ON THE INCOHERENT RADIATION OF RELATIVISTIC ELECTRONS AND POSITRONS IN CRYSTAL

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The incoherent bremsstrahlung of high energy electrons in crystal is caused by thermal spread of atoms from their equilibrium positions in the lattice. In the present article the simulation procedure for the intensity of incoherent radiation based on the quasi-classical formulae of the bremsstrahlung theory is developed. Substantial orientation dependence of the intensity of hard incoherent radiation is demonstrated under the angles of incidence of the particles to the crystallographic plane, close to the critical angle of planar channeling. The dechanneling effect leads to decrease of that orientation dependence. The results of simulation are in a good agreement with the experimental data.

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1. INTRODUCTION

Under motion of fast charged particles in a crystal along one of crystallographic axes or planes the channeling phenomenon is possible, when the particles move in channels formed by atomic strings or planes in the crystal (see, e.g., [1-3]). The redistribution of the particle flux in the crystal takes place under the channeling. Due to this fact both increase and decrease of yields of the processes connected with small impact parameters are possible. This is connected to the fact that positively charged channeling particles couldn't come to small distances to the positively charged atomic nuclei in the lattice, so such particles would collide with the atomic nuclei in the crystal more rare than under absence of the channeling. For negatively charged particles the inverse effect takes place.

Under disorientation of the crystal on small angle to the incident beam the particles performing the above-barrier motion in relation to the atomic strings or planes in the crystal appears together with the channeled particles [3-5]. For the above-barrier particles some redistribution of the particle flux in the crystal also takes place. However, this effect has substantially different form than the redistribution of the flux of channeled particles. Namely, under above-barrier motion near atomic planes in the crystal the positively charged particles "hang" for a relatively long time in the region of atomic nuclei location in the crystallographic planes, whereas negatively charged particles fly across this region faster than positively charged ones. At last, if the angle of the crystal disorientation substantially exceeds the critical angle of channeling the effect of redistribution of the particle flux in the crystal is absent [3].

So, in the case of disorientation of the crystal by the angles of order of the critical channeling angle substantial orientation dependence of yields of the processes connected to small impact parameters must take place. Such orientation dependences have been observed earlier for the nuclear reactions yields, δ-electron yield, and a number of other processes (see, e.g., [2,3,6,7]).

The present paper is devoted to the analysis of orientation dependence of the yield of incoherent radiation of relativistic electrons and positrons under motion of the particles in the crystal near one of crystallographic planes. The simulation procedure for this process based on the model in which the atomic planes in the crystal are treated as rows of atomic strings is presented. In the frames of this model both the coherent scattering of the particles on the uniform atomic string potentials and the incoherent scattering on the thermal vibrations of the lattice atoms are taken into account. So, our procedure permits to take into account the effects of dechanneling and rechanneling of the particles, that makes it applicable to the targets of arbitrary thickness. The results of simulation are compared to the experimental data on the orientation dependence of the radiation yield in the range of photon energies close to the energy of the radiating electron, obtained earlier in KIPT [7].

2. SIMULATION OF THE INCOHERENT RADIATION

Radiation of relativistic electron in matter develops in a large spatial region along the particle's momentum called as the coherence length [3,8]. If the electron collides with a large number of crystal atoms in the coherence length, the effective constant of the interaction of the electron with the lattice atoms may be large in comparison with the unit, so we could use the semiclassical description of the radiation process. In thedipole approximation the spectral density of bremsstrahlung is described by the formula [3]

$$\frac{d\mathrm{E}}{d\omega} = \frac{e^2\omega}{2\pi c^4}\int_{\delta}^{\infty}\frac{dq}{q^2}\left[1+\frac{(\omega)^2}{2\varepsilon\varepsilon'}-2\frac{\delta}{q}\left(1-\frac{\delta}{q}\right)\right]\left|\vec{W}_q\right|^2, \quad (1)$$

where $q = \frac{\varepsilon}{\varepsilon'}\left(\omega - \vec{k}\vec{v}\right)$, k is the wave vector of the radiated photon, ε is the energy of the initial electron, v is its velocity, $\varepsilon' = \varepsilon - \omega$, $\vec{W}_q = \int_{-\infty}^{\infty}\vec{v}_{\perp}(t)e^{icqt}dt$ is the Fourier component of the electron acceleration in the direction orthogonal to v, $\delta = \frac{m^2c^3\omega}{2\varepsilon\varepsilon'}$. Particularly, for the case of radiation of the electron in the field of single atom (using the screened Coulomb potential $U(r) = Ze\frac{e^{-r/R}}{r}$, as the potential of the atom, where Z is the atomic number, R is Thomas-Fermi radius) we have:

$$W_q^{(1)}(\rho_0) = \\ = \frac{2Ze^2c}{\varepsilon}\sqrt{q^2+R^{-2}}K_1\left(\rho_0\sqrt{q^2+R^{-2}}\right)\frac{\rho_0}{\rho_0}, \quad (2)$$

where $K_1(x)$ is the modified Bessel function of the third kind, ρ_0 is the impact parameter. Since the characteristic values of q making the main contribution to the integral (1) are $q \sim \delta << R^{-1}$, we can take $q = 0$ in (2):

$$\vec{W}^{(1)}(\vec{\rho}_0) = \frac{2Ze^2c}{\varepsilon R}K_1\left(\frac{\rho_0}{R}\right)\frac{\rho_0}{\rho_0}. \quad (3)$$

Integrating over q, and after that over the impact parameter, we obtain with logarithmic accuracy the Bethe-Heitler result for radiation efficiency by the unit particle flux in the field of the atom:

$$\omega\frac{d\sigma_{BH}}{d\omega} = \int\frac{d\mathrm{E}}{d\omega}d^2\rho_0 = \quad (4)$$

$$= \int\frac{d\mathrm{E}}{d\omega}d^2\rho_0 = \frac{16}{3}\frac{Z^2e^6}{m^2c^2}\left(1+\frac{3}{4}\frac{(\omega)^2}{\varepsilon\varepsilon'}\right)\ln\left(\frac{mRc}{}\right).$$

Note that the integral over the impact parameter diverges at small values of ρ_0. The divergence results from the use of the dipole approximation, which is valid at $\rho_0 \geq /mc$. We take this constraint into account by introducing the lower limit of integration ($\rho_{\min} = /mc$ that is the Compton wavelength of the electron), so this result is obtained with logarithmic accuracy.

Consider now the radiation of the electron interacting with the crystal that is the system of atoms periodically arranged in space. The case of our interest is the electron incidence onto the crystal under small angle ψ to one of its crystallographic axes (the z axis). It is known [3] that averaging of the equation for the $\left|\vec{W}_q\right|^2$ over the thermal vibrations of atoms in the lattice leads to the split of this value (and so the radiation intensity) into the sum of two terms describing coherent and incoherent effects in radiation:

$$\left\langle\left|\vec{W}_q\right|^2\right\rangle = \quad (5)$$

$$= \sum_{n,m}e^{iqc(t_n-t_m)}\left\langle W_q^{(1)}(\rho_n+\vec{u}_n)\right\rangle\left\langle W_q^{(1)}(\rho_m+\vec{u}_m)\right\rangle + \\ + \sum_n\left\{\left\langle\left|W_q^{(1)}(\vec{\rho}_n+\vec{u}_n)\right|^2\right\rangle - \left|\left\langle W_q^{(1)}(\vec{\rho}_n+\vec{u}_n)\right\rangle\right|^2\right\},$$

where the indexes n and m numerate the atoms under collisions, t_n is the time moment when the electron collides with the n-th atom, $\rho_n = \rho(t_n) - \rho_n^0$ is the impact parameter of the collision with the n-th atom in its equilibrium position ρ_n^0, $\rho(t)$ is the trajectory of the electron in the plane orthogonal to the z axis, and u_n is the thermal shift of the n-th atom from the position of equilibrium.

Consider the situation when the electron moves under small angle θ to one of the crystallographic planes densely packed with atoms (the (x, z) plane). Assume also that the motion takes place under small angle ψ to the z axis, namely $\theta << \psi << 1$. In the range of frequencies

$$\omega >> 2\varepsilon\varepsilon'\theta/m^2c^3a_y, \quad (6)$$

where a_y is the distance between such atomic planes, the incoherent component of radiation is predominant [3]. Like in the case of the radiation on the single atom, we take $q = 0$ and then, substituting the formula for $W^{(1)}$ from (3), we find the following expression for the incoherent part of the quantity of interest:

$$\left|\vec{W}_{incoh}\right|^2 = \frac{4Z^2e^4c^2}{\varepsilon^2R^2}\times \quad (7)$$

$$\times\sum_n\left\{\left\langle\left(K_1\left(\frac{|\vec{\rho}_n+\vec{u}_n|}{R}\right)\right)^2\right\rangle - \right.$$

$$\left. - \left\langle\frac{\vec{\rho}_n+\vec{u}_n}{|\vec{\rho}_n+\vec{u}_n|}K_1\left(\frac{|\vec{\rho}_n+\vec{u}_n|}{R}\right)\right\rangle^2\right\}.$$

It is convenient to compare the efficiency of the incoherent radiation in the crystal with the radiation efficiency in amorphous medium (with equal numbers of collisions with atoms in both cases). The ratio of these two values is equal to

$$\frac{\int d^2\rho_0 \left(\frac{dE}{d\omega}\right)_{incoh}}{N\Delta\omega \frac{d\sigma_{BH}}{d\omega}} = \frac{1}{2\pi N R^2 \ln\left(\frac{mRc}{\hbar}\right)} \times$$

$$\times \int d^2\rho_0 \sum_n \left\{ \left\langle \left(K_1\left(\frac{|\vec{\rho}_n + \vec{u}_n|}{R} \right) \right)^2 \right\rangle - \right. \qquad (8)$$

$$\left. - \left\langle \frac{\vec{\rho}_n + \vec{u}_n}{|\vec{\rho}_n + \vec{u}_n|} K_1\left(\frac{|\vec{\rho}_n + \vec{u}_n|}{R} \right) \right\rangle^2 \right\},$$

where N is the whole number of atoms with which the electron collides under motion through the crystal, integration over $d^2\rho_0$ means the integration over all possible points of incidence of the beam onto the crystal surface. This integration can be effectively reduced to the integration over one elementary cell in the plane (x,y).

Under computing the value

$$F(\vec{\rho}) = \left\langle \left(K_1\left(\frac{|\vec{\rho} + \vec{u}|}{R} \right) \right)^2 \right\rangle - \left\langle \frac{\vec{\rho} + \vec{u}}{|\vec{\rho} + \vec{u}|} K_1\left(\frac{|\vec{\rho} + \vec{u}|}{R} \right) \right\rangle^2$$

we substitute $\rho = \rho_{\min}$ when $\rho < \rho_{\min}$ to take into account the constraint under small distances, like in the case of a single atom. Averaging over thermal vibrations is made by integration with Gaussian distribution. This integration could be carried out only numerically, so the values of the function $F(\rho)$ in (8) are determined by interpolation of the results of numerical integration for the finite set of the values ρ. The impact parameters of the collisions with atoms ρ_n are determined using the simulated trajectory of the electron in the crystal (see below). Integration over $d^2\rho_0$ can be carried out using Monte-Carlo techniques.

3. SIMULATION OF THE PARTICLE TRAJECTORY IN THE CRYSTAL

Consider the motion of the electron in the crystal under small angle ψ to one of crystallographic axes densely packed with atoms. Let this angle is small enough to validity of the approximation of the uniform potential of the atomic string, but much larger than the critical angle of axial channeling. In this case the problem of motion of the electron in the crystal is reduced to the two-dimensional problem of motion in the (x, y) plane, which is perpendicular to the axes of atomic strings. The electron's trajectory $\rho(t)$ is determined by the equation of motion [1-3]

$$\ddot{\vec{\rho}} = -\frac{c^2}{\varepsilon} \frac{\partial}{\partial \vec{\rho}} \sum_s U_R(\vec{\rho} - \vec{\rho}_s^{\,0}),$$

where $U_R(\rho - \rho_s^0)$ is the potential energy of the electron interaction with the uniform potential of the atomic string parallel to the z axis, ρ_s^0 is the position of the string in the (x, y) plane.

The potential of the atomic string decreases rapidly with distance increase, so the curvature of the electron trajectory takes the place mainly on a small, least distant to the string axis part of it. So we can approximate the real trajectory of the electron by the broken line, the angles between parts of which in the (x, y) plane are equal to the angles of scattering (deflection) of the particle in the field of corresponding strings:

$$\Delta\varphi(b) = \pi - 2b \int_{\rho_{\min}}^{\infty} \frac{d\rho / \rho^2}{\sqrt{1 - \frac{U_R(\rho)}{\varepsilon_\perp} - \frac{b^2}{\rho^2}}}, \qquad (9)$$

where b is the impact parameter of the collision with the string, $\varepsilon_\perp = \varepsilon\psi^2/2$, $\rho_{\min}$ is the minimal distance between the electron and the axis of the atomic string (see [3,4]). During the simulation process the successive steps from one collision with the string to the next one are made. On every such step the parameters of the current rectilinear part of the trajectory are used in looking for the coordinates of the next string under collision and the impact parameter of that collision. Realization of this algorithm using MathCAD package is published on the address [9].

Thermal spread of the atoms from their positions of equilibrium in crystal leads to arising the incoherent scattering of the electrons on the thermal vibrations of atoms together with the coherent scattering on the uniform potentials of the atomic strings (see, e.g., [3], §57).

The scattering angle of the electron is given by the formula, which is distinct from Eq. (3) for the value $W^{(1)}$ only by the coefficient c:

$$\vec{\vartheta}^{(1)}(\vec{\rho}_0) = \frac{2Ze^2}{\varepsilon R} K_1\left(\frac{\rho_0}{R}\right) \frac{\rho_0}{\rho_0}.$$

After averaging the squared absolute value of the sum of scattering angles on all atoms of the string over thermal vibrations of atoms, we obtain the formula analogous to (5), and find that the mean squared value of the incoherent scattering angle $\langle \vartheta^2 \rangle_{incoh}$ is described by the formula coinciding to (7) with additional factor c^{-2}.

In the simulation procedure for the electron's trajectory the incoherent scattering is taken into account by addition the random value with Gaussian distribution with the dispersion equal to $\sqrt{\langle \vartheta^2 \rangle_{incoh}}/\psi$ to the azimuth value of the coherent scattering angle (9). Such model is more realistic than [9]; it permits to take into account the dechanneling phenomenon, when the particles moving in the channels formed by atomic planes of the crystal, could leave them because of the scattering on thermal vibrations of the atoms.

4. ORIENTATION DEPENDENCE OF THE INCOHERENT RADIATION OF FAST PARTICLES IN CRYSTAL

Specific parameters of the simulation are chosen in correspondence to conditions of the experiment [7]. In that experiment the photons of energy $\hbar\omega$ = 1.1 GeV emitted by electrons of energy ε = 1.2 GeV incident on the silicon (Z = 14, R = 0,194 Å) crystal under small angle θ to the plane (110) (the (x, z) plane) had registered. Under these conditions the angle of electron incidence to the <001> axis (the z axis) had chosen large enough to ensure the absence of axial channeling:

$$\psi \sim 100\psi_c,$$

where the critical angle of the axial channeling

$$\psi_c = \sqrt{\frac{4Ze^2}{\varepsilon\, a_z}} \approx 3{,}5 \cdot 10^{-4} \text{ rad}$$

(the lattice constant along the <001> axis in silicon crystal is a_z = 5,431 Å). The experimental data (see Fig. 2 in [7]) and the results of simulation are presented on Fig. 1. We see rather good agreement between them.

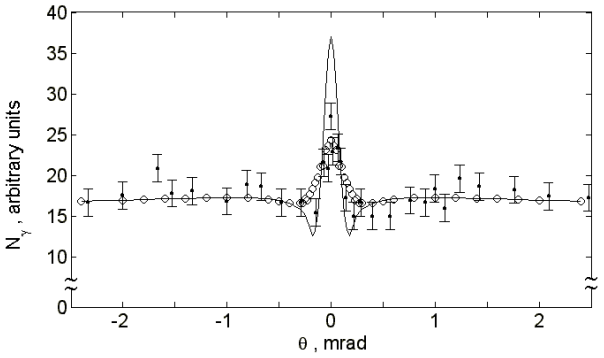

Fig. 1. *Photon yields with the energy $\hbar\omega$ = 1.1 GeV vs the incidence angle of electrons with the energy ε = 1.2 GeV to the (110) plane of a 30 μm thick silicon monocrystal [7] (error bars) and the intensity of incoherent radiation of electrons as a result of the simulation with (circles) and without (solid line) account of the incoherent scattering of the electrons on the thermal vibrations of the lattice atoms*

The character of the orientation dependence of the incoherent radiation is determined by the specialties of the particle's dynamics in the crystal. This fact could be illustrated under comparison of the orientation dependencies of the incoherent radiation by electrons and positrons under the same conditions (see Fig. 2, where we have not take into account the incoherent scattering of the particles on thermal vibrations of atoms for more demonstrability). For the θ values close to zero the planar channeling takes place for the most part of points of incidence of the particle onto the crystal. The electron (negatively charged particle) under planar channeling spends the most part of the time of its motion through the crystal in close vicinity to atomic plane, with small impact parameters of collisions with atoms that leads to the maximum in the efficiency of incoherent radiation. On the other side, the positron under planar channeling spends the most part of the time far from atomic planes that leads to the minimum in the incoherent radiation intensity.

In the case of θ values close to the critical angle of planar channeling θ_c (for the case under discussion $\theta_c \approx 2 \cdot 10^{-4}$ rad) the above-barrier positrons spend the most part of the time in close vicinity to atomic planes that leads to maxima in the incoherent radiation efficiency. Oppositely, the above-barrier electrons rapidly move through atomic planes that leads to minima in the incoherent radiation intensity.

Under $\theta >> \theta_c$ the energy of transverse motion of the particle $\varepsilon_\perp = \varepsilon\theta^2/2$ [3] exceeds by far the height of the potential barrier formed by uniform potential of the atomic plane in the crystal. In this case the trajectory of the particle is almost rectilinear. For such trajectory all possible impact parameters of collisions with atoms are almost equiprobable, like in amorphous medium, and the incoherent radiation efficiency become equal to that in amorphous medium (in accuracy to Debye-Waller factor) and independent on the crystal orientation.

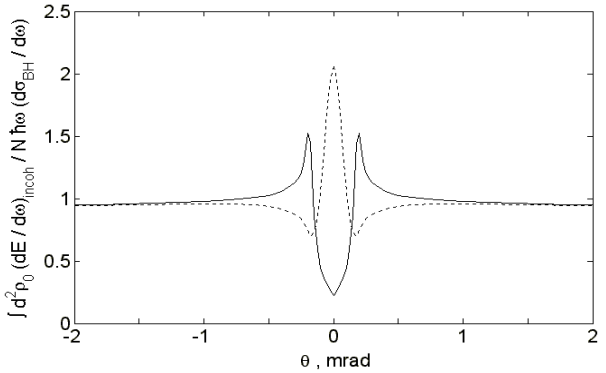

Fig. 2. *The efficiency of incoherent radiation of positrons (solid line) and electrons (dashed line) in the crystal in ratio to that in amorphous medium with the same number N of collisions with atoms as a result of the simulation according to Eq. (8) under the conditions of Fig. 1*

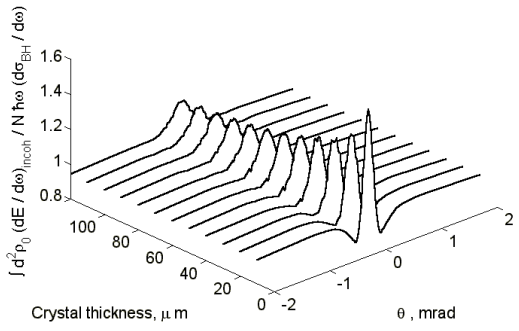

Fig. 3. *Changing of the orientation dependence of the incoherent radiation efficiency by the electrons under the conditions of Fig. 1 with the increase of the crystal thickness*

The electron scattering on the thermal oscillations of the lattice atoms could lead to the dechanneling of the electron, when the motion in the planar channel changes to the above-barrier motion. As a consequence, the maxima and minima described above become less marked (compare the results of simulation with and without account of the incoherent scattering on Fig. 1).

As the crystal thickness grows, the probability of dechanneling of the electrons increases that leads to the gradual decrease of the orientation dependence of the incoherent radiation efficiency (see Fig. 3).

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О НЕКОГЕРЕНТНОМ ИЗЛУЧЕНИИ РЕЛЯТИВИСТСКИХ ЭЛЕКТРОНОВ И ПОЗИТРОНОВ В КРИСТАЛЛЕ

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Некогерентное тормозное излучение электронов высоких энергий в кристалле обусловлено тепловым разбросом атомов относительно их равновесных положений в решетке. В настоящей статье развита процедура моделирования интенсивности некогерентного излучения, основанная на квазиклассических формулах теории тормозного излучения. Показана существенная ориентационная зависимость интенсивности жесткого некогерентного излучения при углах падения частиц на кристаллографическую плоскость, близких к критическому углу плоскостного каналирования. Эффект деканалирования приводит к уменьшению этой ориентационной зависимости. Результаты моделирования находятся в хорошем согласии с данными эксперимента.

ПРО НЕКОГЕРЕНТНЕ ВИПРОМІНЮВАННЯ РЕЛЯТИВИСТСЬКИХ ЕЛЕКТРОНІВ ТА ПОЗИТРОНІВ У КРИСТАЛІ

М.Ф. Шульга, В.В. Сищенко

Некогерентне гальмівне випромінювання електронів високих енергій у кристалі обумовлено термодинамічними флуктуаціями відносно рівноважних положень атомів у гратці. У цієй статті розвинуто процедуру моделювання інтенсивності некогерентного випромінювання, яка основана на квазікласичних формулах теорії гальмівного випромінювання. Показано значну орієнтаційну залежність інтенсивності жорсткого некогерентного випромінювання при кутах падіння частинок на кристалографічну площину, які близкі до критичного кута площинного каналювання. Ефект деканалювання приводить до зменшення цієї орієнтаційної залежністі. Результати моделювання знаходяться у гарному узгодженні з даними експерименту.