

# CALCULATION OF DC ARC PLASMA TORCH VOLTAGE-CURRENT CHARACTERISTICS BASED ON STEENBECK MODEL

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Modern technology of treatment of waste and equilibrium plasma chemistry is based on plasma technology of gas heating using the direct current arc plasma torches. The work is devoted to the problem of the determination of plasma torches parameters and power sources parameters (working voltage and current of plasma torch) at the pre-designing stage. The sequence of calculation of voltage-current characteristics of DC arc plasma torch is proposed. It is shown that the simple Steenbeck model of arc discharge in cylindrical channel makes it possible to carry out this calculation. The results of the calculation are confirmed by the experiments.

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## 1. INTRODUCTION

There are several environmental (ecology related) problems actual today: the treatment of waste (municipal, medical, radioactive), the effective burning of low-calorie coals, some plasma chemistry processes namely the hydrogen sulphide decomposition, gasification of coals and heavy hydrocarbons. The problems listed above require the hot gases (and air in particular) of temperatures about 3000...5000°C for the waste treatment processes and up to 10000...12000°C for plasma chemistry processes. These problems can be solved with the help of plasma technology using the direct current arc plasma torches as a gas heating device, what causes the necessity of the pre-design stage methodology for plasma torch parameters determination.

## 2. ARC DISCHARGE IN PLASMA TORCH

The arc discharge is used in DC arc plasma torch to heat the plasma forming gas (air). The arc discharge in gas at the pressure above ~0.05 MPa is the arc discharge at high pressure. In our case we use the air at atmospheric pressure. There are two areas in the arc discharge: the cathode area and the positive anode column. The electron current of the discharge is caused by the physical processes in the cathode area. The length of the cathode area is small enough (about  $10^{-5}$ ... $10^{-7}$  cm). The cathode potential drop in the arc discharge at the atmospheric pressure is about 10 V. Thus the main thermal power used for gas heating is produced in a positive anode column of arc discharge. The power of electric arc per 1 cm is 1500 W at the intensity of electric field in anode column  $E=15$  V/cm and arc current  $i=100$  A. The heat from the arc channel is transferred by the heat conductivity; the contribution of radiation becomes considerable at ultrahigh pressures. To stabilize the arc (to provide thermal balance) in cylindrical channel of plasma torch it is necessary to cool the channel walls or to blow the arc by the working gas. It's especially effective to twirl the gas flow to stabilize the arc in a plasma torch channel. At atmospheric gas pressure the arc plasma is at equilibrium: the temperatures of components are close to each other. Radiation losses for air nitrogen at atmospheric pressure are about 1...3% of input power. The ionization in the arc discharge occurred by the thermal way irrespective of the input way of energy supply to plasma.

Taking into account all these factors, we shall consider process of gas heating in positive anode column of arc discharge.

## 3. ELENBAAS - HELLER EQUATION

One can obtain the equation of arc column at the following simplifying assumptions. The electric field intensity is constant along the arc (positive anode column); the conductivity is the function of average temperature. The assumptions are true in the case of laminar gas flow in the plasma torch channel. One can write the thermal balance per the unit of the arc length. We can neglect the radiation in the case of gas heating when the gas temperature in the arc is less than 12000...15000K. Last moment certifies that the radiation is not present in the balance. It is usually observed experimentally in the case of air heating at approximately atmospheric pressure and at not too high density of arc current. So, the main losses are determined by heat conductivity and the balance of energy of a column looks like [1]:

$$-\frac{1}{r} \frac{d}{dr}(rJ) + \sigma(T)E^2 = 0, \quad J = -\lambda \frac{dT}{dr}. \quad (1)$$

We have the Elenbaas – Heller equation reduced to the thermal balance equation, where  $r$  is the variable radius,  $J$  is the heat flow per square unit ( $W/cm^2$ ),  $\sigma(T)$  is the electrical conductivity of arc column ( $\Omega^{-1}cm^{-1}$ ),  $\lambda$  is the thermal conductivity of arc column ( $WK^{-1}cm^{-1}$ ). According to (1) the thermal balance consists in the equality of the ohmic heating to the gas thermal conductivity. The arc current is specified as usual:

$$i = E \int_0^R \sigma 2\pi r dr \quad (2)$$

## 4. STEENBECK MODEL

In general case we have  $T=T_c$  at  $r=R$ , where  $R$  is the channel radius,  $T_c$  is the wall temperature, and  $dT/dr=0$  at  $r=0$ . The plasma temperature is much higher than the wall temperature and one can assume  $T_c=0$ .

The anode column conductivity is the function of the plasma temperature [1]:

$$\sigma(T) \sim \exp(-I/2T), \quad (3)$$

where  $I$  is the "effective" ionization potential.

One can assume the electrical conductivity is the delta function of radius  $r$ : the conductivity is constant at

$0 \leq r \leq r_0$  and the conductivity is zero at  $r_0 \leq r \leq R$ , the temperature is described by the parabolic function:

$$T(r) = T_0 - (T_c - T_0) \frac{r^2}{r_0^2} = T_0 - \Delta T \frac{r^2}{r_0^2}, \quad (4)$$

where  $T_0$  is the temperature at the centre of the channel,  $T_c = T|_{r=r_0}$  is the temperature at the boundary of arc current (see Fig.1).

One can see the equation (1) is the balance of specific power per length unit. We can obtain the full power per length unit by the integration of the equation (2) on the full cylinder volume and the dividing of the result by the cylinder length

$$\begin{aligned} W &= \int_0^{r_0} 2\pi r \sigma(T) E^2 dr = \pi r_0^2 \sigma E^2 = \frac{i^2}{\pi r_0^2 \sigma} = \\ &= -2\pi r_0 \lambda \left. \frac{dT}{dr} \right|_{r=r_0} = 4\pi \lambda \Delta T \end{aligned} \quad (5)$$

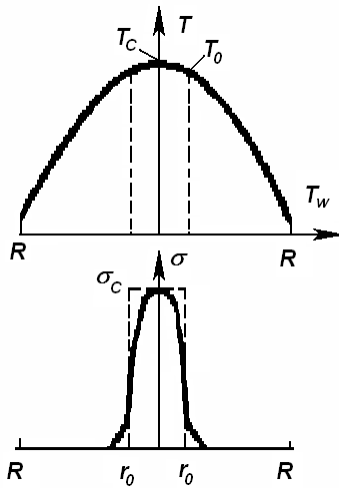


Fig.1. Dependence of temperature and conductivity on the radius of channel

Near the boundary of arc current  $r=r_0$  we can write:

$$\left. \frac{dT}{dr} \right|_{T=T_c} \approx \frac{\sigma(T_c)}{\Delta T}. \quad (6)$$

We note, the equation (6) enable us to obtain the boundary of the arc current.

One can write by the using of equations (5), (6):

$$4\pi \lambda \sigma(T_c) = W \left. \frac{d\sigma}{dT} \right|_{T=T_c}. \quad (7)$$

By the using of the expression (3) we obtain:

$$T_c = \sqrt{\frac{IW}{8\pi\lambda}}. \quad (8)$$

## 5. THE VOLTAGE-CURRENT CHARACTERISTIC OF ANODE POSITIVE COLUMN

Thus we have obtained the expression for the temperature in the centre of the channel through the electrical power per the length unit and effective ionization potential. Knowing the temperature it is possible to evaluate all parameters interesting for us. The temperature gives us the electrical conductivity of arc and we

can find radius of “hot” zone in which the electrical current mainly exists. Really, we can find the radius of arc if we know the arc power per length unit, the arc current and the arc temperature:

$$W = \frac{i^2}{\pi r_0^2 \sigma(T_c)}; \quad r_0 = \sqrt{\frac{i^2}{\pi W \sigma(T_c)}}. \quad (9)$$

Then we find the intensity of electric field of arc (the voltage per length unit):

$$E = \frac{i}{\pi r_0^2 \sigma(T_c)}, \quad (10)$$

$I \approx 6,2 \text{ eV}$ ,  $\lambda \approx 1,5 \cdot 10^{-2} \text{ (WK}^{-1}\text{cm}^{-1}\text{)}$  for air at the atmospheric pressure and at the temperatures  $T \approx 8000 \dots 12000 \text{ K}$  [1]. From (2) the electrical conductivity of the length unit of arc column is

$$\sigma(T) = C \exp(-I/2kT). \quad (11)$$

Or the electrical conductivity of the length unit of the arc in the air is:

$$\sigma(T) = 83 \cdot \exp\left(-\frac{I \cdot e}{2kT}\right), \text{ (}\Omega^{-1}\text{cm}^{-1}\text{)}, \quad (12)$$

where  $k=1.38 \cdot 10^{-23}$  is the Boltzmann constant ( $J.K^{-1}$ ),  $e=1.6 \cdot 10^{-19}$  is the charge of electron (Coulomb).

Let's note that we do not take into account the pre-exponential dependence on temperature here. Such a pre-exponent is given by Saha formula in which the ionization potential is replaced by effective potential  $I \approx 6,2 \text{ eV}$  which we use.

All earlier considered dependences concern to the case of free arc in unlimited space. In the case of gas heating by the arc discharge in plasma torch the arc burns in the cylindrical channel of some radius  $R$ .

In the stationary case there is also the constant heat flow through the cylinder surface of radius  $r$  and length  $L$ .

$$-L2\pi r \lambda \frac{dT}{dr} = WL. \quad (13)$$

We assume the system is homogeneous on surface of cylinder, and also the temperature is equal to  $T_c$  at  $r=r_0$ , the temperature is equal to zero at the radius of the cylinder  $r=R$ . We shall receive:

$$\theta = \int_0^{r_0} \lambda(T) dT = \frac{W}{2\pi} \ln \frac{R}{r_0}. \quad (14)$$

The same power  $W$  per length unit can be expressed by arc current (see (9)):

$$W = \sigma E^2 \pi r_0^2 = \frac{i^2}{\pi r_0^2 \sigma}. \quad (15)$$

We have obtained the dependence of arc radius  $r_0$  on channel radius  $R$  by the integration of equation (14). To find the arc temperature we use the equation (13). It's known, the formulas obtained correspond to Steenbeck's principle of power minimum. By the replacing of temperature derivative on radius according to the approximation (4), we shall obtain:

$$\frac{dT}{dr} = -\frac{\Delta T}{r_0} = -2 \frac{1}{r_0 \left( \frac{1}{\sigma} \frac{d\sigma}{dT} \right)_{r=r_0}} = -4 \frac{T_c^2}{r_0 I}.$$

Thus, we have from (13) (compare with the formula (8)):

$$T_c = \sqrt{\frac{WI}{8\pi k\lambda}}. \quad (16)$$

It is possible to plot the curve of the of electrical field intensity of arc on arc current in the channel of given radius on the base of obtained expressions.

To construct such characteristic it is necessary to do the following calculations:

1) We determine the arc channel temperature by the formula (16) at various values of power per length unit.

2) We find the arc channel radius by the formula (14), assuming the heat conductivity is constant for the given values of arc power per length unit:

$$r_0 = R \times \exp\left(-\frac{2\pi\theta}{W}\right) = R \times \exp\left(-\frac{2\pi\lambda T_c}{W}\right). \quad (17)$$

3) Now we find the electrical conductivity of arc by formula (12) for given values of arc power per length unit.

4) We determine the arc current by the formula obtained from (9) for the electrical conductivity calculated:

$$i = r_0 \sqrt{\pi W \sigma}. \quad (18)$$

5) The electric field of arc is the result of division of power per unit length by arc current:

$$E = \frac{W}{i}. \quad (19)$$

One can see the result of above calculations on the Fig.2.

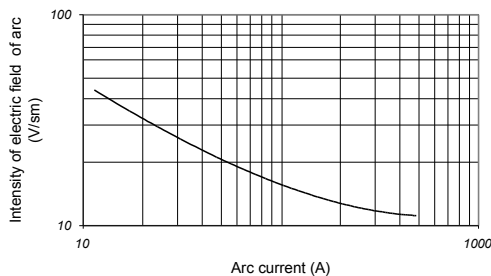


Fig.2. Voltage-current characteristic of arc in the channel calculated by the formulas (16)-(19)

We introduce the flow of enthalpy:

$$I_H = Q \times c_p T_c. \quad (20)$$

Here  $Q$  is the gas flow rate, it's the number of gas (air) particles passing through the system in time unit,  $c_p$  is the thermal capacity of air molecule at constant pressure that is  $7k/2$ . We assume the flow of enthalpy is approximately equal to full power of a plasma torch. We neglect power losses on the cooling of plasma torch channel walls. We can write to within plasma torch effectiveness:

$$I_H = Q \times c_p T_c = WL. \quad (21)$$

In view of (21) we can write down the formulas for electrical power of arc:

$$WL = Q c_p \sqrt{\frac{WI}{8\pi k\lambda}}; \quad W = Q^2 c_p^2 \frac{I}{8\pi k\lambda L^2}. \quad (22)$$

We can calculate the voltage-current characteristics of air arc heating in plasma torch on the base of the above expressions without taking into account the heat exchange by the radiation.

The sequence of calculation of voltage-current characteristics of a positive anode column of arc in plasma torch looks as follows:

## 6. CALCULATION OF THE VOLTAGE-CURRENT CHARACTERISTICS OF FREE LENGTH ARC IN CYLINDRICAL CHANNEL

1. We assume the working gas flow in plasma torch channel  $Q$  (particles per second), or mass gas flow  $G_m$ (g/s)

$$Q = N_A \frac{G_m}{M}, \quad (23)$$

where  $N_A=6.02 \cdot 10^{23}$  is the Avogadro number,  $M$  is the molecular mass of gas ( $M=29$ ). In the case of volume gas flow  $G_V$  (normal  $m^3/h$ ) is:

$$Q = N_A \frac{1000 \cdot G_V}{22.4 \cdot 3600}, \quad (24)$$

2. We assume consistently the series of values of power per length unit of arc anode column  $W$  (W/cm);

3. We calculate the temperature  $T_c$  (K) for given values of  $W$  (W/cm) by the formula (16).

4. We calculate the radius  $r_0$  of arc anode column by the expression (17).

5. The conductivity  $\sigma$  ( $\Omega^{-1}cm^{-1}$ ) of arc column can be calculated by the expression (12).

6. We calculate the arc current  $i$  (A) by the expression (18).

7. The electric field intensity  $E$  (V/cm) of arc column is calculated by the formula (19).

8. The length  $L$  (cm) of free arc depends on gas flow (see (22)):

$$L = Q \cdot c_p \cdot \sqrt{\frac{I}{8\pi k\lambda W}}. \quad (25)$$

9. The voltage ( $V$ ) of anode positive column of arc is:

$$U = L \cdot E. \quad (26)$$

The above calculation sequence enables us to obtain the voltage-current characteristics of plasma torch with free length arc.

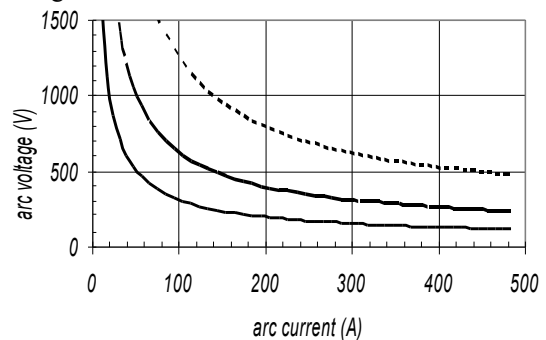


Fig.3. Voltage-current characteristics of free length arc

in the channel  $R=1.25\text{ cm}$

The voltage-current characteristics of free length arc in the channel of radius  $R=1.25\text{ cm}$  for gas flows  $G_V=5\text{ (m}^3\text{/h)}$ ,  $G_V=10\text{ (m}^3\text{/h)}$ ,  $G_V=20\text{ (m}^3\text{/h)}$ ,  $G_V=30\text{ (m}^3\text{/h)}$  and  $G_V=40\text{ (m}^3\text{/h)}$  are shown on Fig.3.

One can see the length of free length arc in plasma torch channel for the above gas flows on the Fig.4. Some lengths of arc are rather long and its may be longer than the physical length of plasma torch channel. It means, the practical interest is to calculate the voltage-current characteristics of anode column of fixed length arc. This length is determined by the design of plasma torch, by the physical length of plasma torch arc channel.

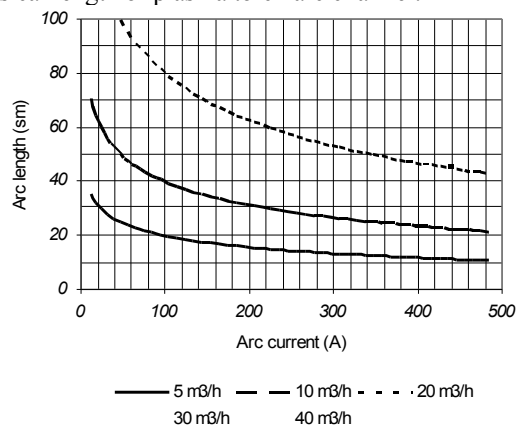


Fig.4. Arc lengths dependences on arc current in the channel  $R=1.25\text{ cm}$

The sequence of calculation of voltage-current characteristics of arc anode positive column of fixed length in plasma torch is similar to the above and it looks as follows:

### 7. CALCULATION OF VOLTAGE-CURRENT CHARACTERISTICS OF FIXED LENGTH ARC IN CYLINDRICAL CHANNEL OF PLASMA TORCH

1. We determine the arc length  $L\text{ (cm)}$  as the parameter of plasma torch design.
2. We assume the working gas flow in plasma torch channel  $Q\text{ (particles per second)}$ , or mass gas flow  $G_m\text{ (g/s)}$  or volume gas flow  $G_V\text{ (m}^3\text{/h)}$ .
3. We assume consistently the series of values of power per length unit of arc anode positive column  $W\text{ (W/cm)}$ ;

4. One can calculate the arc temperature  $(K)$  by the using of expressions (16) and (25):

$$T_c = \frac{W \cdot L}{Q \cdot c_p} \quad (27)$$

5. The arc column radius  $r_0$  is determined by the expression (17).

6. The conductivity  $\sigma(T)$  of arc column is the function of arc temperature (12).

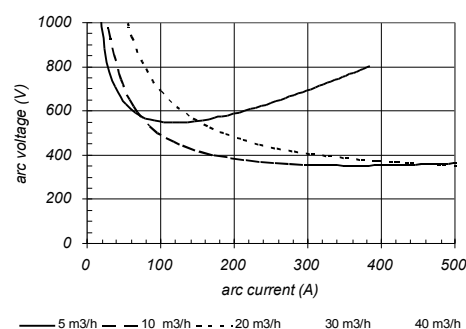


Fig.5. Voltage-current characteristics of fixed length arc  $L=30\text{ cm}$  in plasma torch channel of radius  $R=1.25\text{ cm}$

7. The arc current  $i$  depends on the conductivity  $\sigma(T)$  (18).

8. One can calculate the electrical field intensity of the arc by formula (19).

9. The arc voltage  $U$  is a result of multiplication of electrical field intensity  $E$  and arc length  $L$  (26).

The Fig.5 shows the voltage-current characteristics of fixed length arc  $L=30\text{ cm}$  in plasma torch channel of radius  $R=1.25\text{ cm}$  for gas flows  $G_V=5\text{ (m}^3\text{/h)}$ ,  $G_V=10\text{ (m}^3\text{/h)}$ ,  $G_V=20\text{ (m}^3\text{/h)}$ ,  $G_V=30\text{ (m}^3\text{/h)}$  and  $G_V=40\text{ (m}^3\text{/h)}$ .

### 8. EXPERIMENTAL AND THEORETICAL VOLTAGE-CURRENT CHARACTERISTICS

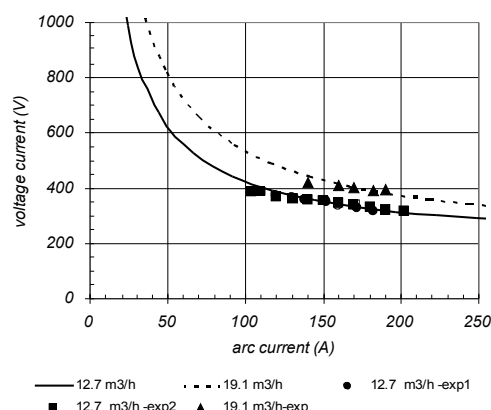


Fig.6. Experimental (points) and theoretical (continuous lines) voltage-current characteristic of plasma torch  $L=27\text{ cm}$ ,  $R=1.25\text{ cm}$ . The effective ionization potential was assumed  $4.85\text{ eV}$

The experimental voltage-current characteristics were measured in the experiments with plasma torch EAH-200. The experimental results differ from the above calculated voltage-current characteristics. One can explain the differences by the conditions of arc discharge in plasma torch: there is some metal vapor in the working gas (air). This vapor metal is the result of the evaporation of cathode material. It results in the decreasing of the “effective” ionization potential.

One can see the results of the experiments and calculation of voltage-current characteristics on Fig.6. The “effective” ionization potential is assumed to be equal to  $4.85\text{ eV}$  (but not the above  $6.2\text{ eV}$ ).

### 9. DISCUSSION

The above calculation sequence of parameters of anode positive column of arc discharge based on Steenbeck model enables us to understand better the behavior of arc discharge in plasma torch channel on various working regimes. For example, in the case of free length arc the arc temperature and the arc channel radius do not depend on the gas flow in the channel, but the arc length (and the arc resistance) is proportional to the gas flow in the plasma torch channel (see section 6). In the case of fixed length arc the temperature of arc channel is determined by the gas flow (see expression (27)), and the arc channel radius do not depend on arc current (see expressions (17) and (27)) and it depends on gas flow. The model presented can describe such phenomenon as shunting of arc: the arc of free length may be shorter than the plasma torch channel length (see curves  $5 \text{ m}^3/\text{h}$  on Figs.4 and 5). There is the possibility of such changing of plasma torch regime as the decreasing of gas flow from the regime at which the arc of free length is longer than the plasma torch channel length (curves 20, 30 and  $40 \text{ m}^3/\text{h}$  on Fig.5) to the regime at which the arc of free length is shorter than the plasma torch channel (curve  $5 \text{ m}^3/\text{h}$  on Fig.5). In this case there is the possibility of spontaneous change of plasma torch regime from mode "with fixed length arc" to mode "with free length arc" at the small gas flows, it corresponds to the shunting of arc.

At the same time there are some problems with the using of the above model to obtain the voltage-current characteristics in wide range of plasma torch geometry and gas flows. The voltage-current characteristics obtained as a result of calculations can differ essentially (10...20%) with the characteristics received by measuring in real experiment.

There are some reasons of this:

1) The geometry of plasma torch channel:

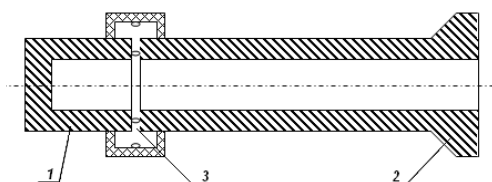


Fig.7. Plasma torch channel geometry: 1 – cathode; 2 – anode; 3 – inlet of gas flow

One can see the geometry of plasma torch channel on Fig.7. The gas flows into inter-electrode gap: gas flow is not constant along the plasma torch channel; gas flow in cathode channel is less than gas flow in anode channel.

## РАСЧЕТ ВОЛЬТ-АМПЕРНЫХ ХАРАКТЕРИСТИК ПЛАЗМЕННОЙ ГОРЕЛКИ ДУГИ ПОСТОЯННОГО ТОКА, ОСНОВАННЫЙ НА МОДЕЛИ ШТЕЙНБЕКА

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Современная технология обработки отходов и равновесной плазмохимии основана на плазменной технологии нагрева газа, используя плазменные горелки дуги постоянного тока. Работа посвящена проблеме определения параметров плазменных горелок и параметров источников питания (рабочее напряжение и ток плазменной горелки) на стадии проектирования. Предложена последовательность расчета вольт-амперных характеристик плазменной горелки дуги постоянного тока. Показано, что простая модель Штейнбека дугового разряда в цилиндрическом канале делает возможным провести этот расчет. Результаты расчета подтверждены экспериментами.

## РОЗРАХУНОК ВОЛЬТ-АМПЕРНИХ ХАРАКТЕРИСТИК ПЛАЗМОВОЇ ГОРІЛКИ ДУГИ СТАЛОГО

2) The gas temperature increases in the process of gas heating and moving to the outlet of anode. Thermal conductivity  $\lambda$  is the function of gas temperature.

3) The length of real arc can exceed the length of plasma torch channel.

One can see the outlet of plasma torch on Fig.8. The circuit of electrical current in plasma torch is realized by the arc filament. The straight line of arc filament on Fig.8 is about four diameters of plasma torch channel and back line of arc filament is about four diameters of plasma torch channel too. Adding the length of this extra-channel part of the arc filament to the plasma torch channel length, which is about 10...15 diameters, we will obtain the real arc length (for the case presented on Fig.8) that is significantly longer than the plasma torch channel. Thus the real arc resistance can differ from that we calculate from the above formulas.

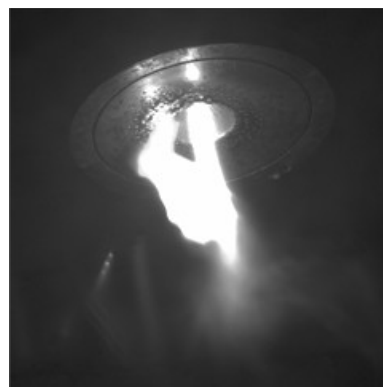


Fig.8. Arc filament at the outlet of plasma torch

At the same time the above sequence of calculation allows to determine the probable voltage-current characteristics of plasma torch at the stage of a choice of its sizes. The given model also allows us to choose the necessary parameters of current source for a plasma torch power supply.

## CONCLUSIONS

We can conclude from the above: the offered model of calculation of voltage-current characteristics can be interesting for the specialists engaged in development of plasma torches intended for gas heating in various applications.

## ACKNOWLEDGMENT

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## **СТРУМУ, ЩО ОСНОВАНИЙ НА МОДЕЛІ ШТЕЙНБЕКА**

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Сучасна технологія обробки відходів та рівноважної плазмохімії основана на плазмовій технології нагріву газу, використовуючи плазмові горілки дуги сталого струму. Робота присвячена проблемі визначення параметрів плазмових горілок та параметрів джерел живлення (робочої напруги та струму плазмової горілки) на стадії проектування. Запропонована послідовність розрахунку вольт-амперних характеристик плазмової горілки дуги сталого струму. Показано, що проста модель Штейнбека дугового розряду в циліндричному каналі робить можливим провести цей розрахунок. Результати розрахунку підтверджені експериментами.