SYNCHROTRON RADIATION OF RELATIVISTIC ELECTRONS MOVING AT SMALL PITCH-ANGLES IN INHOMOGENEOUS MAGNETIC FIELD

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Synchrotron radiation of relativistic electrons moving in an inhomogeneous magnetic field at small pitch-angles is considered. The trajectory of an ultrarelativistic electron is obtained taking into account a curvature and radial inhomogeneity of magnetic field lines. The general formulae describing the radiation of an electron moving at pitch-angles from 0 to $\pi/2$ are derived. The range of instantaneous characteristic frequencies appears instead a single characteristic frequency. The applicability of the formulae for runaway electrons in the tokamak is evaluated. The condition for using of the obtained formulae is derived.

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1. INTRODUCTION

The synchrotron radiation is widely used in various fields of science and engineering. However, the formulae that take into account curvature of magnetic force lines are only obtained recently.

The general formulae for synchrotron radiation of ultrarelativistic charged particles moving along a spiral trajectory in curved magnetic force lines were firstly obtained in [1]. The radiation mechanism was called as synchrocurvature to underline that the curvature of magnetic force lines was taken into account. In [2], the formulae were generalized to take into account drift in an inhomogeneous magnetic field.

The radiation of relativistic electrons moving along a curved spiral trajectory at small pitch-angles was considered in [3,4]. The synchrotron radiation spectrum of runaway electrons in tokamak was obtained in [3]. The spectral angular distribution, spectrum and polarization characteristics have been derived in [4]. It is shown in [6] that the same radiation spectrum has been considered in [3] and in [4]. In limiting cases the formulae [1-5] turn to the classical synchrotron and curvature radiation.

The particle trajectory has been taken in drift approximation in [1,2]. The equations of motion for charged particles in circular magnetic force lines have been solved in [4,5] to derive the particle trajectory more exactly. It turned out that there was the regime of motion when the pitch-angle and the curvature radius of the trajectory is varying while the particle rotates around magnetic line. Consequently, the instantaneous characteristic frequency in given direction is changed, and we have the range of characteristic frequencies instead of a single characteristic frequency.

At the same time, the model of circular magnetic force lines has been used in [4,5]. Such a field does not satisfy the Maxwell equations in vacuum. So necessary to consider the general radiation mechanism for an relativistic electron moving in an inhomogeneous magnetic field, with the corrections caused by the magnetic field gradient and the pitch-angle variability taken into account. The curved magnetic field with non-constant magnitude is considered in present paper. For this case the electron motion can be analysed by using the more precise trajectory than the drift one. The general trajectory will allow generalizing the former radiation formulae and finding out the limits of their applicability. In order to obtain a general expression for the radiation spectrum Schwinger's formula is used. Using this formula the radiation spectrum of the electron in the inhomogeneous magnetic field can be obtained, once the curvature radius (or the acceleration) of the particle trajectory is known.

The present paper is organized as follows. The motion of relativistic electrons in curved inhomogeneous magnetic fields is considered in Section 2, using methods of the theory of nonlinear systems. The trajectory position vector is expanded as a power series in small parameter $\varepsilon = r_B/R << 1$, where r_B is the Larmor radius, R is the magnetic force line. The radiation mechanism is described in Section 3. The validity criterion of the obtained formulae is discussed in Section 4. The solution of motion equations is outlined in Appendix.

2. TRAJECTORY OF A CHARGED PARTICLE

To find the trajectory of a charged particle the guiding center approach is used in [1,2]. The pitch-angle which defines as the angle between the particle velocity vector and the magnetic field vector is constant in such model. It is necessary to have more exact trajectory that the phenomenon of changing pitch-angles can take into account.

Let us assume that a magnetic field has lines of force being curved with radius R, and the magnitude of magnetic field is depends on the radial coordinate. Suppose that Cartesian (x,y)-coordinates are in the osculating plane of the magnetic field lines, and z-axis coinciding with the axis of cylindrical magnetic surface. If we introduce cylindrical coordinates (r,φ,z), the magnetic field vector can be expressed as

$$\mathbf{B} = B_0\left(\frac{R}{r}\right)\left(-\sin\varphi\ \mathbf{i} + \cos\varphi\ \mathbf{j}\right), \qquad (1)$$

where φ is the polar angle in (x,y)-plane, $\mathbf{i}$, $\mathbf{j}$ are the basis vectors of Cartesian frame. Here, in contrast to [1] and [4,5], the radial dependence of the magnetic field is included. (Also, the direction of the magnetic field differs from adopted in [4,5] by replacement $\mathbf{B} \to -\mathbf{B}$.)

The particle of charge e, mass m, velocity v and Lorentz-factor $\gamma = (1 - v^2/c^2)^{-1/2} >> 1$ moves along curved spiral trajectory in the magnetic field (1). The angular velocity Ω corresponding this motion is defined as $\Omega \equiv v_{\|}/R$, where $v_{\|}$ is the velocity of the guiding center along the magnetic line with curvature radius *R*. The radius of Larmor's circle r_{B} is much less than R, $\varepsilon = r_{\mathrm{B}}/R << 1$.

The equations of motion of the charged particle in the magnetic field (1) are solved in Appendix. The asymptotic expansion of the radial component of the position vector $\mathbf{r}(r,\varphi,z)$ of trajectory, in which the terms proportional to $(r_{\mathrm{B}}/R)^5 << 1$ are dropped, has the form

$$r = R\left\{1 + \varepsilon\cos\psi + \varepsilon^2\left[\frac{3}{4} + \delta^2 - \left(\frac{1}{4} + \frac{1}{3}\delta^2\right)\cos 2\psi\right] + \right.$$
$$\left. + \varepsilon^3\left(\frac{5}{48} + \frac{7}{24}\delta^2 + \frac{1}{12}\delta^4\right)\cos 3\psi + \varepsilon^4 z_4(\psi)\right\}, \quad (2)$$

where $\psi = \bar{\omega} t$, $\delta = \Omega/\omega_0$, $\omega_0^2 = \omega_{\mathrm{B}}^2 + 2\Omega^2$,

$$\omega_{\mathrm{B}} = \frac{eB_0}{mc\gamma},$$

$$\bar{\omega}^2 = \omega_0^2\left[1 - \frac{1}{2}\varepsilon^2\left(1 + 2\delta^2 + \frac{20}{3}\delta^4\right) + \varepsilon^4\alpha_4\right].$$

The expressions of $z_4(\psi)$ and α_4 are given in Appendix. These terms make possible to calculate with enough accuracy the absolute values of velocity and acceleration, φ and z components can be found after expression (2) is substituted into Eqs (A2). It's worse to mention that in (2) the assumption about small δ is not made yet.

The square of velocity is

$$v^2 = \Omega^2 R^2 + \frac{\Omega^4 R^2}{\omega_{\mathrm{B}}^2} + \omega_0^2 r_B^2 - \varepsilon^4\omega_0^2 R^2\left(\frac{1}{8} + \frac{1}{12}\delta^2 + \frac{37}{18}\delta^4\right). \quad (3)$$

The magnitude of the acceleration is

$$a^2 = \Omega^4 R^2 + \omega_0^4 r_B^2 + 2\Omega^2 R\omega_0^2 r_B\cos\psi - \varepsilon 6\Omega^4 R^2\cos\psi -$$
$$- 4\varepsilon\Omega^2 R\omega_0^2 r_B\left(1 - \frac{3}{2}\delta^2 + \frac{3}{2}\delta^4\right) -$$
$$- 4\varepsilon\Omega^2 R\omega_0^2 r_B\left(1 - \frac{7}{3}\delta^2 - \frac{1}{2}\delta^4\right)\cos 2\psi -$$
$$- \varepsilon^3\omega_0^4 R^2\left(\frac{7}{4} - \frac{27}{4}\delta^2 + \frac{209}{12}\delta^4 - \frac{85}{3}\delta^6\right)\cos\psi -$$
$$- \varepsilon^3\omega_0^4 R^2\left(\frac{1}{4} - \frac{155}{24}\delta^2 + \frac{263}{24}\delta^4 + \frac{29}{4}\delta^6 + \frac{1}{2}\delta^8\right)\cos 3\psi. \quad (4)$$

The corrections of the first order (on ε) in comparison with the first and second terms have been remained in Equation (4).

It should be emphasized that Equation (4) differs from the corresponding expression obtained in [2]. They coincide only at small and large pitch-angles.

2.1. THE ANGLE BETWEEN PARTICLE'S VELOCITY AND MAGNETIC FIELD

It was shown in [5] that for an ultrarelativistic electron moving in a circular magnetic field there is the regime of motion, with varying pitch-angles. Let's show that this property is also saved for the motion within the magnetic field lines (1). The instantaneous characteristic frequency in every given direction appears as consequence of changing curvature of the particle trajectory.

The components of the particle velocity have the form

$$\dot{r} = -\omega_0 r_B\sin\psi + \varepsilon\omega_0 r_B\left(\frac{1}{2} + \frac{2}{3}\delta^2\right)\sin 2\psi, \quad (5)$$
$$r\dot{\varphi} = \Omega R\left[1 - \varepsilon\cos\psi + \varepsilon^2\left(-\frac{1}{4} - \delta^2 + \left(\frac{3}{4} + \frac{1}{3}\delta^2\right)\cos 2\psi\right)\right]$$
$$\dot{z} = \underline{\frac{\Omega^2 R}{\omega_{\mathrm{B}}}} + \omega_{\mathrm{B}} r_B\cos\psi + \varepsilon\omega_{\mathrm{B}} r_B\left[\underline{\frac{1}{2} + \delta^2} - \left(\frac{1}{2} + \frac{1}{3}\delta^2\right)\cos 2\psi\right]$$

It is visible from expression (5) that the transversal velocity of guiding center is consisted of the centrifugal drift velocity and the drift velocity caused by the magnetic field gradient (underlined terms in (5)).

The velocity vector, which is transverse to the magnetic field line, can be written as $\mathbf{v}_\perp = \dot{r}\mathbf{e}_r + \dot{z}\mathbf{e}_z$. Its square is

$$v_\perp^2 = \frac{\Omega^4 R^2}{\omega_{\mathrm{B}}^2} + 2\Omega^2 R r_B\cos\psi + \omega_{\mathrm{B}}^2 r_B^2 + \Omega^2 r_B^2\left(2 + 2\delta^2 - \left(2 + \frac{2}{3}\delta^2\right)\cos 2\psi\right) \quad (6)$$

The angle between a velocity vector and magnetic field, called also as a pitch-angle α, is defined as $\mathrm{tg}\alpha = |\mathbf{v}_\perp|/|v_\||$.

There are two limiting regimes of motion: i) the value of transverse velocity is close to the speed of light $\omega_{\mathrm{B}} r_{\mathrm{B}} \to c$, thus $\Omega R/\omega_{\mathrm{B}} r_{\mathrm{B}} = \delta/\varepsilon << 1$, and ii) the transverse velocity is no relativistic, and the longitudinal velocity goes to the speed of light $\Omega R \to c$, so $\Omega R/\omega_{\mathrm{B}} r_{\mathrm{B}} = \delta/\varepsilon >> 1$. In case i), the ratio of Larmor velocity to the velocity of centrifugal drift is $q \equiv \frac{\omega_{\mathrm{B}} r_{\mathrm{B}}}{\Omega^2 R/\omega_{\mathrm{B}}} = \frac{\varepsilon}{\delta^2} >> 1$, $v_\perp \approx \omega_{\mathrm{B}} r_{\mathrm{B}}$ and the pitch-angle α is constant. In case ii), all terms are essential in upper line of Equation (6) ; this line can be written as

$$v_\perp^2 = \frac{\Omega^4 R^2}{\omega_{\mathrm{B}}^2}\left(1 + 2q\cos\psi + q^2\right). \quad (7)$$

As is seen from (7), the pitch-angle changes periodically at $\delta^2 \sim \varepsilon$ (or $q \sim 1$). The amplitude of pitch-angle oscillations is $\mathrm{tg}\alpha \sim \delta$.

From here follows, that at $\delta^2 \sim \varepsilon < \delta$ the magnitude of pitch-angle is not saved. The curvature radius of the trajectory is also changes from point to point that results in appearance of a range of characteristic frequencies.

At the same time, as it follows from (5), the angle α_{D} between the velocity vector and the direction of drift trajectory saves constant value and is equal to

$\mathrm{tg}\alpha_{\mathrm{D}} = \omega_{\mathrm{B}} r_{\mathrm{B}}/v_{\|} = \varepsilon/\delta$. Thus, the angle under which the particle trajectory is wound onto the drift trajectory has constant value. Possible, the angle α_{D} should be used instead of a pitch-angle.

To calculate the acceleration components, Equations (5) should be substituted into the right hand sides of Equations (A1).

3. RADIATION SPECTRUM

The total energy radiated per unit time $P = \mathrm{d}\mathcal{E}/\mathrm{d}t$ can be calculated from general expression [8]

$$P = \frac{\mathrm{d}\mathcal{E}}{\mathrm{d}t} = \frac{2e^2}{3c^3}\frac{a^2 - (\mathbf{a}\times\mathbf{v})^2/c^2}{(1 - v^2/c^2)^2} = \frac{2e^2}{3c^3}\gamma^4 a^2, \qquad (8)$$

where it have been taken into account that the acceleration and velocity vectors are perpendicular.

The power averaged over time is of interest. Substituting (4) in (8) and averaging over time, we obtain

$$P = \frac{2e^2}{3c^3}\gamma^4\left(\Omega^4 R^2 + \omega_{\mathrm{B}}^4\left(1 - 6\delta^4\right)r_B^2\right) \qquad (9)$$

It can be seen that the total energy losses are consist of two contributions such as the losses owing to acceleration of the particle moving along a circular force line, and the losses due to the acceleration at Larmor rotation around a magnetic force line. For small longitudinal velocities $\Omega R << c$ and even in the case of relativistic longitudinal velocities for $\delta^2 < \varepsilon < \delta$ (this inequality is equivalent $q >> 1$), Equation (9) has the same form as the classical formula of power losses in a homogeneous magnetic field [8]

$$P = \frac{2e^4 B_0^2}{3m^2c^5}\gamma^2 v_\perp^2. \qquad (10)$$

For $\varepsilon << \delta^2$ (or $q << 1$) and $v_\| \to c$, we obtain from (8) the formula of the curvature radiation power

$$P = \frac{2e^2}{3c}\gamma^4\frac{c^2}{R^2}. \qquad (11)$$

Let's calculate the spectral distribution of the radiation emitted by an electron in inhomogeneous magnetic field. The total radiated power per unit frequency emitted by a relativistic electron is given by expression [9]

$$\frac{dP}{d\omega} = -\frac{e^2\omega}{\pi}\int_{-\infty}^{\infty} dt d\tau\left[1 - \frac{\mathbf{v}(t+\tau)\mathbf{v}(t)}{c^2}\right]\cos\omega\tau \times \times\frac{\sin\omega\,|\mathbf{r}(t+\tau) - \mathbf{r}(t)|/c}{|\mathbf{r}(t+\tau) - \mathbf{r}(t)|}, \qquad (12)$$

where $\mathbf{r}(t)$ and $\mathbf{v}(t)$ are the position and velocity vectors of the electron at time *t*, given by Equations (2) and (5).

Using the Frenet formulas of the natural trihedral, the next expressions can be obtained [6]

$$|\mathbf{r}(t+\tau) - \mathbf{r}(t)| = \tau\left(1 - \frac{\tau^2}{24}k^2v^2\right), \qquad (13)$$

$$1 - \frac{\mathbf{v}(t+\tau)\mathbf{v}(t)}{c^2} \cong 1/\gamma^2 + \frac{\tau^2}{2}k^2v^2, \qquad (14)$$

$$kv^2 = |\mathbf{a}|, \qquad (15)$$

where k is the curvature of the trajectory (2), and the velocity is given by Equation (5). To obtain the equations (13), (14) we have taken into account that the acceleration is perpendicular to velocity, $\mathbf{va} = 0$. Here, contrary to the trajectories used in [1,2], the curvature of trajectory (2) depends on time $k = k(t)$.

Using (13), (14), the radiation spectrum can be represented in the form

$$\frac{dP(t)}{d\omega} = \frac{e^2\omega}{\pi\gamma^2 v}\left\{\int_0^\infty\frac{d\tau}{\tau}\left(1 + \frac{\tau^2}{2}\gamma^2k^2v^2\right)\times \times\sin\frac{\omega\tau}{2\gamma^2}\left(1 + \frac{\tau^2}{12}\gamma^2k^2v^2\right) - \int_0^\infty\frac{d\tau}{\tau}\left(1 + \frac{\tau^2}{2}\gamma^2k^2v^2\right)\sin 2\omega\tau\right\}, \qquad (16)$$

In comparison with [2, 6], equation (16) contains the refined expressions of the trajectory curvature and particle velocity.

Introducing a new variable $x = \tau\gamma k v/2$, using the formula from [9]

$$\int_0^\infty(1 + 2x^2)\sin\frac{3}{2}y(x + \frac{x^3}{3})\frac{dx}{x} - \frac{\pi}{2} = \frac{1}{\sqrt{3}}\int_y^\infty dx K_{5/3}(x),$$

where $K_{5/3}(x)$ denotes the MacDonald function of order 5/3, Equation (16) can be expressed as

$$\frac{dP(t)}{d\omega} = \frac{\sqrt{3}e^2}{2\pi}\gamma k\, y\int_y^\infty K_{5/3}(x)dx, \qquad (17)$$

where $y = \frac{2}{3}\frac{\omega}{\gamma^3 kv}$. (Another method for a representation of the radiation spectrum through the Bessel function of the zeroth order $J_0(x)$ and its derivative $J_0'(x)$ has been developed in [3].) As $v \approx c$, the radiation spectrum is entirely determined by the instant curvature of trajectory or, according (15), its acceleration at the given moment. After substituting Equation (4) in (17), we can transform Equation (17) in the form

$$\frac{dP(t)}{d\omega} = \frac{\sqrt{3}e^2\gamma}{2\pi v^2}\left(\Omega^4R^2 + \omega_{\mathrm{B}}^4 r_{\mathrm{B}}^2 + 2\Omega^2R\omega_{\mathrm{B}}^2 r_{\mathrm{B}}\cos\psi\right)^{1/2}F\left(\frac{\omega}{\omega_C}\right) \qquad (18)$$

where $F(y) = y\int_y^\infty K_{5/3}(x)dx$.

The characteristic radiation frequency is

$$\omega_C = \frac{3}{2}\frac{\gamma^3}{c}\left(\Omega^4R^2 + \omega_{\mathrm{B}}^4 r_{\mathrm{B}}^2 + 2\Omega^2R\omega_{\mathrm{B}}^2 r_{\mathrm{B}}\cos\psi\right)^{1/2}. \qquad (19)$$

For $q = \varepsilon/\delta^2 >> 1$, from Equation (19) we have the synchrotron characteristic frequency $\omega_{\mathrm{synch}} = \frac{3eB\sin\alpha}{2mc}\gamma^2$, where α is the pitch-angle [9]. This characteristic frequency is maintained both in the case of nonrelativistic $\delta << \varepsilon$ and relativistic $\delta >> \varepsilon$ longitudinal velocities if the inequality $q >> 1$ is fulfilled. In this case, from Equation (19) we obtain the spectral distribution of the synchrotron radiation [9]. For $q = \varepsilon/\delta^2 << 1$, we obtain the characteristic frequency $\omega_{\rm curv} = \frac{3}{2}\gamma^3\frac{c}{R}$ and the spectral distribution of the curvature radiation. In the limiting cases, the characteristic radiation frequency is constant.

As it follows from Equation (19), for $q \sim 1$ the characteristic frequency is dependent on time. The band of characteristic frequencies arises instead the single characteristic frequency,

$$\frac{3}{2}\gamma^3\frac{c}{R}|1-q| < \omega_C < \frac{3}{2}\gamma^3\frac{c}{R}(1+q). \qquad (20)$$

It is interesting to have an averaged spectral power. For this purpose, it is necessary to average Equation (17) over the time period $2\pi/\bar{\omega}$; however, neglecting the corrections $\sim\varepsilon^2$ to the effective frequency $\bar{\omega}$, it is possible to put $\bar{\omega} = \omega_B$. Equation (17) is transformed to

$$\frac{dP(t)}{d\omega} = \frac{\sqrt{3}e^2}{4\pi}\int_0^\infty \frac{d(\omega_B t)}{\pi}\gamma k\, y\int_y^\infty K_{5/3}(x)dx, \qquad (21)$$

where $y = \omega/\omega_C$.

The equation (21) has the same form as obtained in [5], but here the improved expression for the acceleration of the electron enters.

To integrate with respect to $|\omega_B|t$ in (21), we introduce the variable $z = 1 + q^2 + 2q\cos\omega_B t$ and change the order of integration. Then [6]

$$\frac{dP}{d\omega} = \frac{P_{\rm C}}{\bar{\omega}_{\rm C}} f(y_{\rm C}, q), \qquad (22)$$

$$f(y_{\rm C}, q) = \frac{9\sqrt{3}}{8\pi} y_{\rm C}\left\{\int_{\frac{y_{\rm C}}{|1-q|}}^\infty dx K_{5/3}(x) + \right.$$

$$\left. + \frac{1}{\pi}\int_{\frac{y_{\rm C}}{1+q}}^{\frac{y_{\rm C}}{|1-q|}} dx K_{5/3}(x)\left(\frac{\pi}{2} + \arcsin\frac{1+q^2-y_{\rm C}^2/x^2}{2q}\right)\right\},$$

where $P_{\rm C} = \frac{2}{3}\frac{e^2}{c}\gamma^4\beta_\parallel^2\Omega^2$ is the total power emitted by a charged particle moving with velocity $v_\parallel$ along a circular orbit of radius R, $y_{\rm C} = \omega/\bar{\omega}_C$ $\bar{\omega}_{\rm C} = (3/2)\gamma^3\Omega\beta_\parallel/\beta$.

Integrating in (22) with respect to frequency, we obtain the total emitted power

$$P = \frac{2}{3}\frac{e^2}{c}\gamma^4\beta_\parallel^2\Omega^2\left(1+q^2\right). \qquad (23)$$

It is visible that Eq. (23) coincides with Eq. (9).

4. DISCUSSION

As it follows from the preceding consideration, the general formula of synchrotron radiation (22) should be used, when the velocity of a centrifugal drift is comparable to the velocity of Larmor gyration or $q \sim 1$.

Let's compare the obtained formulae for the total energy losses and radiation spectrum with the expressions from other papers. The expression for total power losses (9) coincides (to within small parameters) with obtained in [4,5] and differs from the relevant formula in [1,2]. The main difference arises in the field of parameters, in which the pitch-angle is variable. In this case, the velocity of centrifugal drift is comparable to the velocity of Larmor rotation $q \sim 1$.

In Fig.1 we present the comparison of the behavior of the total energy losses as a function of the ratio $v_\perp/v_\parallel = \omega_B r_{\rm B}/\Omega R$, obtained from equation (9), with the synchrotron and the curvature radiation mechanism. The magnitude of magnetic field, the curvature radius, and the energy of runaway electrons have been taken as in the tokamak TEXTOR [3]. From this figure we can notice that for pitch-angles $\alpha < 10^{-1}$ the using of classical synchrotron radiation mechanism is not sufficient. Thus, if the parameter $q = \frac{\omega_B R}{v_\parallel}\frac{v_\perp}{v_\parallel} \sim 1$, then for description of the radiation of ultrarelativistic electrons moving in inhomogeneous magnetic field we must take into account the curvature of magnetic lines.

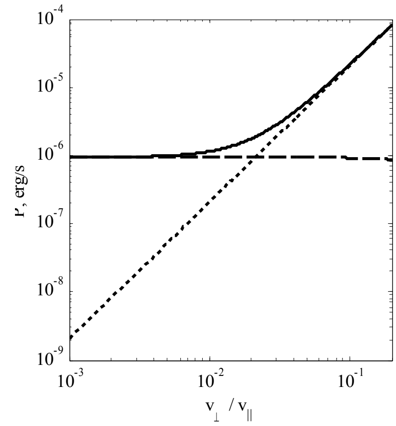

Fig.1. Comparison of the total energy losses described by Equation (9)(solid curve) with the energy losses of the synchrotron radiation (dotted curve) and curvature radiation (dashed curve) for $B = 2\cdot10^4$ G, $\gamma = 50$, $R = 175$ cm

The spectral distribution of the radiation is given by Equation (22). In Fig.2, the spectral distribution of the total radiation intensity $dP/d\omega$ for an ultrarelativistic electron moving in a curved magnetic field is represented as a function of the frequency ω, for the synchrotron radiation with effective Larmor radius and for the synchrocurvature radiation of [2]. The solid curve is described by Equation (22), the dashed curve corresponds to [2, eq. (23)]. The difference between these curves is due to that in [2] the trajectory of particle has constant radius of curvature $\approx R(1+q)$, whereas the curvature radius of trajectory (5) changes from $R|1-q|$ up to $R(1+q)$, and the spectrum (22) is obtained by averaging the contributions in radiation from different points of the trajectory. The dotted curve represents the spectrum of the classical synchrotron radiation of a relativistic electron in a circle trajectory with radius $R\sqrt{1+q^2}$. Such synchrotron radiation has also the total radiation power (9). The values $B \sim 10^4$, $\gamma \sim 100$ and $q = 1.2$ have been taken.

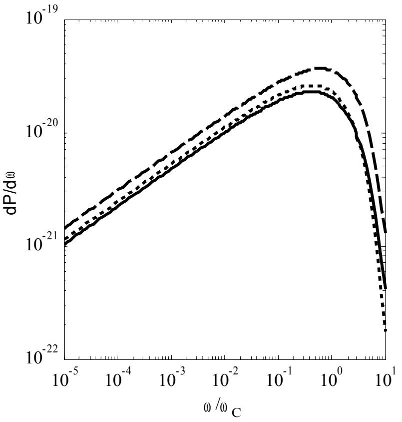

Fig.2. The spectrum of the radiation emitted by a relativistic electron moving in a curved inhomogeneous magnetic field (solid curve, Eq.(22)), of the synchrocurvature mechanism [2] (dashed curve) and of the effective synchrotron radiation (dotted curve)

Thus, the trajectory of an ultrarelativistic electron in a curved inhomogeneous magnetic field is obtained. The total power and the spectral distribution of the radiation emitted by the relativistic charged particle in the curved magnetic field have been derived, using this trajectory. The obtained formulae in the lowest order of smallness coincide with the formulae obtained earlier in [4,5] for a circular magnetic force line. It is natural, as the synchrotron radiation in the given direction goes from a small segment of the trajectory, which with sufficient accuracy can be approximated by an incircle. The account of a radial inhomogeneity of the magnetic field has allowed to find limits of applicability of synchrocurvature radiation mechanism.

The criterion of necessity to take into account the curvature of magnetic lines is found out. The application of this criterion for the tokamak TEXTOR shows that it is easily fulfilled.

Thus, the curvature of a magnetic force line needs to be taken into account, when parameter $q = \omega_B^2 r_B / \Omega^2 R \sim 1$. These conditions are easily realized both in the cosmic space and in the experimental machines.

5. APPENDIX

The equations of particle motion in magnetic field (1) have the form

$$\ddot{r} + r\ddot{\varphi} = -\omega_B \frac{R}{r}\dot{z}\,,\quad r\ddot{\varphi} + 2\dot{r}\dot{\varphi} = 0\,,\quad \ddot{z} = \omega_B \frac{R}{r}\dot{r}\,, \qquad \text{(A1)}$$

where $\omega_B = \dfrac{eB_0}{\gamma mc}$ is the cyclotron frequency.

The system of Equations (A1) has two integrals which correspond to a generalized angular momentum and generalized momentum along z,

$$r^2\dot{\varphi} = M\,,\quad \dot{z} - \omega_B R \ln\left(\frac{r}{R}\right) = V_Z\,, \qquad \text{(A2)}$$

where M and V_Z are the constants of integration.

Substituting (A2) into (A1), assigning $M \equiv \Omega R^2$, introducing new variable $r = R(1+\varepsilon x)$.
and expanding the obtained expression into powers of $\varepsilon = r_B / R << 1$, we obtain the equation

$$\ddot{x} + \omega_0^2 x = \varepsilon\left(\frac{3}{2} + 2\delta^2\right)x^2 - \varepsilon^2\left(\frac{11}{6} + \frac{16}{3}\delta^2\right)x^3 + \varepsilon^3\left(\frac{25}{12} + \frac{59}{6}\delta^2\right)x^4 - \varepsilon^4\left(\frac{137}{60} + \frac{463}{30}\delta^2\right)x^5, \qquad \text{(A3)}$$

where $\omega_0^2 = \omega_B^2 + 2\Omega^2$, $\delta = \Omega / \omega_0$. It should be pointed out that the frequency ω_0 differs from the corresponding frequency in [4] by the factor at Ω^2. This difference is caused by the radial inhomogeneity of the magnetic field (1). The equation (A3) can be solved by the method offered in [6]. Assume that the solution of Equation (A3) has the form

$$x = z(\psi)\,,\ \psi = \overline{\omega} t + \theta \qquad \text{(A4)}$$

After substituting (A4) into Equation (A3) and expanding the solution in a power series

$$z = \sum_{n=0}^{\infty} \varepsilon^n z_n(\psi)\,,\quad \overline{\omega}^{-2} = \sum_{n=0}^{\infty} \varepsilon^n \alpha_n\,,$$

Equation (A3) can be written as a system of equations with unknown functions $z_n(\psi)$ [6]; solving this system (with initial conditions $r(0) = r_B$, $\dot{r}(0) = 0$, $\varphi(0) = \pi/2$ $z(0) = 0$, $M, V_Z = const$), we find out the above mentioned expressions (2), (5).

Where $V_Z \equiv v_D = \dfrac{\Omega^2 R}{\omega_B}$ is the velocity of centrifugal drift

$$z_4 = -\left(\frac{3}{64} + \frac{1}{24}\delta^2 + \frac{23}{12}\delta^4 - \frac{19}{9}\delta^6\right) + \left(\frac{23}{288} + \frac{41}{144}\delta^2 + \frac{71}{72}\delta^4 - \frac{59}{54}\delta^6\right)\cos 2\psi - \left(\frac{31}{576} + \frac{169}{720}\delta^2 + \frac{11}{72}\delta^4 + \frac{1}{54}\delta^6\right)\cos 4\psi$$

$$\alpha_4 = \frac{7}{32}\left(1 + \frac{76}{21}\delta^2 + \frac{1124}{63}\delta^4 + \frac{1264}{21}\delta^6 - \frac{5360}{189}\delta^8\right).$$

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СИНХРОТРОННОЕ ИЗЛУЧЕНИЕ РЕЛЯТИВИСТСКИХ ЭЛЕКТРОНОВ, ДВИЖУЩИХСЯ ПОД МАЛЫМИ ПИТЧ-УГЛАМИ В НЕОДНОРОДНОМ МАГНИТНОМ ПОЛЕ

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Рассмотрено синхротронное излучение релятивистских электронов, движущихся под малыми питч-углами в неоднородном магнитном поле. Траектория ультрарелятивистского электрона получена с учетом кривизны и радиальной неоднородности линий магнитного поля. Выведены общие формулы, описывающие излучение электрона, движущегося при питч-углах от 0 до π/2. Вместо отдельной характерной частоты возникает область характеристических частот. Оценена применимость формул для убегающих электронов в токамаке. Выведены условия для использования полученных формул.

СІНХРОТРОННЕ ВИПРОМІНЮВАННЯ РЕЛЯТИВІСТСЬКИХ ЕЛЕКТРОНІВ, ЩО РУХАЮТЬСЯ ПІД МАЛИМИ ПІТЧ-КУТАМИ У НЕОДНОРІДНОМУ МАГНІТНОМУ ПОЛІ

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Розглянуте синхротронне випромінювання релятивістських електронів, що рухаються під малими пітч-кутами у неоднорідному магнітному полі. Траєкторія ультрарелятивістського електрона отримана з урахуванням кривизни і радіальної неоднорідності ліній магнітного поля. Виведені загальні формули, які описують випромінювання електрона, що рухається при пітч-кутах від 0 до π/2. Замість окремої характерної частоти виникає область характеристичних частот. Проведена оцінка застосовності формул для збігаючих електронів у токамаці. Виведені умови для застосування отриманих формул.