CALCULATION OF NON-UNIFORM SOLENOIDS FOR FOCUSING AND ACCELERATION OF CHARGED PARTICLES

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INTRODUCTION

For many problems of forming, acceleration and focusing charged particles it is need to create non-uniform axially symmetric stationary magnetic fields with specified dependence on longitudinal coordinate within the given accuracy. Usually such fields are created by sectional solenoids consisted of coaxially located sections (coils). The magnetic field intensity created by each section depends on its parameters generally nonlinearly. The parameters to be manipulated are geometrical dimensions of the sections, their mutual location (the section coordinates), current density, etc.

In previous works related to the above-mentioned problem, the sections' current densities were used as some unknown parameters. This is the simplest way to solve the problem of parameters since current density is a linear parameter of the equation for a solenoid magnetic field intensity. The method proposed in [1] is based on a functional minimization allowing to determine the optimum (in sense of minimum standard deviation of created field from required one) current density distribution in all sections of the solenoid. However, if the number of sections is large enough (more then 20), the set of the linear equations – the problem reduced – becomes mal-conditioned which leads to a loss of accuracy in the calculations needed. Hereupon, the load distribution among the sections appears to be far from the optimum desired. Current loading to the edge sections turns out to be many times as large as that of all the others, resulting in thriftless consumption of magnet wire and necessity of forced cooling the outside sections. Power supply of the solenoid is also complicated since setting up the current density distribution among the sections requires either to have an individual power source for each section or to place a rheostat along with each section.

In this work the more complex problem is solved by means of using Tikhonov′s regularization method [2]: the sections parameters of a specific kind are calculated. These can be for example a winding thickness, length or a section internal radius etc., i.e. the values of magnetic field intensity are depending, generally speaking, in the nonlinear way. The method described here is illustrated by solving the problem in case of defining the winding thickness for a solenoid with rectangular cross-sections.

THEORY

Let us consider a sectional solenoid of n coaxial sections with an arbitrary cross-section. It is necessary to create a magnetic field at the solenoid axis in an interval $[a, b]$ within the given accuracy δ, when the field intensity is prescribed by a function $f(z)$. Assume that field intensity, created by the i-th section at the point , is described by the function $H_i(N_i, z)$, which in turn depends on a cross-section of the geometrical configuration, its size and location towards the measuring point. Then magnetic field created by the solenoid on its axis can be formulated as $B(z) = \sum_{i=1}^{n} H_i(N_i, z)$. We shall estimate deviation of $B(z)$ from $f(z)$ in square metric, i.e. by the formula: $\rho(B, f) = \left\{ \int_a^b [B(z) - f(z)]^2 dz \right\}^{1/2}$. This problem pertains to the class of incorrect ones in sense that within given accuracy it has not single-valued solution; moreover, if a sections' number is large enough, its solution becomes unstable towards small variations of initial data. So, from a set of solutions matching $\rho(B, f) \le \delta$ where δ is a given number, one has to pick up the only solution optimized on a certain criteria (a solenoid volume or power consumption can serve as such). The problem is formulated as follows: to find an optimum set of parameters ($N_1, N_2,..., N_n$) under which the functional

$$F(N_1,...,N_n,\beta) = \int_a^b \left[\sum_{i=1}^{n} H_i(N_i, z) - f(z) \right]^2 dz + \beta\,\Omega\,(N_1,...,N_n) \quad (1)$$

reaches its greatest lower bound [2]. Here $\Omega(N_1,...N_n)$ is stabilizing functional determined by the optimization criteria, β is a parameter of regularization. The conditions, under which $\frac{\partial F(N_1,...,N_n,\beta)}{\partial N_k} = 0$, lead us to a set of nonlinear equations:

$$\int_a^b \left[\sum_{i=1}^{n} H_i(N_i, z) - f(z) \right] \frac{\partial H_k}{\partial N_k} dz + \beta \frac{\partial \Omega}{\partial N_k} = 0, \quad (2)$$

that, in turn, can be solved by one of the gradient methods (see [3]).

CALCULATIONS

Solution of the problem on solenoid calculation includes following stages: 1) choice of values determined by a device parameters, heat removal conditions and other factors; 2) choice of initial estimates for both N_i^0 and the regularization parameter β to be ensured that an iterative process converges; 3) solving the set of equations (1) within given accuracy; 4) updating of the initial estimate (current value of a desirable parameter is accepted as such) and that of the parameter β (if the iterative process converges too slowly, β would decrease); 5) repeated solving of (2) until a satisfactory result has been found.

In this section the method offered has been used in case of a homogeneous magnetic field to be produced by a solenoid built of coils with rectangular cross-section. As a desirable parameter N_i the winding thickness $d_i = R_i - r_i$ has been accepted. In this case the approximating functions $H_i(d_i, z)$ and their first derivatives are:

$$H_i(z) = \frac{2\pi J_i}{c}\left[(z-\zeta_i)\ln\frac{R_i+\sqrt{R_i^2+(z-\zeta_i)^2}}{r_i+\sqrt{r_i^2+(z-\zeta_i)^2}} - (z-a_i-\zeta_i)\ln\frac{R_i+\sqrt{R_i^2+(z-a_i-\zeta_i)^2}}{r_i+\sqrt{r_i^2+(z-a_i-\zeta_i)^2}}\right],$$

$$\frac{\partial H_i}{\partial d_i} = \frac{2\pi J_i}{c}\left[\frac{z-\zeta_i}{\sqrt{R_i^2+(z-\zeta_i)^2}} - \frac{z-a_i-\zeta_i}{\sqrt{R_i^2+(z-a_i-\zeta_i)^2}}\right]$$

where J_i is the current density [GS]; ζ_i is the section left edge coordinate [cm]; r_i is the section inner radius [cm]; R_i is its outer radius [cm]; a_i is the length of the section [cm]; c is the light velocity [cm/s].

As a stabilizing functional the square of the solution Euclidean norm have been used: $\Omega(d_1,...,d_n) = \sum_{j=1}^{n}\left(d_j - d_j^0\right)^2$, where N_i^0 is an initial value of the i-th section parameter to be found. Such choice actually corresponds to the criteria of minimum space of wire the solenoid is wound by. As a pattern to be calculated we picked up the solenoid design described in [1] , where the problem of current density definition had been solved under $\beta = 0$. The solenoid length is equal to 727 mm, its inner radius is 85 mm. Current density is the same through all sections and equal to 2 A/mm^2. A homogeneous magnetic field of intensity $f(z) = 1$ kOe is created along the length of 436 mm (60% of the solenoid length). The set of equations (2) was solved by the Newton method under following initial values of parameters: the winding thickness was $d_i^0 = 0$ for all sections, the parameter of regularization β was altered from 10^5 to 0, absolute accuracy of winding thickness calculated value was $\varepsilon = 10^{-4}$ cm (i.e. the iterative processes were to stop when $\beta = 0$ and the next correction to d_i was less than ε for all the sections taken into account). The regularization parameter β was modified by means of its dividing to a constant factor $k = 2$ in the course of its gaining the accuracy prescribed, and it was made equal to zero when getting the value below $\beta = 10^{-10}$.

The Table 1 shows the meanings of the mean-square deviation $\rho(B, f)$, maximum of the relative deviation $\Delta = \max\left(B(z) - f(z)\right)/f(z)$ and volume of the conductor with which the solenoid is reeled up when being splitted into various number of sections m. The figures for the value, which is inversely proportional to the conditioned number $\mathrm{cond}\mathbf{A} = \|\mathbf{A}\|_1 \|\mathbf{A}^{-1}\|_1$ of a matrix $\mathbf{A}$ with components $a_{ij} = \frac{\partial^2 F}{\partial d_i \partial d_j}$, where $\|\mathbf{A}\|_1 = \max_{1\le j\le m}\sum_{i=1}^{m}|a_{ij}|$, related to the last iteration are also given. As we can see from the Table, upon improvement of the field approximation the solenoid volume (and weight) tends to grow.

Table 1.

m	ρ, Oe·cm$^{1/2}$	Δ	V, cm^3	1/cond **A**
4	18.1	$1.09\cdot10^{-2}$	$2.12\cdot10^{4}$	$8.22\cdot10^{-2}$
6	3.13	$2.16\cdot10^{-3}$	$2.19\cdot10^{4}$	$4.74\cdot10^{-3}$
8	0.549	$4.31\cdot10^{-4}$	$2.26\cdot10^{4}$	$1.67\cdot10^{-4}$
10	$8.56\cdot10^{-2}$	$7.50\cdot10^{-5}$	$2.35\cdot10^{4}$	$3.95\cdot10^{-6}$
12	$1.16\cdot10^{-2}$	$1.10\cdot10^{-5}$	$2.50\cdot10^{4}$	$6.64\cdot10^{-8}$
14	$1.24\cdot10^{-3}$	$1.29\cdot10^{-6}$	$2.80\cdot10^{4}$	$9.57\cdot10^{-10}$
16	$1.19\cdot10^{-4}$	$1.32\cdot10^{-7}$	$3.34\cdot10^{4}$	$1.40\cdot10^{-11}$
18	$1.38\cdot10^{-5}$	$1.47\cdot10^{-8}$	$4.20\cdot10^{4}$	$2.00\cdot10^{-13}$
18	$4.13\cdot10^{-3}$	$5.63\cdot10^{-6}$	$2.41\cdot10^{4}$	$1.56\cdot10^{-8}$

It should be noted that as far as the regularization parameter final figure gets equal to zero, these results correspond to the best magnetic field approximation in sense of the lowest root-mean-square deviation. However, the results like that cannot always be taken as satisfactory. For example, the Fig. 1 pictures a magnetic field design for the solenoid with 18 coils, $m = 18$ (as the solenoid is symmetric, only a half of it is shown). As we see from the picture, the edge sections manufacturing would most likely be a toilsome affair, and the magnetic field approximation of such accuracy as 10^{-8} in most cases is hardly required.

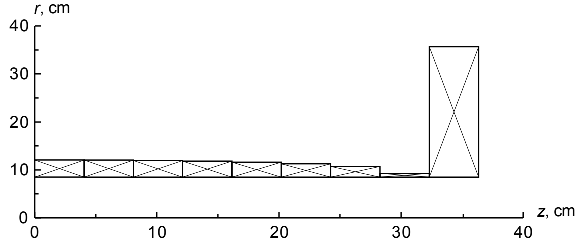

Fig. 1.

In order to optimize the solution towards both parameters – accuracy of a magnetic field approximation and the solenoid volume – the iterative process was terminated when a root-mean-square discrepancy $\rho(B, f)$ reached value of $\delta = 4\cdot10^{-3}$. The relative error of the field approximation thus amounted to $\Delta = 5.63\cdot10^{-6}$, as the solenoid volume got to $V = 2.41\cdot10^4$ cm^3. These results are shown in the bottom line of Table 1, and the solenoid optimum configuration corresponding to the solution being found is pictured in Fig. 2. It goes without saying that the solution of ours is not optimum in sense of the lowest root-mean-square deviation, however it has allowed to reduce both dimension of edge sections and almost twice – volume of the solenoid as a whole.

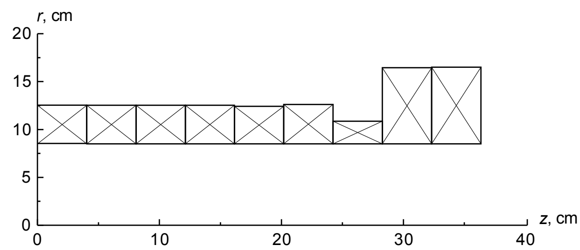

Fig. 2.

Let us note that current density in all sections was installed identical that excludes the edge sections overloading. Due to this, magnet wire might be consumed more rationally and the power consumption be reduced in comparison with solenoids of different kind where a required magnetic field is provided by the proper current density distribution over sections. Power supply of the solenoid might also be simplified as, at common current for all sections, they could be connected in series and fed up by a common power source. All this eventually determines the solenoid dimensions, weight and cost.

APPLICATIONS

As an example, let us consider the results of modeling a concrete solenoid with prescribed magnetic field distribution on its length. The appropriate calculations had been carried out within a work on two-beam electron-ion accelerator design [5] and working out of adiabatic plasma lenses designed for 5 MeV proton beam focusing [6].

Two-beam electron-ion accelerator. To ensure electron-cyclotron resonance happening inside a resonator for a wave excited by a driving electron beam with natural oscillations of a non-uniform slow-wave structure, it is necessary to place the resonator in a non-uniform magnetic field, the diagram of which is shown in a Fig. 3. The relative accuracy of a field approximation Δ should be less than 1%.

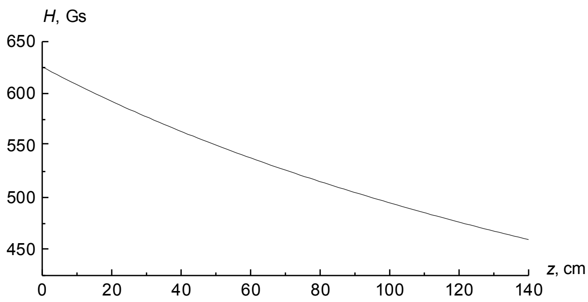

Fig. 3.

The configuration of solenoid creating a field in need, is shown in Fig. 4. Its main parameters are as follows: length- 200 cm, internal radius- 39 cm, sections length -10 cm, number of sections- 15, average current density over the winding cross-section – 2 A/mm^2. The parameter to be calculated was thickness of winding. The field approximation relative error has not exceeded $\Delta = 10^{-3}$ over length of 140 cm, i.e. 70% of the solenoid length.

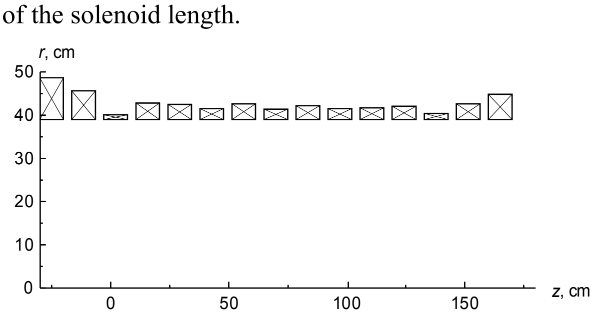

Fig. 4.

Adiabatic plasma lens. In devices like this a radius of the focusing current channel is determined by an external magnetic field topography. For the greater efficiency of a current lens a focused proton beam should fill a focusing current channel as good as possible. To this effect one has to create such external magnetic field over the device length that the focused protons trajectories would coincide with its magnetic lines of force. The diagram of such field is shown in Fig. , and configuration of the solenoid creating it is in Fig. . Its main parameters are as follows: length – 114 cm, internal radius – 10 cm, length of each section – 11.5 cm, number of the sections – 10, average current density over cross-section of winding – 2 A/mm^2. The parameter to be calculated was thickness of winding. The relative error of the field approximation has not

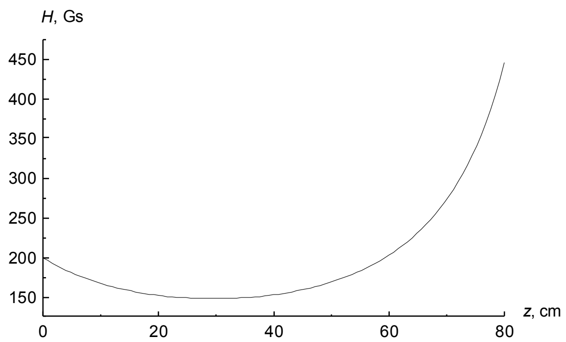

Fig. 5

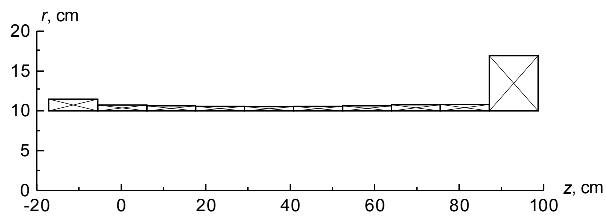

Fig. 6

exceeded $\Delta = 2.48 \cdot 10^{-3}$ over the length of 80 cm, i.e. 70 % of the solenoid length.

REFERENCES

1. Lugansky L.B. Calculation of a Solenoid with a Magnetic Field Given Distribution. JTP, 1985, V. 55, No.7, P.1263–1271.
2. Tikhonov A.N., Arsenin V.Ya. Methods of Incorrect Problems Solution. Moscow. Nauka, 1986.
3. Korn G., Korn T. Guide to Mathematics for Research Workers and Engineers. Moscow. Nauka, 1984. 831 P.
4. Butenko V.I. Calculation of a Solenoid Creating a Magnetic Field with Given Dependence on Longitudinal Coordinate. JTP, 1992, V. 62, No.7, P.157–165.
5. V. Butenko, B. Ivanov, V. Ognivenko, I. Onishchenko, V. Prishchepov, A. Yegorov. Development of a Collective Linear Ion Accelerator with RF Field Excitation Using Doppler Effect. VANT, ser. Fizika plazmy, 1999, No. 3,4(3,4), P. 212.
6. V. Belan, V. Butenko, B. Ivanov, V. Kiselev, A. Kitsenko, A. Linnik, I. Onishchenko, V. Prishchepov, A. Yegorov. Plasma Lens with a Current Density Depended on External Magnetic Field. VANT, ser. Fizika plazmy, 1999, No. 3,4(3,4), P. 233.