electron) should depend on variables $\varepsilon_{\perp\alpha}, \mu_{\varphi\alpha}, v_z$, where

$$\varepsilon_{\perp\alpha} = \frac{m_\alpha v_\perp^2}{2} + e\Phi(r),\ \mu_{\varphi\alpha} = \frac{m_\alpha r^2 \omega_{c\alpha}}{2} + m_\alpha v_\varphi r. \quad (2)$$

To determine the particular form of the dependence on variables (2) it is necessary to know the conditions under which the plasma was produced. One of the various possible methods of plasma production in crossed E_r and B_z fields is a sharp application of an electric field to plasma produced in advance in a magnetic field. Yet another method is the ionization of neutral gas in the system in which the crossed fields already exist. Such a scenario of plasma production takes place, for example, in a Penning discharge. We assume both ways to be identical ones. Further by saying the phrase "the appearance of a charged particle in crossed fields" we shall imply either sharp application of a radial electric field in plasma or ionization of the neutral atoms.

Let assume that the particle of α specie appears in the crossed fields with the initial values of the radial coordinate r_0 and velocity components $v_{r0}, v_{\varphi 0}, v_{z0}$. The probability that, moving in crossed fields, the particle resides in the phase volume $d\varepsilon_{\perp\alpha} d\mu_{\varphi\alpha} dv_z$ is equal to:

$$dp = \delta(\varepsilon_{\perp\alpha} - \frac{m_\alpha v_{\perp 0}^2}{2} - e\Phi(r_0))\delta(\mu_{\varphi\alpha} - \frac{m_\alpha r_0^2 \omega_{c\alpha}}{2} - m_\alpha v_{\varphi 0} r_0)\delta(v_z - v_{z0}) d\varepsilon_{\perp\alpha} d\mu_{\varphi\alpha} dv_z, \quad (3)$$

where $v_{\perp 0}^2 = v_{r0}^2 + v_{\varphi 0}^2$. Such a form of the dependence of the probability density is a consequence of the conservation laws of energy and angular momentum of a particle. If before the particles appearance in crossed fields they have a certain distribution on the coordinates and velocities $f_\alpha(r_0, v_{r0}, v_{\varphi 0}, v_{z0})$, then the probability of the particle to belong to the phase volume $d\varepsilon_{\perp\alpha} d\mu_{\varphi\alpha} dv_z\, r_0 dr_0\, dv_{r0}\, dv_{\varphi 0}\, dv_{z0}$ is equal to:

$$dP = f(r_0, v_{\perp 0}, v_{z0}) dp\, r_0 dr_0\, dv_{r0}\, dv_{\varphi 0}\, dv_{z0}. \quad (4)$$

To obtain the particle distribution function, it is necessary to integrate the expression (4) over the variables $r_0, v_{r0}, v_{\varphi 0}, v_{z0}$. Let suppose that the distribution function $f_\alpha(r_0, v_{r0}, v_{\varphi 0}, v_{z0})$ is given by the following expression:

$$f_\alpha(r_0, v_{r0}, v_{\varphi 0}, v_{z0}) = \frac{n_0}{(2\pi)^{3/2} v_{T0}^3} \exp\left(-\frac{v_{\perp 0}^2}{2v_{T0}^2} - \frac{v_{z0}^2}{2v_{T0}^2}\right) \times Y(r_0 - r_a), \quad (5)$$

where n_0 and v_{T0} are the density and the thermal velocity of the particles before their appearance in the crossed fields. If the n_0 and v_{T0} do not depend on the radius, then the distribution function of plasma particles in crossed fields is as follows:

$$f_{0\alpha}(\varepsilon_{\perp\alpha}, \mu_{\varphi\alpha}, v_z) = \frac{n_0}{(2\pi)^{1/2} m_\alpha^2 v_{T0}^3} I_0\left(\frac{2\Omega_\alpha(\Omega_\alpha + \omega_{c\alpha})}{m_\alpha v_{T0}^2 (2\Omega_\alpha + \omega_{c\alpha})^2} \times \sqrt{\left(\varepsilon_\perp + (\Omega_\alpha + \omega_{c\alpha})\mu_\varphi\right)\left(\varepsilon_\perp - \Omega_\alpha \mu_\varphi\right)}\right)$$
$$\times \exp\left(-\frac{\Omega_\alpha^2 \left(\varepsilon_\perp + (\Omega_\alpha + \omega_{c\alpha})\mu_\varphi\right)}{m_\alpha v_{T0}^2 (2\Omega_\alpha + \omega_{c\alpha})^2} - \frac{(\Omega_\alpha + \omega_{c\alpha})^2 \left(\varepsilon_\perp - \Omega_\alpha \mu_\varphi\right)}{m_\alpha v_{T0}^2 (2\Omega_\alpha + \omega_{c\alpha})^2} - \frac{v_z^2}{2v_{T0}^2}\right)$$
$$\times Y\left(\frac{1}{(2\Omega_\alpha + \omega_{c\alpha})}\left(\sqrt{\frac{2}{m_\alpha}\left(\varepsilon_\perp + (\Omega_\alpha + \omega_{c\alpha})\mu_\varphi\right)} + \sqrt{\frac{2}{m_\alpha}\left(\varepsilon_\perp - \Omega_\alpha \mu_\varphi\right)}\right) - r_a\right), \quad (6)$$

where $I_0(x)$ is the Bessel modified function, Ω_α is the angular velocity of the particles drift motion in crossed fields:

$$\Omega_\alpha = \frac{\omega_{c\alpha}}{2}\left(-1 + \sqrt{1 + \frac{8e\Phi_0}{m_\alpha \omega_{c\alpha}^2 r_a^2}}\right). \quad (7)$$

For the quadratic dependence of the potential on the radius, Ω_α is constant. In the frame of reference rotating with the angular velocity Ω_α, a particle of specie α makes a circular motion like Larmor rotation in the magnetic field. The modified cyclotron frequency $\omega_{c\alpha *} = (2\Omega_\alpha + \omega_{c\alpha})$ plays the role of the cyclotron frequency in this case. The Larmor radius ρ_α as well as the radial coordinate of the center of the Larmor circle R_α (variables of the guiding center of the particle) are related to the variables $\varepsilon_{\perp\alpha}$ and $\mu_{\varphi\alpha}$ by the following relations [1]

$$\rho_\alpha = \frac{1}{(2\Omega_\alpha + \omega_{c\alpha})}\sqrt{\frac{2}{m_\alpha}\left(\varepsilon_\perp - \Omega_\alpha \mu_\varphi\right)}, \quad (8)$$

$$R_\alpha = \frac{1}{(2\Omega_\alpha + \omega_{c\alpha})}\sqrt{\frac{2}{m_\alpha}\left(\varepsilon_\perp + (\Omega_\alpha + \omega_{c\alpha})\mu_\varphi\right)}. \quad (9)$$

Note that ρ_α and R_α are also integrals of motion. Distribution function in the variables of the guiding center has the following form

$$f_{0\alpha}(\rho_\alpha, R_\alpha, v_z) = \frac{n_0}{(2\pi)^{1/2} v_{T0}^3} I_0\left(\frac{\Omega_\alpha(\Omega_\alpha + \omega_{c\alpha})}{v_{T0}^2} R_\alpha \rho_\alpha\right) \times \exp\left(-\frac{\Omega_\alpha^2 R_\alpha^2}{2v_{T0}^2} - \frac{(\Omega_\alpha + \omega_{c\alpha})^2 \rho_\alpha^2}{2v_{T0}^2} - \frac{v_z^2}{2v_{T0}^2}\right) \times Y(R_\alpha + \rho_\alpha - r_0). \quad (10)$$

Equation (10) presents the wanted distribution function of particles in crossed fields with an arbitrary relationship between the energy of their motion in the electric field and their energy of thermal motion.

2. EXPRESSIONS FOR THE DISTRIBUTION FUNCTION IN THE LIMITING CASES

Now we consider the limiting cases of the distribution function for different limiting ratios of the thermal velocity v_{T0} and the velocity of the drift motion of the guiding center $R_\alpha \Omega_\alpha$.

First, let assume that $v_{T0} \sim 0$, while the anode potential Φ_0 is high enough so that the following inequality is valid for the most of the plasma volume (except of the paraxial region)

$$R_\alpha \rho_\alpha \Omega_\alpha (\Omega_\alpha + \omega_{c\alpha}) > v_{T0}^2. \quad (11)$$

Then, we can use the asymptotic form for the large values of the Bessel functions argument, $I_0(x) \sim e^x / \sqrt{2\pi x}$. In this case the distribution function (10) has the form

$$f_{0\alpha}(\rho_\alpha, R_\alpha, v_z) = \frac{n_0}{2\pi v_{T0}^2 \sqrt{\Omega_\alpha(\Omega_\alpha + \omega_{c\alpha}) R_\alpha \rho_\alpha}} \times \exp\left(-\frac{\left(\Omega_\alpha R_\alpha - (\Omega_\alpha + \omega_{c\alpha})\rho_\alpha\right)^2}{2v_{T0}^2} - \frac{v_z^2}{2v_{T0}^2}\right) \times Y(R_\alpha + \rho_\alpha - r_0). \quad (12)$$

It follows from Eq. (12), that the main contribution to the equilibrium distribution function comes from particles, which satisfy the condition:

$$\Omega_\alpha R_\alpha \sim (\Omega_\alpha + \omega_{c\alpha})\rho_\alpha > v_{T0}. \quad (13)$$

Inequality (13) determines the approximation of a strong radial electric field at which the thermal velocity of the particles before their appearance in crossed fields is much less than the velocity of the particle drift in crossed fields, $\Omega_\alpha > \omega_{c\alpha}$. Note that the approximation of a strong radial electric field (13) differs from that approach, taken in [2, 3], where the criterion of a strong electric field is also the inequality $V_{T0} < \Omega_\alpha R_\alpha$ (besides the condition $v_{T0} = 0$). In contrast to the latter condition, the criterion (13) admits the same order of magnitude for Ω_α and $\omega_{c\alpha}$. In the limiting case $v_{T0} = 0$, the distribution function (12) degenerates into δ-function:

$$f_{0\alpha}(\rho_\alpha, R_\alpha) \sim \delta\left((\Omega_\alpha R_\alpha - (\Omega_\alpha + \omega_{c\alpha})\rho_\alpha)^2\right). \quad (14)$$

Similar ion distribution function in the form of δ-function was considered in [2, 3], where the problem of the excitation of the ion cyclotron instability of the plasma in crossed B and E_r fields has been solved.

Let assume now that the inequality opposite to (13) holds,

$$R_\alpha \rho_\alpha \Omega_\alpha (\Omega_\alpha + \omega_{c\alpha}) < v_{T0}^2. \quad (15)$$

This inequality corresponds to the case of a weak electric field. In this case, the distribution function (10) takes the form:

$$f_{0\alpha}(\rho_\alpha, R_\alpha, v_z) = \frac{n_0}{(2\pi)^{1/2} v_{T0}^3} \times \exp\left(-\frac{\Omega_\alpha^2 R_\alpha^2}{2v_{T0}^2} - \frac{(\Omega_\alpha + \omega_{c\alpha})^2 \rho_\alpha^2}{2v_{T0}^2} - \frac{v_z^2}{2v_{T0}^2}\right) \times Y(R_\alpha + \rho_\alpha - r_0). \quad (16)$$

The smallness of the velocity of the drift motion in comparison to the thermal velocity, $\Omega_\alpha R_\alpha < v_{T0}$, (the approximation of weak radial electric field) is sufficient condition for the inequality (15). In fact, the inequality $\Omega_\alpha R_\alpha < v_{T0}$ is equivalent to the condition $\Omega_\alpha < \omega_{c\alpha}$, that is also a criterion of a weak electric field. We also note that despite the uniform ionization along the radius, the distribution function (16) is Gaussian on radial coordinate of the guiding centre.

CONCLUSIONS

The expression for the equilibrium distribution function for plasma particles in crossed longitudinal magnetic and radial electric field is derived using the probabilistic approach. This expression takes into account non-zero initial velocity of the atoms before the ionization and includes the product of the modified Bessel function and exponential (11) whose arguments are the coordinates of the guiding centre.

Limiting expressions are obtained from the general expression for the distribution function for the cases of strong (12) and weak (16) radial electric field. These expressions are consistent with the previously obtained expressions.

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ФУНКЦИЯ РАСПРЕДЕЛЕНИЯ ЧАСТИЦ ПЛАЗМЫ В ПРОДОЛЬНОМ МАГНИТНОМ И РАДИАЛЬНОМ ЭЛЕКТРИЧЕСКОМ ПОЛЯХ

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Решается задача об определении функции распределения частиц в плазме, которая создается в скрещенных продольном магнитном и радиальном электрическом полях. Предполагается, что функция распределения по скоростям частиц остаточного газа до его ионизации, или начальной плазмы, прежде чем она появляется в скрещенных полях, является максвелловской с ненулевой тепловой скоростью. Образовавшиеся частицы плазмы затем движутся в скрещенных полях без столкновений. Полученная функция распределения записана в координатах ведущего центра. Получены также выражения для функции распределения в предельных случаях сильного и слабого электрических полей.

ФУНКЦІЯ РОЗПОДІЛУ ЧАСТИНОК ПЛАЗМИ В ПОЗДОВЖНЬОМУ МАГНІТНОМУ І РАДІАЛЬНОМУ ЕЛЕКТРИЧНОМУ ПОЛЯХ

Д.В. Чібісов

Розв'язано задачу про визначення функції розподілу частинок у плазмі, яка створюється в схрещених поздовжньому магнітному і радіальному електричному полях. Вважається, що функція розподілу за швидкостями частинок залишкового газу до його іонізації, або початкової плазми, перш ніж вона з'являлася в схрещених полях, є максвелівською з ненульовою тепловою швидкістю. Частинки плазми, що утворилися, рухаються далі в схрещених полях без зіткнень. Здобуту функцію розподілу записано в координатах ведучого центра. Здобуто також вирази для функції розподілу в граничних випадках сильного та слабкого електричних полів.