MATHEMATICAL MODEL OF INHOMOGENEOUS CAVITY CHAIN

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Mathematical model of inhomogeneous chain of cylindrical cavities is developed. Coupling coefficients in the inhomogeneous cavity chain can be calculated with definite accuracy for the structure with arbitrary parameters. Influence of non-resonant fields and "long-range" couplings on the characteristics of the structure is taken into account by calculation of the coupling coefficients.

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1. INTRODUCTION

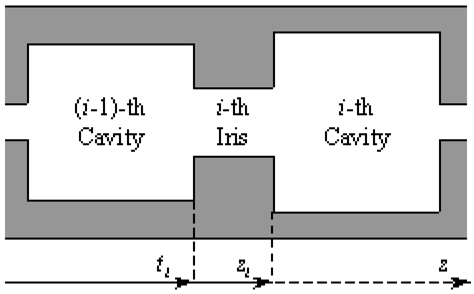
The equivalent circuit analysis is widely used for description of the behavior of accelerating structures (see, for example, [1-3]). It proves to be useful at the stage of primary study of electrodynamic properties of the structure and its conceptual design. Using this approach (equivalent circuit analysis) the technique of consecutive cell-tuning in homogeneous and strongly inhomogeneous structures is developed (see [4,5]). Using the technique of consecutive cell-tuning the comparative analysis of "random" and constant gradient, quasi-constant gradient structures has been done in [5]. Next parameters were chosen for comparison: energy gain, field gradient and damping. It was shown that "random" structures are comparable with constant gradient and quasi-constant gradient ones. However, justification of using of equivalent circuit analysis for description of chains of coupled cavities must be made with the help of rigorous electrodynamic methods.

In [6] precise equations describing the RF-coupling of two cavities is obtained. The method of partial cross-over regions is used. On the base of these equations the dependence of coupling coefficients versus frequency, iris radius, etc. is calculated. The analytical solution of these equations for various limited cases is presented in [7]. It is shown that in the case of iris radius tending to zero and infinitely small disk thickness ($a \to 0$, $t = 0$) the obtained equations agree with those in [2] obtained on the base of quasi-static approach. In [8] the method of partial cross-over regions is used to describe an infinitely long chain of identical cylindrical cavities coupled through irises. The dependence of the phase shift per period versus frequency is calculated for structures with various parameters.

Using of the method of partial cross-over regions for description of inhomogeneous chain of cylindrical cavities reduces to clumsy formulas and difficult calculations. In this work we propose more simple, from our point of view, method for description of inhomogeneous structures. This method is based on rigorous electrodynamic approach as well.

2. ELECTRODYNAMIC APPROACH

Inhomogeneous accelerating structure consists of an array of ideally conducting co-axial cylindrical cavities coupled through dividing irises with radii a_i and thickness t. The radii and lengths of the cavities we denote by b_i and d_i (see Fig.).



Longitudinal cross-section of inhomogeneous chain of cylindrical cavities

The field functions in the region $z_i < z < z_i+d_i$ are represented in terms of cavity modes:

$$E^{(i)}_{z,mp} = \sum_m \sum_p A^{(i)}_{mp} J_0(k^{b(i)}_{\perp m} r)\cos(k^{(i)}_p (z - z_i)),$$

$$E^{(i)}_{r,mp} = \sum_m \sum_p A^{(i)}_{mp} \frac{k^{(i)}_p}{k^{b(i)}_{\perp m}} J_1(k^{b(i)}_{\perp m} r)\sin(k^{(i)}_p (z - z_i)),$$

$$H^{(i)}_{\varphi,mp} = \tag{1}$$

$$= \sum_m \sum_p B^{(i)}_{mp} \frac{-i\varepsilon_0 \omega^{(i)}_{mp}}{k^{b(i)}_{\perp m}} J_1(k^{b(i)}_{\perp m} r)\cos(k^{(i)}_p (z - z_i))$$

where $A^{(i)}_{mp}$ is the amplitude and $\omega^{(i)}_{mp}$ is the resonant frequency of axially-symmetric E_{0mp}-mode in the i-th cavity; $B^{(i)}_{mp} = A^{(i)}_{mp}\omega \big/ \omega^{(i)}_{mp}$.

The field functions in the region $t_i < z < t_i+t$ (disk aperture) can be expressed in terms of modes of uniform waveguide of cylindrical symmetry:

$$E_{z,s}^{(i)} = \sum_s J_0\left(k_{\perp s}^{a(i)} r\right) \left[C_{1s}^{(i)} \exp(-h_s^{(i)}(z-t_i)) + \right.$$
$$\left. + C_{2s}^{(i)} \exp(h_s^{(i)}(z-t_i))\right],$$
$$E_{r,s}^{(i)} = \sum_s J_1\left(k_{\perp s}^{a(i)} r\right)\left[C_{1s}^{(i)} \exp(-h_s^{(i)}(z-t_i)) - \right.$$
$$\left. - C_{2s}^{(i)} \exp(h_s^{(i)}(z-t_i))\right] \frac{h_s^{(i)}}{k_{\perp s}^{a(i)}}, \tag{2}$$
$$H_{\varphi,s}^{(i)} = \sum_s J_1\left(k_{\perp s}^{a(i)} r\right)\left[C_{1s}^{(i)} \exp(-h_s^{(i)}(z-t_i)) + \right.$$
$$\left. + C_{2s}^{(i)} \exp(h_s^{(i)}(z-t_i))\right] \frac{-i\varepsilon_0 \omega}{k_{\perp s}^{a(i)}}.$$

Tangential components of electric field at the left and right boundaries of the i-th disk aperture are expanded into the series with the complete set of the first order Bessel functions:

$$E_{r+}^{(i)} = \sum_s w_{s+}^{(i)} J_1\left(k_{\perp s}^{a(i)} r\right),$$
$$E_{r-}^{(i)} = \sum_s w_{s-}^{(i)} J_1\left(k_{\perp s}^{a(i)} r\right) \tag{3}$$

where $0 < r < a_i$. Sign (+) refers to the right boundary of the disk and sign (-) – to the left one, correspondingly. As the tangential component of electric field is continuous at the both boundaries of the i-th disk, coefficients C and w are connected via the following equations:

$$C_{1s}^{(i)} = \frac{k_{\perp s}^{a(i)}}{h_s^{(i)}} \frac{w_{s-}^{(i)} \exp(h_s^{(i)} t) - w_{s+}^{(i)}}{2sh(h_s^{(i)} t)},$$
$$C_{2s}^{(i)} = \frac{k_{\perp s}^{a(i)}}{h_s^{(i)}} \frac{w_{s-}^{(i)} \exp(-h_s^{(i)} t) - w_{s+}^{(i)}}{2sh(h_s^{(i)} t)}. \tag{4}$$

Coefficients in expansion (1) are determined by the tangential components of electric field $E_{r+}^{(i)}$ and $E_{r-}^{(i+1)}$:

$$A_{mp}^{(i)}\left(\omega^2 - \omega_{mp}^{(i)2}\right) = \frac{2\pi\omega_{mp}^{(i)}}{iN_{mp}^{(i)}} \times \tag{5}$$
$$\times \left[\int_0^{a_{i+1}} E_{r-}^{(i+1)} H_\varphi^{(i)*}(r, t_{i+1}) r dr - \int_0^{a_i} E_{r+}^{(i)} H_\varphi^{(i)*}(r, z_i) r dr\right]$$

where $N_{mp}^{(i)}$ is the norm of E_{0mp}-mode in the i-th cavity.

By matching tangential component of magnetic field in the both planes of the i-th disk aperture ($0 < r < a_i$), one can write the equations for the coefficients $A_{mp}^{(i-1)}$, $A_{mp}^{(i)}$, $w_{s-}^{(i)}$, $w_{s+}^{(i)}$:

$$\sum_m \sum_p \frac{(-1)^p \omega J_1\left(k_{\perp m}^{b(i-1)} r\right)}{k_{\perp m}^{b(i-1)}} A_{mp}^{(i-1)} =$$
$$= \sum_s \frac{\omega J_1\left(k_{\perp s}^{a(i)} r\right)}{h_s^{(i)}} \frac{w_{s-}^{(i)} ch\left(h_s^{(i)} t\right) - w_{s+}^{(i)}}{sh\left(h_s^{(i)} t\right)}, \tag{6.1}$$

$$\sum_m \sum_p \frac{\omega J_1\left(k_{\perp m}^{b(i)} r\right)}{k_{\perp m}^{b(i)}} A_{mp}^{(i)} =$$
$$= \sum_s \frac{\omega J_1\left(k_{\perp s}^{a(i)} r\right)}{h_s^{(i)}} \frac{w_{s-}^{(i)} - w_{s+}^{(i)} ch\left(h_s^{(i)} t\right)}{sh\left(h_s^{(i)} t\right)}. \tag{6.2}$$

The left and right parts of Eqs.(6) are functions of variable r. Since the functions are equal the expansion coefficients of these functions with the complete set of orthogonal functions $J_1\left(k_{\perp n}^{a(i)} r\right)$ are equal too:

$$\sum_m \sum_p \frac{(-1)^p \theta_m^{(i-1,+)} J_0\left(\theta_m^{(i-1,+)}\right)}{k_{\perp m}^{b(i-1)}\left(\lambda_n^2 - \theta_m^{(i-1,+)2}\right)} A_{mp}^{(i-1)} =$$
$$= \frac{J_1(\lambda_n)}{2h_n^{(i)}} \frac{w_{n-}^{(i)} ch\left(h_n^{(i)} t\right) - w_{n+}^{(i)}}{sh\left(h_n^{(i)} t\right)}, \tag{7.1}$$

$$\sum_m \sum_p \frac{\theta_m^{(i,-)} J_0\left(\theta_m^{(i,-)}\right)}{k_{\perp m}^{b(i)}\left(\lambda_n^2 - \theta_m^{(i,-)2}\right)} A_{mp}^{(i)} =$$
$$= \frac{J_1(\lambda_n)}{2h_n^{(i)}} \frac{w_{n-}^{(i)} - w_{n+}^{(i)} ch\left(h_n^{(i)} t\right)}{sh\left(h_n^{(i)} t\right)} \tag{7.2}$$

with $\theta_m^{(i-1,+)} = k_{\perp m}^{b(i-1)} a_i$, $\theta_m^{(i,-)} = k_{\perp m}^{b(i)} a_i$, $n = 1, 2, \ldots$.

Eqs. (4,5) let us to express all the coefficients A_{mp}, except one, for example, A_{10}, via $w_{s\pm}$. Then Eqs. (7) turn into the following inhomogeneous set of algebraic equations for $w_{s\pm}$:

$$\widetilde{w}_{n+}^{(i)} + 2\sum_s \left(f_n^{(i)}\left[T_{ns}^{(i,-)} \widetilde{w}_{s+}^{(i)} - \widetilde{T}_{ns}^{(i,+)} \widetilde{w}_{s-}^{(i+1)}\right] + \right.$$
$$\left. + \widetilde{f}_n^{(i)}\left[T_{ns}^{(i-1,+)} \widetilde{w}_{s-}^{(i)} - \widetilde{T}_{ns}^{(i-1,-)} \widetilde{w}_{s+}^{(i-1)}\right] \right) = \tag{8.1}$$
$$= 6\pi\left(\widetilde{f}_n^{(i)} \frac{A_{10}^{(i-1)} J_0\left(\theta_1^{i-1,+}\right)}{\lambda_n^2 - \theta_1^{(i-1,+)2}} - f_n^{(i)} \frac{A_{10}^{(i)} J_0\left(\theta_1^{i,-}\right)}{\lambda_n^2 - \theta_1^{(i,-)2}} \right),$$

$$\widetilde{w}_{n-}^{(i)} + 2\sum_s \left(f_n^{(i)}\left[T_{ns}^{(i-1,+)} \widetilde{w}_{s-}^{(i)} - \widetilde{T}_{ns}^{(i-1,-)} \widetilde{w}_{s+}^{(i-1)}\right] + \right.$$
$$\left. + \widetilde{f}_n^{(i)}\left[T_{ns}^{(i,-)} \widetilde{w}_{s+}^{(i)} - \widetilde{T}_{ns}^{(i,+)} \widetilde{w}_{s-}^{(i+1)}\right] \right) = \tag{8.2}$$
$$= 6\pi\left(f_n^{(i)} \frac{A_{10}^{(i-1)} J_0\left(\theta_1^{i-1,+}\right)}{\lambda_n^2 - \theta_1^{(i-1,+)2}} - \widetilde{f}_n^{(i)} \frac{A_{10}^{(i)} J_0\left(\theta_1^{i,-}\right)}{\lambda_n^2 - \theta_1^{(i,-)2}} \right)$$

where

$$T_{ns}^{(i,\pm)} = \frac{\pi a_{i+1,i}}{b_i} \sum_m \frac{\theta_m^{(i,\pm)3} J_0^2\left(\theta_m^{(i,\pm)}\right) E_m^{(i,\pm)} \chi_m^{-1}}{\left(\lambda_n^2 - \theta_m^{(i,\pm)2}\right)\left(\lambda_s^2 - \theta_m^{(i,\pm)2}\right)},$$

$$\widetilde{T}_{ns}^{(i,\pm)} =$$
$$= \frac{\pi a_{i+1,i}}{b_i} \sum_m \frac{\theta_m^{(i,\pm)2} \theta_m^{(i,\mp)} J_0\left(\theta_m^{(i,\pm)}\right) J_0\left(\theta_m^{(i,\mp)}\right) \widetilde{E}_m^{(i,\mp)}}{\chi_m \left(\lambda_n^2 - \theta_m^{(i,\mp)2}\right)\left(\lambda_s^2 - \theta_m^{(i,\pm)2}\right)},$$

$$f_n^{(i)} = h_n^{(i)} a_i ch(h_n^{(i)} t) / sh(h_n^{(i)} t),$$
$$\widetilde{f}_n^{(i)} = h_n^{(i)} a_i / sh(h_n^{(i)} t),$$

$$E_m^{(i,\pm)} = \begin{cases} \dfrac{cth\left(d_i\sqrt{\theta_m^{(i,\pm)2}-\Omega_{i+1,i}^2}\,/\,a_{i+1,i}\right)}{\sqrt{\theta_m^{(i,\pm)2}-\Omega_{i+1,i}^2}}, & m\neq 1\\[2ex] \dfrac{cth\left(d_i\sqrt{\theta_m^{(i,\pm)2}-\Omega_{i+1,i}^2}\,/\,a_{i+1,i}\right)}{\sqrt{\theta_m^{(i,\pm)2}-\Omega_{i+1,i}^2}}-\Delta, & m=1\end{cases},$$

$$\widetilde{E}_m^{(i,\pm)} = \begin{cases} \dfrac{sh^{-1}\left(d_i\sqrt{\theta_m^{(i,\pm)2}-\Omega_{i+1,i}^2}\,/\,a_{i+1,i}\right)}{\sqrt{\theta_m^{(i,\pm)2}-\Omega_{i+1,i}^2}}, & m\neq 1\\[2ex] \dfrac{sh^{-1}\left(d_i\sqrt{\theta_m^{(i,\pm)2}-\Omega_{i+1,i}^2}\,/\,a_{i+1,i}\right)}{\sqrt{\theta_m^{(i,\pm)2}-\Omega_{i+1,i}^2}}-\Delta, & m=1\end{cases},$$

$$\Delta = a_{i+1,i}/d_i\left(\theta_1^{(i,\pm)2}-\Omega_{i+1,i}^2\right),\qquad \Omega_{i+1,i}=a_{i+1,i}\omega/c,$$

$$\chi_m = J_1^2(\lambda_m)\lambda_m\pi/2,\quad \widetilde{w}_{s\pm}^{(i)} = 3\pi J_1(\lambda_s)w_{s\pm}^{(i)}.$$

Since $w_{s\pm}$ are the expansion coefficients of tangential at the disk aperture cross-section component of the electric field, Eqs. (8) are the interaction equations for the fields defined in the circular regions. There are several interesting results that may be obtained from Eqs. (8). One is that the fields of only four circular regions interact directly: at the left and right boundaries of the i-th disk aperture $\widetilde{w}_{s-}^{(i)}$ and $\widetilde{w}_{s+}^{(i)}$, at the right boundary of the (i–1)-th disk aperture $\widetilde{w}_{s+}^{(i-1)}$ and at the left boundary of the (i+1)-th disk aperture $\widetilde{w}_{s-}^{(i+1)}$. It follows from the fact that i–th cavity contacts directly only with two neighboring cavities: (i–1)-th and (i+1)-th. Another important result coming from Eqs. (8) is that the interaction of fields in adjacent apertures is described by the terms which contain factors $\widetilde{T}_{ns}^{(i,+)}$, $\widetilde{T}_{ns}^{(i-1,-)}$. It can be shown that $\widetilde{T}_{ns}^{(i,+)}$, $\widetilde{T}_{ns}^{(i-1,-)} \to 0$ when $a_i \to 0$ and $t = 0$. At the same time factors $T_{ns}^{(i,-)}$ and $T_{ns}^{(i-1,+)}$ in terms, which describe fields interaction at the left and right boundaries of single aperture, tend to constant values independent of a_i when $a_i \to 0$ and $t = 0$.

The solution to the set of linear algebraic equations (8) can be written in the following form:

$$\widetilde{w}_{s\pm}^{(j)} = \sum_{n=-N}^{N}\zeta_{s\pm}^{(j,i+n)}A_{10}^{(i+n)}. \tag{9}$$

The choice of the value of N depends on the fact how many couplings of the i-th cavity with another ones we want to take into account. Index j takes the values i–N+1, i–N+2, ... , i–1, i, i+1, ... , i+ N, correspondingly. For paired couplings (each cavity is coupled only with adjacent ones) $N = 1$. Substituting expression (9) in Eqs. (8) one can obtain inhomogeneous set of linear algebraic equations for $\zeta_{s\pm}^{(j,i+n)}$:

$$\zeta_{n+}^{(j,i+n)} + 2\sum_s\left(f_n^{(i)}\left[T_{ns}^{(i,-)}\zeta_{s+}^{(j,i+n)} - \widetilde{T}_{ns}^{(i,+)}\zeta_{s-}^{(j+1,i+n)}\right] + \widetilde{f}_n^{(i)}\left[T_{ns}^{(i-1,+)}\zeta_{s-}^{(j,i+n)} - \widetilde{T}_{ns}^{(i-1,-)}\zeta_{s+}^{(j-1,i+n)}\right]\right) =$$
$$= 6\pi\left(\widetilde{f}_n^{(i)}\frac{\delta_{i-1,i+n}J_0\left(\theta_1^{i-1,+}\right)}{\lambda_n^2-\theta_1^{(i-1,+)2}} - f_n^{(i)}\frac{\delta_{i,i+n}J_0\left(\theta_1^{i,-}\right)}{\lambda_n^2-\theta_1^{(i,-)2}}\right), \tag{10.1}$$

$$\zeta_{n-}^{(j,i+n)} + 2\sum_s\left(f_n^{(i)}\left[T_{ns}^{(i-1,+)}\zeta_{s-}^{(j,i+n)} - \widetilde{T}_{ns}^{(i-1,-)}\zeta_{s+}^{(j-1,i+n)}\right] + \widetilde{f}_n^{(i)}\left[T_{ns}^{(i,-)}\zeta_{s+}^{(j,i+n)} - \widetilde{T}_{ns}^{(i,+)}\zeta_{s-}^{(j+1,i+n)}\right]\right) =$$
$$= 6\pi\left(f_n^{(i)}\frac{\delta_{i-1,i+n}J_0\left(\theta_1^{i-1,+}\right)}{\lambda_n^2-\theta_1^{(i-1,+)2}} - \widetilde{f}_n^{(i)}\frac{\delta_{i,i+n}J_0\left(\theta_1^{i,-}\right)}{\lambda_n^2-\theta_1^{(i,-)2}}\right) \tag{10.2}$$

where $n = -N \ldots 0 \ldots N$; $j = i$–N+1, i–N+2, ... , i–1, i, i+1, ... , i+ N.

From Eq. (9) one can deduce that the electric field tangential component in the circular regions, through which i-th cavity is connected with other elements of the structure under consideration, are only determined via the E_{010}-mode amplitudes in the cavities. Eq. (5) for E_{010}-mode amplitude in the i-th cavity will have the form:

$$A_{10}^{(i)}\left(\omega^2-\omega_{10}^{(i)2}\right) = \omega_{10}^{(i)2}\sum_{n=-N}^{N}A_{10}^{(i+n)}\times \left(\varepsilon_+^{(i)}\Lambda_{i+}^{(i+1,i+n)} - \varepsilon_-^{(i)}\Lambda_{i-}^{(i,i+n)}\right) \tag{11}$$

where $\varepsilon_\pm^{(i)}$ are the well known coupling coefficients derived on the basis of quasi-static approximation (see, for example, [2]) which are given by

$$\varepsilon_\pm^{(i)} = \frac{2}{3\pi J_1^2(\lambda_1)}\frac{a_{i+1,i}^3}{b_i^2 d_i}. \tag{12}$$

The coupling coefficients Λ have frequency dependence. They are given by

$$\Lambda_{i+}^{(i+1,i+n)} = J_0\left(\theta_1^{(i,+)}\right)\sum_s\frac{\zeta_{s-}^{(i+1,i+n)}}{\lambda_s^2-\theta_1^{(i,+)2}}, \tag{13.1}$$

$$\Lambda_{i-}^{(i,i+n)} = J_0\left(\theta_1^{(i,-)}\right)\sum_s\frac{\zeta_{s+}^{(i,i+n)}}{\lambda_s^2-\theta_1^{(i,-)2}}. \tag{13.2}$$

It is necessary to choice the number of terms in sum on n in Eq. (11) equal the number of equations (10). Thus, the problem of coupled cavities has been rigorously reduced to the problem of the coupling of electric fields which are determined in circular regions.

Eqs. (11) are similar to the equations of equivalent circuit analysis. Only one equation corresponds to each cavity. Existence of infinite number of cavity modes besides E_{010}-mode (non-resonant fields) affects on the form of coupling coefficients Λ. Besides the term which contains factor $A_{10}^{(i)}$ the number of terms in the right-hand side of Eqs. (11) depends on the fact how many couplings of the considered cavity with another ones we want to take into account.

3. CONCLUSIONS

Mathematical model of inhomogeneous chain of cylindrical cavities is developed. This model combines the explicitness of the model of equivalent coupled circuit chain with the possibility to control rigorously the influence of non-resonant fields and "long-range" coupling of cavities on the structure characteristics. Coupling coefficients can be calculated with definite accuracy for the structure with arbitrary parameters. Influence of non-resonant fields and "long-range" coupling on the characteristics of the structure is taken into account by calculation of the coupling coefficients.

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МАТЕМАТИЧЕСКАЯ МОДЕЛЬ НЕОДНОРОДНОЙ ЦЕПОЧКИ СВЯЗАННЫХ ЦИЛИНДРИЧЕСКИХ РЕЗОНАТОРОВ

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Разработана математическая модель для описания неоднородной цепочки связанных цилиндрических резонаторов. Коэффициенты связи в неоднородной цепочке резонаторов могут быть рассчитаны с заданной точностью для структуры с произвольными параметрами. Влияние нерезонансных полей и "дальних" взаимодействий на характеристики структуры учитывается при расчете коэффициентов связи.

МАТЕМАТИЧНА МОДЕЛЬ НЕОДНОРІДНОГО ЛАНЦЮЖКА ЗВ'ЯЗАНИХ ЦИЛІНДРИЧНИХ РЕЗОНАТОРІВ

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Розроблено математичну модель для опису неоднорідного ланцюжка зв'язаних циліндричних резонаторів. Коефіцієнти зв'язку у неоднорідному ланцюжку резонаторів можуть бути розраховані з певною точністю для структури з будь-якими параметрами. Вплив нерезонансних полів та "далеких" взаємодій на характеристики структури враховується при розрахуванні коефіцієнтів зв'язку.