INVESTIGATION OF NUCLEAR CHRONOMETER TIME DECAY CHARACTERISTICS FOR REVISION OF AGE OF ASTROPHYSICS OBJECTS

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The new algorithm for the determinations of the characteristics functions of the energy distribution, decay probability, decay intensities and life times of the excited levels is proposed. Quantum-mechanical motivation of the needing of the time characteristics revising for the nuclear-chronometers it is given. Calculations for concrete decay events of the nuclei ^{238}U, ^{232}Th, ^{235}U at room and stars temperatures, with and without account of the Doppler effect were conducted.

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1.INTRODUCTION

Since the statistic low of the radioactive decay it is considered absolutely exact, work of the "nuclear clock", which are used for measurement gap time in the geologies, archeologies, astrophysicist, is founded on its principle.

Main principle of the technology nuclear chronometers consists in correlations measurement of the mother and daughter nuclear in the greater volume of the matter. Such principle is founded on tacit suggestion that velocity of the radioactive decay is constant and does not depend on physical and chemical states of the ambience, in which are found radioactive nuclei. However it was currently realized, that thereof rules in some cases there are exceptions.

So, change of the chemical state of the decay atom and the thermodynamic influences bring to the observable change of the electron seizure velocity and internal conversion. And in unusual states of the strong ionizing, for instance, in depths of the stars, more strong effects of the velocities increase can exist even for α- and β-decay. This is necessary to take into account in radioactive chronology.

Besides, up to recently time, in all known method nucleus chronometry took into consideration only life time of the main levels of the decay nuclear. But duration of the nucleus radiation processes seizes not only main, but also many excited states of the synthesized nuclear, which were formed. Account these factor can greatly change the final results estimation for all time interval, characterizing evolution decay chain, and updated factors of "nuclear clock" can correspond to greatly smaller values of the real processes duration for the nuclear-chronometers decay. Signifies that the "age" of object in which occurs decay should be smaller [1].

2. DECAY EVOLUTIONS

Purpose given work was a development of the algorithm for the account of the decay of the necessary amount excited states of the radioactive nuclei and their heat motion.

For description of the decay evolution and determinations its time characteristics (probability, intensities, life time) in these work quantum-mechanical approach, founded on Krylov-Fock theorem [2], generalized for the mixed states (when decay process of the ensemble of the particles simultaneously goes with its formation by the nucleus syntheses or decay of the previous state) [3] is used. Such approach allows taking into account mentioned above factors.

For simplification of the analysis limit ideal event of the long life α-active nuclear, which in determined initial time moment (t = 0) portioned in two states (main and first excited).

Then, in accordance with Krylov-Fock theorem decay functions L(t) and $L_0(t)$, characterizing decay of certain initial (first excited) and following (main) states accordingly, are presented in the manner of

$$L(t) = |p(t)|^2 / |p(0)|^2 , \tag{1}$$

$$L_0(t) = |p_0(t)|^2 / |p_0(0)|^2 , \tag{2}$$

were

$$p_0(t) = \int_0^\infty |G_0(\varepsilon)|^2 \exp(-i\varepsilon t/\hbar) d\varepsilon , \tag{3}$$

$$p(t) = \int_0^\infty |G(\varepsilon)|^2 \exp(-i\varepsilon t/\hbar) d\varepsilon \; - \tag{4}$$

characteristics functions of the energy distribution energy at the main and first states. Marking width of the main and first excited levels of the α-decay by Γ_α^0 and Γ_α^1 accordingly, and width of the γ decay by Γ_γ, the full width of the main and the first excited state are presented by

$\Gamma_0 = \Gamma_\alpha^0$; $\Gamma_1 = \Gamma_\alpha^1 + \Gamma_\gamma$.

Then characteristics functions for the main and first excited states will be determined by appropriation energy spectrum of a distributions

$$|G_0(\varepsilon)|^2 = const\left[(\varepsilon_1 - \varepsilon)^2 + \Gamma_1^2/4\right]^{-1} \times$$
$$\times \left[(\varepsilon_0 - \varepsilon)^2 + \Gamma_0^2/4\right]^{-1}, \tag{5}$$

$$|G(\varepsilon)|^2 = const \times \left[(\varepsilon_1 - \varepsilon)^2 + \Gamma_1^2/4\right]^{-1} . \tag{6}$$

Here ε_0, ε_1 и ε - energy of the main and first excited states and actual system energy, which consists of the internal motion energy of the mother nuclear and kinetic

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energy of the heat motion. Probability of the decay will be defined by

$$W_n(t) = 1 - L_n(t)\,, \qquad (7)$$

and velocities of the decay with provision of the each channel contribution -

$$\rho_n(t) = \Gamma_i^n / \Gamma_n \, dW_n(t)/dt\,,$$

were $n = 0{,}1$. (8)

Intensity of the decay it is possible to define as

$$I(t) = \int_0^t dt' \rho_0(t)\rho_1(t - t') \qquad (9)$$

For determination of the radioactive nuclear life time and period of its half-life $T_{1/2}$, it is possible to use known formulas

$$\langle \tau \rangle = \int tI(t)dt \Big/ \int I(t)dt\,,\ T_{1/2} = \tau ln2\,. \qquad (10)$$

So, offered approach allows taking into account decay not only main, but also previous state of the nuclear. Such decay can occur on two channels. As can be seen from energy distribution (5) and (6), decay of the first exited state has purely exponential nature. Decay of the main state already has not an exponential nature since it is assigned by multiplying of the two exponents. This allows expecting that account of the decay each previous exited level in decay chains will contribute certain additional distortion in the exponential nature of resulting energy distribution. So, final estimations of the time intervals can be changed.

3. DESCRIPTION OF THE DATA

First this approach (for the approbation reason) was used for theoretical description of the γ-quantum time distribution, which is resonance scattering by nuclei ^{57}Fe at Messbauer experiments [4].

Since according to scheme of such experiment [4], the decay process of the scattering with excitation for nuclear ^{57}Fe goes simultaneously with their resonance excitement by the γ-quantum, which flies from source, for description of the decay evolutions we use generalized Krylov-Fock theorem (1) – (4), where

$$|G_0(\varepsilon)|^2 = const\left[(\varepsilon - \varepsilon_0)^2 + \Gamma^2/4\right]^{-1}$$

is spectrum of the excitement energy distribution at the nuclear of a source, and

$$|G(\varepsilon)|^2 = const\left[(\varepsilon - \varepsilon_\gamma)^2 + \Gamma^2/4\right]^{-1} \times$$
$$\times \left[(\varepsilon - \Delta\varepsilon - \varepsilon_\gamma)^2 + \Gamma^2/4\right]^{-1}$$

is spectrum of the excitement energy distribution at the ^{57}Fe nuclear of the scattering matter; the energy shifting $\Delta\varepsilon$ is adjusted by the velocities driftage device of the source; $\Gamma = \hbar/\tau$.

Calculating $p(t)$ and $p_0(t)$ on formulas (3) and (4) by means of theorems about deduction, we shall get in accordance with (9)

$$I(t) = const\{\exp(-t/\tau)\}(t/\tau +$$
$$+ (\Gamma/\Delta\varepsilon)\left[\sin(t\Delta\varepsilon/h + \varphi) - \sin\varphi\right]$$
$$\{-\cos(t\Delta\varepsilon/\hbar + \varphi) + \cos\varphi\}, \qquad (9a)$$

where

$$\varphi = -arctg(\Gamma/2\Delta\varepsilon)\,.$$

Fig. 1 express results of the calculations by the formula (9a), with $\Delta\varepsilon = 3\Gamma$ (curves 1), which are normalized on upper points of the experimental data in comparison with results of the theoretical calculation [4] (curves 2,3), which are executed within the framework of classical theory of the electromagnetic radiation interaction with a matter. As it is seen, results of our calculation well agree with experiment that is indicative about the correctness using of such approach for determination of the time decay characteristics.

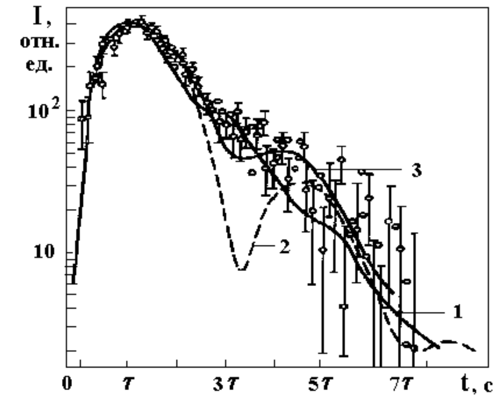

Fig. 1. *Time distribution of the γ-quantum, resonance scattered by ^{57}Fe nuclear ($\Delta\varepsilon = 3\Gamma$)*

4. DOPPLER EXPANSION

As is well known, Messbauer effect occurs without Doppler expansion of the lines, but under usual decay of the excited nuclear levels within matter with a very high temperature (for instance, in depths of the stars), Doppler effect can play observable role. So, for more strict description of the decay evolutions it is necessary to take into account the Maxwell function of the distribution on energy

$$f(\varepsilon,\varepsilon_\gamma)d\varepsilon = -\frac{1}{\sqrt{\pi}D}\exp(-\frac{(\varepsilon - \varepsilon_r)^2}{D^2})d\varepsilon\,,$$

where $\varepsilon_r = \varepsilon_\gamma^2 / 2\mu c^2$ - kinetic energy of the return of the nuclear after the releases or the absorptions of the γ-quantum, ε_γ - γ-quantum energy; $D = 2\sqrt{\varepsilon_r kT}$ is the Doppler width for the resonance release or absorption. Then real functions of the g-quantum energy distribution with provision of the Doppler expansion we shall present in the manner of

$$|G_n(\varepsilon)|^2{}_{dop} = \int_0^{+\infty} |G_n(\varepsilon)|^2 \int_0^{\infty} \left(f(\varepsilon,\varepsilon_\gamma)d\varepsilon_\gamma\right)d\varepsilon$$

where $\varepsilon_\gamma = \varepsilon - \varepsilon_0 - \varepsilon_r + D$, n = 0,1. As a result for $G_0(\varepsilon)$ and $G_1(\varepsilon)$ shall get the following expressions:

$$\left|G_0(\varepsilon)\right|^2_{dop} = \frac{const}{\sqrt{\pi}D}\left\{\int_0^{\bar{\varepsilon}} \frac{e^{-b^2(\varepsilon-\varepsilon_r)^2}d\varepsilon}{((\varepsilon_1-\varepsilon)^2+a_1^2)((\varepsilon_0-\varepsilon)^2+a_0^2)} - \right.$$

$$\left. - \int_0^{\bar{\varepsilon}} \frac{e^{-b^2\varepsilon^2}d\varepsilon}{(\varepsilon^2+a_1^2)(\varepsilon^2+a_0^2)} + \frac{\pi}{2(a_0^2-a_1^2)}\left[\frac{e^{b^2a_1^2}}{a_1} - \frac{e^{b^2a_0^2}}{a_0}\right]\right\},$$

$$\left|G_1(\varepsilon)\right|^2_{dop} = \frac{const}{\sqrt{\pi}D}\left\{\int_{-\infty}^{\bar{\varepsilon}} \frac{e^{-b^2(\varepsilon'+\varepsilon_1-\varepsilon_r)^2}d\varepsilon'}{\varepsilon'^2+a^2} - \int_0^{\bar{\varepsilon}} \frac{e^{-b^2\varepsilon'^2}d\varepsilon'}{\varepsilon'^2+a^2} + \frac{e^{b^2a^2}}{2a}\right\}.$$

The offered method possible to generalize on event greater amount excited states. For this matter function $G_n(e)$ shall present as $\left|G_n(\varepsilon)\right|^2 = const\prod_{i=1}^{N}\left[(\varepsilon_i-\varepsilon)^2+\Gamma_i^2/4\right]^{-1}$, where $i \in [1,n]$, i is considered level, and similar image we shall average on energy. As a result of multiple transformations we shall get following expression:

$$\left|G_n(\varepsilon)\right|^2_{dop} = \frac{const}{\sqrt{\pi}D}\left\{\int_0^{\bar{\varepsilon}} \frac{e^{-b^2(\varepsilon-\varepsilon_r)^2}d\varepsilon}{\prod\limits_{i=0}^{N}((\varepsilon_i-\varepsilon)^2+a_i^2)} - \int_0^{\bar{\varepsilon}} \frac{e^{-b^2\varepsilon^2}d\varepsilon}{\prod\limits_{i=0}^{N}(\varepsilon^2+a_i^2)} + \frac{\pi}{2}\sum_{i=1}^{N}\frac{e^{b^2a_i^2}}{a_i\prod\limits_{\substack{i=0,\\ j\neq i}}^{N}(a_i^2-a_j^2)}\right\}\times$$

$$\times\int_0^{\bar{\varepsilon}} \frac{e^{-b^2\varepsilon^2}d\varepsilon}{\prod\limits_{i=0}^{N}(\varepsilon^2+a_i^2)} + \frac{\pi}{2}\sum_{i=1}^{N}\frac{e^{b^2a_i^2}}{a_i\prod\limits_{\substack{i=0,\\ j\neq i}}^{N}(a_i^2-a_j^2)}.$$

Here $\Gamma_1^2/4 = a^2$, $1/D^2 = b^2$, $\Gamma_0^2/4 = a_0^2$, $\varepsilon' = \varepsilon - \varepsilon_1$ - changes at calculation integral; a_0, a_1 and a are special points ($\varepsilon_0 = ia_0$; $\varepsilon_1 = ia_1$; $\varepsilon_{i0} = ia_i$).

As it is seen, the generalized Krylov-Fock theorem allows to take into account not only Doppler effect, but also necessities amount of the excited states of the radioactive nuclear, appearing in nuclear-syntheses process, that, certainly, must positively influence upon accuracy of the estimation of the intensities and decay velocities of the radioactive nuclear-chronometers, and, signifies, upon estimation of the different objects age by the methods of nuclear chronometry.

5. RESULTS OF THE CALCULATIONS

To realize, what influence renders Doppler effect on the main features of decay, they were organized corresponding calculations under room and under stars temperatures for the radioactive nuclear ^{238}U, ^{235}U and ^{232}Th, which are broadly used in large-scale nucleus chronometry for the dating of the astrophysical objects age. For simplification of the calculation was considered event of the consequent decay only from the first excited and the main states. Calculations were conducted by numerical methods and have shown that in the case of, when Doppler effect was not taken into account, the graphics of the intensities of the decay nuclei have maxima in pointes, corresponding to the table values of the given level life-time. This is indicative of that the offered method is correct. As example on Fig. 2 is brought graphic of the decay intensities dependencies from the time for the first excited level of the ^{238}U nuclear disregarding of the Doppler effect ($\tau_{eksp} = 2{,}93\cdot10^{-10}$c, $\tau_{teor} = 2{,}92\cdot10^{-10}$s).

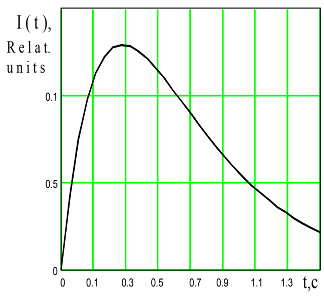

***Fig. 2.** Decay intensities vs time for the excited nuclear ^{238}U disregarding Doppler effect*

Calculations conducted with provision of the Doppler effect have demonstrated that under room temperature it practically does not render influences on the decay velocity and life time of the excited level ($\tau_{eksp} \approx \tau_{teor}$). But under stars temperature ($T = 3\cdot10^8$), as was expected, probability and intensity of decay noticeably increased and, accordingly, decreased the life time. As example on Fig. 3 is brought the graphic of the decay intensities dependencies from the time for the first excited level for the ^{238}U nuclear with provision of the Doppler effect under stars temperature. In this case, $\tau_{teor} = 4{,}6\cdot10^{-11}$ s, when turning to the stars temperatures, the life time of the first excited level of the ^{238}U decreased approximately in 6,4 times.

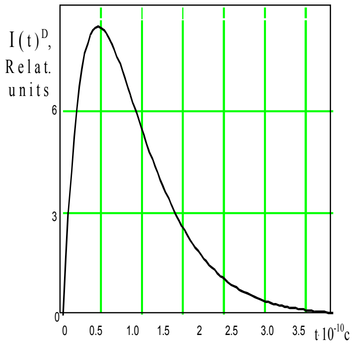

Fig. 3. *Decay intensities vs time for the excited nuclear ^{238}U with accounted of the Doppler effect*

The similar calculations of the life time for the first excited level ^{235}U and ^{232}Th, called on for with provision of the Doppler effect, have demonstrated its reduction under stars temperature in compare with experimental data in 5,7 times for ^{235}U and in 7,2 times for ^{232}Th. Follows to expect that account of the greater number of the excited states will give else greater reduction of the life time.

Given work is a first stage of the complex quantum theoretical research of the decay time features. Analytical expressions are received in this work for the time characteristics of decay for the long life α-active nuclear, which in determined initial time moment portioned in two states (main and first excited) with provision of the Doppler effect. Generalization of the getting expressions is made on event greater amount of the excited states. Calculations called on for concrete nuclear under room and stars temperatures, have demonstrated significant increase of the decay velocities under stars temperatures and possibility of the observable speedup of decay because of presence of the excited states.

Designed method allows at decay processes to take into account Doppler effect and necessities amount of the excited states. Such method conducts corresponding calculations for the different nuclear and for the different temperature, that permits its practical application in nucleus chronometry, for instance for more exact determination of the astrophysical objects age and decay time of the nuclear waste.

6. CONCLUSIONS

Modern methods of the nucleus chronometry founded on classical belief about constancy of the decay velocities and not taking into account influences upon decay time features of the different factors (in particular, the heat motion of the nuclear and decay of the previous states), give only possible upper estimation limites of the objects age. So such estimations must be revised within the framework of more general quantum theory.

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ИССЛЕДОВАНИЕ ВРЕМЕННЫХ ХАРАКТЕРИСТИК РАСПАДА ЯДЕР-ХРОНОМЕТРОВ С ЦЕЛЬЮ УТОЧНЕНИЯ ВОЗРАСТА АСТРОФИЗИЧЕСКИХ ОБЪЕКТОВ

Н.Л. Дорошко, М.Э. Долинская

Предложен новый алгоритм для определения характеристических функций распределения по энергии, вероятности распада, интенсивности распада и времени жизни возбужденных состояний уровней. Дано квантовомеханическое обоснование необходимости пересмотра временных характеристик ядер-хронометров. Проведены расчеты для конкретных случаев распада ядер ^{238}U, ^{232}Th, ^{235}U при комнатных и звездных температурах с учетом эффекта Доплера и при его отсутствии.

ДОСЛІДЖЕННЯ ЧАСОВИХ ХАРАКТЕРИСТИК РОЗПАДУ ЯДЕР-ХРОНОМЕТРІВ З МЕТОЮ УТОЧНЕННЯ ВІКУ АСТРОФІЗИЧНИХ ОБ'ЄКТІВ

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Запропоновано новий алгоритм для визначення характеристичних функцій розподілу по енергії, ймовірностей розпаду, інтенсивності розпаду і часу життя збуджених станів рівнів. Зроблено квантовомеханічне обгрунтування необхідності перегляду часових характеристик ядер-хронометрів. Проведені розрахунки для конкретных випадків розпаду ядер ^{238}U, ^{232}Th, ^{235}U при кімнатних і зоряних температурах з урахуванням ефекту Доплера та за його відсутності.