MODE HYBRIDIZATION IN A GYROTRON CAVITY WITH BACKGROUND PLASMA

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It is shown that a formation of hybrid modes (mode hybridization) in a gyrotron cavity occurs as their frequencies approach coincident points induced by background plasma. Such process, which is accompanied by a drastic change in fields and Q-values of the cavity modes, is studied with respect to some gyrotron parameters. Estimations are presented for several high-power continuous-wave (cw) gyrotrons.

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INTRODUCTION

High-power cw gyrotrons are among the main sources of microwave radiation for fusion applications [1]. Neutralization of the beam space charge is one of the characteristic processes in their operation. It indicates the presence of low-density plasma inside the cavities of high-power cw gyrotrons.

When plasma appears in a gyrotron cavity, the cavity modes become hybrid [2]. With increasing plasma density, hybrid modes of different types reveal common features. Those of their fields, which are negligibly small in the case of an empty cavity, begin to grow and can affect unpredictably the beam dynamics (and thus gyrotron efficiency). Furthermore, the similarity of the fields of hybrid modes can give rise to competition between them. Such plasma-induced effects were not studied previously [2, 3] and their impact on gyrotron operation still remains unknown.

The paper describes the mechanism of hybrid mode formation caused by plasma in a gyrotron cavity. Here the plasma is assumed to be uniform. The rough estimates of the effect produced by a non-uniform plasma have recently been presented in [4].

1. EIGENFREQUENCY PROBLEM FOR THE PLASMA-FILLED GYROTRON CAVITY

Consider electromagnetic waves $\left(\mathbf{E}(\mathbf{r},t),\mathbf{B}(\mathbf{r},t)\right) = \left(\mathbf{E}(\mathbf{r}),\mathbf{B}(\mathbf{r})\right)\exp(-i\omega t)$ in a gyrotron cavity completely filled with an uniform plasma. The cavity represents a section of a circular metallic waveguide with slowly varying radius ($\left|dR(z)/dz\right| << 1$) and is placed in an external magnetic field B_0 directed along the cavity axis z. The cold and collisionless plasma with immobile ions is considered. A justification of such plasma model has been presented in [2].

The eigenfrequency problem for the plasma-filled gyrotron cavity can be solved by the method of separation of variables ($\mathbf{r} = (r,\varphi,z)$)

$$\left(B_z,\partial E_z/\partial z\right) = \left(h_z(r,z),\hat{e}_z(r,z)\right) f(z)\exp(il\varphi) .$$

Then Maxwell's equations yield

$$\left(\Delta_\perp - k_z^2(z) + \frac{\varepsilon_1^2-\varepsilon_2^2}{\varepsilon_1}k^2\right)h_z = -\frac{\varepsilon_2\varepsilon_3 k}{\varepsilon_1}\hat{e}_z , \qquad (1)$$

$$\left(\Delta_\perp - \frac{\varepsilon_3}{\varepsilon_1}k_z^2(z) + \varepsilon_3 k^2\right)\hat{e}_z = -k_z^2(z)\frac{\varepsilon_2 k}{\varepsilon_1}h_z , \qquad (2)$$

$$\hat{e}_z\Big|_{r=R(z)} = 0 , \; \left[\frac{\chi^2(z)}{\varepsilon_2 k^2}\frac{dh_z}{dr} + \frac{1}{k}\frac{d\hat{e}_z}{dr} - \frac{lh_z}{r}\right]_{r=R(z)} = 0 , \qquad (3)$$

where $k=\omega/c$, $\varepsilon_1 = 1-\omega_p^2/(\omega^2-\omega_H^2)$, $\varepsilon_2 = -\omega_p^2\omega_H/\left(\omega(\omega^2-\omega_H^2)\right)$, $\varepsilon_3 = 1-\omega_p^2/\omega^2$, ω_p and ω_H are the plasma and the cyclotron frequencies, respectively, $\chi^2(z) = k_z^2(z) - \varepsilon_1 k^2$ and the second condition in (3) yields zero value of E_φ on the cavity wall.

Here we introduce

$$k_z^2(z) = -f''(z)/f(z) , \qquad (4)$$

and take into account conditions ($a = (h_z,\hat{e}_z,k_z^2)$)

$$\left|\frac{1}{a}\frac{\partial a}{\partial z}\right| << \left|\frac{f'}{f}\right| , \left|\frac{1}{a}\frac{\partial a}{\partial z}\frac{f'}{f}\right| << \left|\frac{f''}{f}\right| , \left|\frac{1}{a}\frac{\partial^2 a}{\partial z^2}\right| << \left|\frac{f''}{f}\right| ,$$

which are valid for $\left|dR(z)/dz\right| << 1$.

The coupled equations (1) and (2) show that the waves in a plasma-filled gyrotron cavity are always hybrid ($h_z \neq 0$ and $\hat{e}_z \neq 0$). For a fixed value of z these equations, along with the boundary conditions (3) on the cavity wall, transform to the boundary-value problem for a plasma-filled waveguide with constant radius $R_0 = R(z) = const$ [5]. Therefore, fields in such a smooth waveguide have the same transverse structure as those in a fixed cavity cross-section. Moreover, the relation between $k_z^2(z)$ and ω can be found from the known [5-7] dispersion equation of a magnetized plasma-filled waveguide $D(\omega,k_z) = 0$, where k_z is the axial wavenumber. Hence the function $k_z^2(z)$ introduced in equation (4) for the wave amplitude $f(z)$ becomes known at a given frequency ω. The aim then is to find a suitable complex ω that fulfills the wave boundary (radiation) conditions at both ends of the gyrotron cavity

$$f'(z) \pm ik_z(z)f(z)\Big|_{z=0,L} = 0 , \qquad (5)$$

where L is the cavity length.

Note that the eigenfrequency problem (4) and (5) for a plasma-filled gyrotron cavity has the same form as for the vacuum case [1]. The only change is a different type of the dispersion equation $D(\omega,k_z) = 0$. Under the condition $k_z^2(z)/k^2 << 1$ ($0 \le z \le L$), it is convenient to write this equation as $D(\omega,k_z) = D_1(\omega,k_z)D_2(\omega,k_z) -$

$-\beta(\omega,k_z)=0$, where $|\beta(\omega,k_z)|<<1$ (see notation in [6, 7]). The above condition is generally valid for all gyrotrons operating near cutoff [1].

An analysis of the dispersion equation clarifies the mechanism of hybrid mode formation in a smooth plasma-filled waveguide (and gyrotron cavity). At $k_z=0$, when function $\beta(\omega,k_z)$ reduces to zero, this equation splits into independent equations $D_1(\omega,0)=0$ and $D_2(\omega,0)=0$ for cutoff frequencies of TM waves ($e_z/h_z=\infty$, $e_z=\hat{e}_z/ik_z$) and TE waves ($e_z/h_z=0$), respectively [5]. With condition $k_z^2/k^2<<1$, the waveguide modes become quasi-TM ($|e_z/h_z|>>1$) and quasi-TE ($|e_z/h_z|<<1$) modes. Their frequencies $\omega\;k_z$ are nearly always close to the solutions of equations $D_1(\omega,k_z)=0$ and $D_2(\omega,k_z)=0$. Exceptions are the vicinities of simultaneous solutions to these equations, where the modes become maximally hybrid ($|e_z/h_z|\sim 1$) and their dispersion curves undergo reconnections [6, 7]. The simultaneous solution of equations $D_1(\omega,k_z)=0$ and $D_2(\omega,k_z)=0$ induced by plasma will be called the coincident frequency [8] of TE and TM waves. Hence hybridization of modes occurs as their frequencies approach the coincident frequency. A similar mechanism of mode hybridization has previously been observed in [8-10].

2. MODE HYBRIDIZATION IN A GYROTRON CAVITY WITH A BACKGROUND PLASMA

As an example, consider mode hybridization in the cavity of the 1-MW 140-GHz cw gyrotron of Karlsruhe Institute of Technology (KIT) [11]. The operating cavity mode is TE$_{28,8}$, $R(z=L/2)=2.048$ cm, $L=4.6$ cm, $B_0=5.56$ T.

Fig. 1,a demonstrates the plasma influence on the frequencies of the TE$_{28,8}$ and TM$_{28,7}$ modes (hereinafter the prefix "quasi-" is omitted). As can be seen, plasma brings these frequencies closer to each other. Such process is accompanied by the mode hybridization (Fig. 1,b): the TE mode reveals the features of the TM mode (its value of $|e_z/h_z|$ increases). The reverse process is observed for the TM mode (its value of $|e_z/h_z|$ decreases). Both modes become maximally hybrid ($|e_z/h_z|\sim 1$) at a plasma density of $\omega_p=\omega_{p0}\approx 0.11\omega_H$. Here their frequencies undergo reconnection (Fig. 1,a).

Such plasma density corresponds to a coincident frequency of TE and TM modes. Its value can be approximately found from the equality between the cutoff frequencies of TE and TM modes in a plasma-filled waveguide with central radius of the gyrotron cavity (see Fig. 1,a). For low-density plasma ($\omega_p^2/|\omega_H^2-\omega^2|<<1$), this equality results in

$$\omega_{p0}^2 = \;\omega_{2c}^2-\omega_{1c}^2\;\left(1-\frac{\omega_{1c}^2}{\omega_H^2}\right)\approx\frac{4\pi f_0 c\;\;\chi_2-\chi_1}{R_0}\;1-\gamma_0^{-2}\;\;,(6)$$

where $\omega_{1c}=\chi_1 c/R_0$ and $\omega_{2c}=\chi_2 c/R_0$ are the cutoff frequencies of TM and TE modes, respectively, in an empty circular waveguide, $f_0\approx\omega_{2c}/2\pi\approx\omega_H\gamma_0^{-1}/2\pi$ is the operating frequency, and $\gamma_0=\;1-\beta_z^2-\beta_\perp^2\;^{-1/2}$ is the initial beam relativistic factor. For the 1-MW 140-GHz cw gyrotron, (6) yields $\omega_{p0}/\omega_H=0.105$.

Hybridization of the modes is also accompanied by a change in their diffractive Q-values. The Q-values of TE and TM modes approach each other with increasing plasma density (Fig. 1,c). The resonance change in the Q-values [9] occurs when the modes become maximally hybrid. According to [8, 9], this can be explained by the strong TE-TM polarization mixing ($|e_z/h_z|:\;1$) in the cavity. Notice that plasma shows just a small effect on the wave amplitude $f(z)$. Therefore, the change in $f(z)$ only slightly affects the Q-value.

Hybrid modes with similar frequencies, fields, and Q-values can strongly compete with each other. To avoid such mode competition, a study of mode hybridization with respect to gyrotron parameters is required.

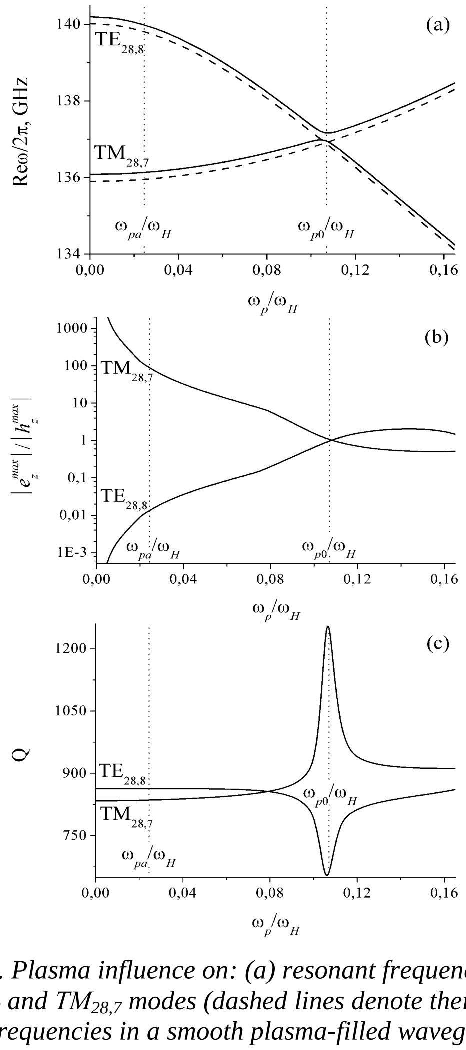

Fig. 1. Plasma influence on: (a) resonant frequencies of TE$_{28,8}$ and TM$_{28,7}$ modes (dashed lines denote their cutoff frequencies in a smooth plasma-filled waveguide with radius R_0=2.048 cm); (b) the ratio of the first radial maxima of $|e_z\;\;r,L\;|$ and $|h_z\;\;r,L\;|$ for TE$_{28,8}$ and TM$_{28,7}$ modes; (c) their Q-values ($Q=\mathrm{Re}\,\omega/2\,\mathrm{Im}\,\omega$)

Equation (6) is useful in such a study. According to (6), the plasma density ω_{p0} is lower the higher is the cavity radius, and the lower are operating frequency and beam energy E_b ($\gamma_0 = 1 + E_b / m_e c^2$). Besides, it depends on the difference $\chi_2 - \chi_1$ between the transverse eigenvalues of the operating TE and the preceding TM modes. However, for all high-power gyrotrons, which operate in very high-order modes ($\chi_2 >> 1$), this difference is almost constant and close to $\pi/2$.

The risk of mode hybridization is maximal, if the value of ω_{p0} coincides with the actual Langmuir frequency ω_{pa} for a low-density plasma in the gyrotron cavity. The density of such plasma approximately equals the density of the beam electrons n_b ($\omega_{pa}^2 \approx 4\pi e^2 n_b / m_e$) [2] and can be found from the relation $\omega_{pa}^2 \approx I_b \omega_H c / I_A r_b \beta_z \beta_\perp$ [1], where I_b and r_b are the beam current and radius, respectively, and I_A is the Alfven current. For the 1-MW 140-GHz cw gyrotron ($I_b = 41$ A, $E_b = 81$ keV, $r_b = 1.01$ cm, $\beta_\perp / \beta_z = 1.3$), an estimation yields $\omega_{p0} / \omega_{pa} = 4.37$. Hence the actual density of the background plasma initiates weak mode hybridization in the cavity of this gyrotron (see also Fig. 1,b). This effect may become stronger for more powerful gyrotrons. In particular, for the 2-MW 170 GHz gyrotron at KIT [11], we have $\omega_{p0} / \omega_{pa} = 2.67$.

CONCLUSIONS

The paper describes the mechanism of mode hybridization caused by plasma in a gyrotron cavity. It is shown that plasma of certain density initiates coincidence between the frequencies of a TE-TM mode pair. Hybridization of the modes occurs as their frequencies approach the coincidence point and becomes maximal in its vicinity. Maximally hybrid modes represent the strong TE-TM polarization mixture. Their Q-values differ significantly from those for modes in an empty cavity.

Mode hybridization has also been studied with respect to some gyrotron parameters. It is shown that this effect is stronger for the gyrotron cavity with the denser frequency spectrum, which is excited by the electron beam with the higher current and the lower energy.

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ГИБРИДИЗАЦИЯ МОД В РЕЗОНАТОРЕ ГИРОТРОНА С ФОНОВОЙ ПЛАЗМОЙ

В.И. Щербинин, Г.И. Загинайлов

Показано, что формирование гибридных мод (гибридизация мод) в резонаторе гиротрона происходит по мере приближения их частот к точкам совпадения, которые формируются фоновой плазмой. Такой процесс, который сопровождается существенным изменением полей и добротностей мод резонатора, исследован в зависимости от параметров гиротрона. Оценки приведены для ряда мощных непрерывных гиротронов.

ГІБРИДИЗАЦІЯ МОД У РЕЗОНАТОРІ ГІРОТРОНА ІЗ ФОНОВОЮ ПЛАЗМОЮ

В.І. Щербінін, Г.І. Загинайлов

Показано, що формування гібридних мод (гібридизація мод) у резонаторі гіротрона відбувається разом із наближенням їхніх частот до точок збігу, які формуються фоновою плазмою. Такий процес, який супроводжується значною зміною полів та добротностей мод резонатора, було досліджено в залежності від параметрів гіротрона. Оцінки наведено для ряду потужних безперервних гіротронів.