CHANGE TIME OF IDENTICAL PARTICLES TUNNELING THROUGH THE RECTANGULAR BARRIER AT THEIR EXCHANGE INTERACTION

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Work is devoted to studying the influence of exchange effects on a time of simultaneous crossing by identical particles through the rectangular quantum barrier. It is shown, that such effects essentially influence on the tunneling parameters. A change of identical particles tunneling time is first computation taking into account their exchange interaction in the field of rectangular quantum barrier.

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1. INTRODUCTION

In given work research of influence of identical particles exchange interaction on time of their simultaneous tunneling through the rectangular quantum barrier is conducted.

Tunneling is one of the most essential quantum-mechanical phenomenas, with which face the developers of modern devices nanotechnology, quantum optics, physicists of the condensed matter and etc. Research of tunneling interesting as in theoretical aspect, by virtue of some unusual features of signal passing through area, "forbidden" with positions of the laws classical physicists (in them impulses of particles accept imaginary values)), so and in practical. The practical interest is stimulated by perspective of development on base of these phenomena new devices, which, possible, will possess the feature, not having analogue amongst material of surrounding us nature.

Attention to problem of the tunneling especially increased in connection with opening of the Hartman effect (1962.). This effect is direct consequence of tunneling Vigner's time definition [1].

As is well known, there are a few definition of tunneling time. Vigner's time (or phase time) is defined as time of the intersection by maximum of the wave package the region of quantum potential barrier:

$$\tau_\phi = \hbar \frac{\partial \phi}{\partial E},$$

where: ϕ – is a phase of a wave transmitted through a barrier, E – energy of particle, incident on a barrier.

Hartman T.E., using the traditional approach in definition of tunneling parameters, has established that Vigner's time of tunneling through a rectangular barrier is determined by the following formula (Hartman's formula):

$$\tau_\phi = \frac{\hbar}{\sqrt{E(U_0 - E)}},$$

where: U_0 – is height of a barrier.

A few important consequences follow from this formula [1].

1. Phase time of tunneling does not depend explicit on weight of a particle, but only from energy.

2. Time of tunneling does not depend on width of a barrier and, hence, at barrier wide enough and at big energies, speed of a particle can exceed speed of light.

The last statement conflicts with the generally accepted position about impossibility of exceeding of light velocity by particles.

There are some experimental results testifying to possibility of display of Hartman's effect [1]. Therefore there is a question, whether has place such effect at scattering of identical particles on each other?

The similar task frequently arises in experiments on scattering in nuclear physics. The simplified model of such process is simultaneous crossing by identical particles the area of a rectangular barrier at their movement in opposite directions. For Coulomb potential such task has been solved by Mott N.F. (Mott's formula) [2].

At research of processes of identical particles tunneling in view of their exchange interaction it is convenient to consider tunneling as a limiting case of elastic scattering.

Particles can have spin (protons, electrons, etc.), or have not spin, as in case of scattering of alpha particles on alpha particles.

Scattering of identical particles has the features [3] which are finding out their fundamental difference from not of identical particles.

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This difference is found out from the circuit resulted on Fig.1.

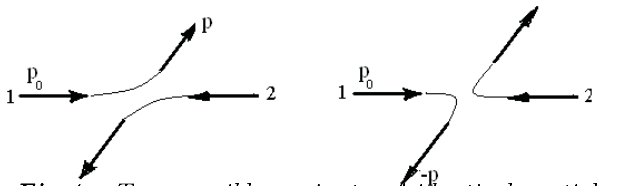

Fig.1. *Two possible variants of identical particles scattering in system of mass center which are not distinct experimentally [3]*

As particles are not distinct from each other, and we cannot establish, which from particles gets in one of counters, at calculation of scattering section it is necessary to summarize amplitudes of probabilities of both processes specified on Fig.1. It means that can be revealed interference constituent, the contribution of which to the section of dispersion will depend on the spin state and from what particles disperse: bosons or fermions

For two identical spinless Bose particles in system of center of mass the effective scattering section of one particle in direction of spatial angle $d\Omega = \sin\theta d\theta d\varphi$ looks like [1]:

$$\sigma(p \leftarrow p_0) = \left|f(p \leftarrow p_0) + f(-p \leftarrow p_0)\right|^2 . \qquad (1)$$

Because events, proper to the hit of particle 1 or particles 2 in some one of counters experimentally not distinguished (they are adequate to amplitudes of probabilities f of events, presented by each from components in a formula (1)), for spherically symmetric interactions have:

$$\begin{aligned} \sigma &= \left|f(E,\theta) + f(E,\pi-\theta)\right|^2 = \\ &= \left|f(E,\theta)\right|^2 + \left|f(E,\pi-\theta)\right|^2 + \\ &\quad + 2f(E,\theta)f^*(E,\pi-\theta) . \end{aligned} \qquad (2)$$

For two not identical particles we would have expression:

$$\sigma = \left|f(E,\theta)\right|^2 + \left|f(E,\pi-\theta)\right|^2 .$$

Thus, for identical particles in section of scattering there is the additional component responsible for quant mechanical effect of interference, observable experimentally in case of scattering only for identical particles. The Mott's formula [2] describing scattering of identical particles in a field of Coulomb potential, also contains additional component, arising by virtue of exchange interaction of particles. For two identical fermions with spin 1/2, scattering in the field of potential, not dependent from spins, wave functions of two-partial system look like:

$$\Psi = \Psi(x_1, x_2)\chi ,$$

where: χ is spinor, responding to singlet state with a spin $s = 1$, $m = +1, 0, -1$ or triplet state with a spin $s = 0$.

In order that a complete wave function was antisymmetric, its coordinate part must be antisymmetric for the triplet state and symmetric in the singlet state.Thus, two identical fermions with a spin in the singlet state are examined at processes of scattering similar two identical spinless Bose particles [3].

For triplet state of polarized fermions a coordinate function must be antisymmetric.

At scattering of non-polarized beams of fermions have [3]:

$$\begin{aligned} \sigma_{non-polarized} &= \frac{1}{4}\sigma_{singlet} + \frac{3}{4}\sigma_{triplet} = \\ &= \left|f(E,\theta)\right|^2 + \left|f(E,\pi-\theta)\right|^2 + \\ &\quad - Re f(E,\theta)f^*(E,\pi-\theta) . \end{aligned}$$

Thus, for non-polarized fermions as well as for triplet state, we have antisymmetric coordinate wave function. As follows from [2], the wave function describing collision of two particles in system of the center of mass can be representing by the following expression:

$$u(\vec{r})_{r\to\infty} \to e^{ikz} + r^{-1}f(\theta,\varphi)e^{i\vec{k}\vec{r}} , \qquad (3)$$

where: r, θ, – spherical coordinates of a vector $\vec{r}$. For identical particles, taking into account necessity symmetrization of wave function of scattering, asymptotic expressions for symmetric and antisymmetric wave functions should be written down as follows:

$$\Psi = (e^{ikz} \pm e^{-ikz}) + \left[f(\theta,\varphi) \pm f(\pi-\theta,\varphi+\pi)\right] r^{-1} e^{i\vec{k}\vec{r}} , \qquad (4)$$

where "+" corresponds to symmetric function, "-" - to antisymmetric function. As follows from (2) at account of identity of particles at their scattering in field of a quantum barrier, to section of scattering of two particles it is necessary to bring in the addition determined third element in formula (2). Just the same difference in the formula of scattering and stipulates a change time of tunneling for identical particles by virtue of their exchange interaction.

2. AN ESTIMATIONS OF THE TIME OF IDENTICAL PARTICLES INTERACTION IN FIELD OF A RECTANGULAR POTENTIAL BARRIER

The circuit of two particles tunneling in one-dimensional variant is represented on Fig.2.

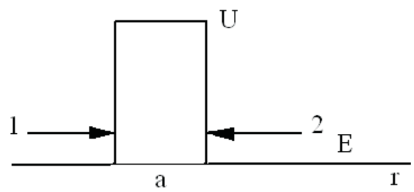

Fig.2. *The circuit of two identical particles interaction in field of a rectangular potential barrier*

If there is a scattering potential, asymptotic form of wave function looks like [4]:

$$r\Psi = \sum_l P_l(\cos\theta) g_l(r) \cong \sum_l A_l P_l(\cos\theta)\sin(kr+\Delta_l)\,, \tag{5}$$

where: $\Delta_l = \delta_l - l\pi/2$. From the scattering theory with use of a method of partial waves [4] follows, that cross-section of scattering not identical particles is described by the formula:

$$\sigma = |f(\theta)|^2 = \frac{1}{k^2}\left|\sum_l \frac{(2l+1)}{2} P_l(\cos\theta)(e^{2i\delta_l}-1)\right|^2 . \tag{6}$$

This formula shows a dependence of cross-section on a phase δ_l. At $l = 0$ angular dependence is absent, and we have the following expression:

$$\sigma = \frac{1}{k^2}\frac{(e^{2i\delta_l}-1)^2}{4} . \tag{7}$$

Taking into account this expression, we shall have:

$$Re(f_b f_c^*) = \frac{1}{4k^2}(e^{2i\delta}-1)(e^{-2i\delta}-1) = \frac{1}{2k^2}(1-\cos 2\delta)\,. \tag{8}$$

Thus, the function describing scattering of two identical particles, will have a next view:

$$\Psi = (e^{ikz} \pm e^{-ikz}) + r^{-1}e^{ikr}\sqrt{\frac{1}{2k^2(1-\cos 2\delta)}} = $$
$$= (e^{ikz} \pm e^{-ikz}) + r^{-1}e^{ikr}\frac{1}{k\sqrt{2}}\sin\delta\,. \tag{9}$$

For small values δ (as it is underlined in [4], at small values k the phase also is small): $\sin\delta \cong \delta \cong e^{\delta} - 1$.

Then we have:

$$\Psi = (e^{ikz} \pm e^{-ikz}) + r^{-1}e^{ikr}\frac{1}{k\sqrt{2}}(e^{\delta}-1)\,. \tag{10}$$

Thus, for scattering of the identical particles described by symmetric wave function, additional shift of a phase appeared approximately equal δ (while usually - for not identical particles, its value 2δ [4]). The additive to time of tunneling in view of exchange interaction of the identical particles described by symmetric wave function, according to [4] makes:

$$\tau_\phi = \frac{1}{\nu_g}\frac{d}{dk}\delta = \frac{m}{\hbar k}\frac{d}{dk}\delta\,. \tag{11}$$

Expressions for δ are resulted in the educational literature for various forms of potential. Having substituted these expressions in (11) it is possible to define the interesting us size of the additive addition to time of tunneling.

In [2, 5] it is shown, that for a barrier with potential

$$V(r) = \begin{cases} U, & r \le a, \\ 0, & r > a. \end{cases}$$

the equation, describing behavior of one particle inside a barrier, looks like:

$$\left(\frac{d^2}{dr^2} + K^2\right)R_{0l}(r) = 0\,, \quad R_{0l}(0) = 0\,, \tag{12}$$

were:

$$K^2 = k^2 - K_0^2\,, \quad K_0^2 = 2mU/\hbar^2\,.$$

Outside of a barrier the decision looks like:

$$R_{0l}(r) = C\sin(kr+\delta_0)\,. \tag{13}$$

Inside of a barrier:

$$\begin{aligned} R_{0l} &= C_l \sin Kr\,, \quad if \quad k \ge K_0\,, \\ R_{0l} &= C_l \sin Qr\,, \quad if \quad k \le K_0\,, \end{aligned} \tag{14}$$

were: $Q = \sqrt{K_0^2 - k^2}$.

If energy of incident particles are small, $Q \approx K_0$ and from a condition of equality of logarithmic derivative functions (13) and (14) it is received (at $ka \ll 1$):

$$\delta_0 = arctg(KD) - ka\,, \tag{15}$$

were:

$$D = \frac{th(Qa)}{Q} \approx \frac{th(K_0 a)}{K_0}\,.$$

From the equation (15) we find:

$$\frac{d\delta_0}{dk} = \frac{D}{k^2D^2+1} - a\,.$$

Then $\Delta\tau_\phi$ (change of particles tunneling time because of their exchange interaction), according to (11) will be expressed by the formula:

$$\Delta\tau_\phi = \frac{m}{hk(E)}\left(\frac{D}{k^2D^2+1} - a\right)\,. \tag{16}$$

For calculations we shall take:

$$D = \frac{th(K_0 a)}{K_0}\,; \quad k^2 = 2mE/\hbar^2\,;$$

$a = 10^{-10}\,cm$; $U = 10^{-11}\,erg$; $E = 2\cdot 10^{-12}\,erg$.

Having substituted these values in (16) it is possible to receive dependences of impulses incident simultaneously on a barrier of identical particles from energy for particles with different masses and to choose the proper region (where $ka \ll 1$) for which the conducted calculations will be correct [2].

On Fig.3 calculation dependences of a delay time $|\Delta\tau_\phi|$ on energy of identical particles are presented. Calculations are lead for particles with mass of electrons (a), protons (b) and α-particles (c). As is obvious from the resulted dependences, exchange interaction reduces absolute value of tunneling time and dependence $|\Delta\tau_\phi|$ on mass is found out. Taking into account the requirement of symmetrization, mentioned above, we shall note, that at scattering of α-particles $|\Delta\tau_\phi|$ negatively, and of fermion's particles - it is necessary to take into account a spin state, since particles can be in singlet or triplet state.

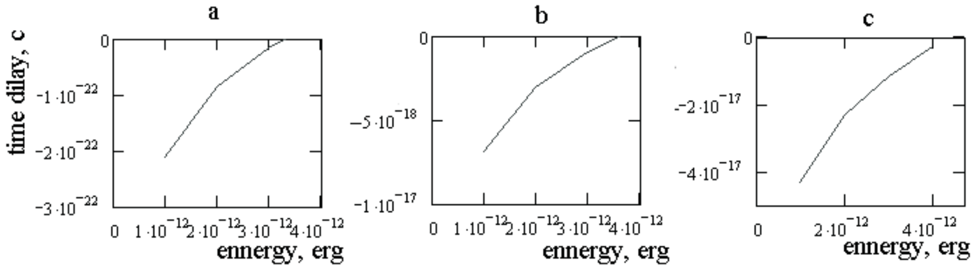

Fig.3. *Dependences of delay time* $|\Delta\tau_\phi|$ *on energy identical particles with mass of electrons (a), protons (b), and α-particles (c) at a condition* $ka \ll 1$

Fig.4 illustrates dependence $|\Delta\tau_\phi|$ on width of barrier for particles with mass of proton.

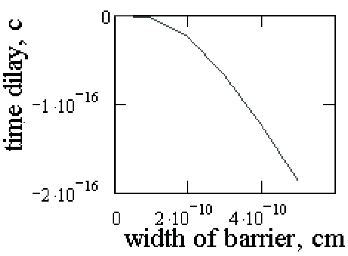

Fig.4. *Change time of tunneling at depending on width of a potential barrier to particles with mass of a proton*

As is obvious from figures 3-4, on change of tunneling time influence both mass of particles, and width of a potential quantum barrier. Thus, the conducted calculations within the framework of applied approaching do not confirm existence of Hartman's effect in case of account of identical particles interaction.

3. CONCLUSIONS

In work the influence of exchange effects on time of simultaneous tunneling identical particles through a rectangular quantum barrier is investigated. It is shown, that such effects essentially influence on a parameters of tunneling. A change of tunneling time of identical particles is first expected taking into account their exchange interaction in a field of rectangular quantum barrier. It is discovered, that additional tunneling time (it can be negative or positive depending on a spin state of particles and, whether scattering bosons or fermions) depends on mass of particles and width of barrier. It means that Hartman's effect does not take place in this case. The received results can be exploited in practical calculations and at designing the devices using processes of quantum tunnel transitions.

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ИЗМЕНЕНИЕ ВРЕМЕНИ ТУННЕЛИРОВАНИЯ ТОЖДЕСТВЕННЫХ ЧАСТИЦ ЧЕРЕЗ ПРЯМОУГОЛЬНЫЙ БАРЬЕР ПРИ ИХ ОБМЕННОМ ВЗАИМОДЕЙСТВИИ

Л. С. Марценюк

Работа посвящена изучению влияния обменных эффектов на время одновременного туннелирования тождественных частиц через прямоугольный квантовый барьер. Показано, что такие эффекты существенно влияют на параметры туннелирования. Впервые рассчитано изменение времени туннелирования тождественных частиц с учетом их обменного взаимодействия в поле прямоугольного квантового барьера.

ЗМІНА ЧАСУ ТУННЕЛЮВАННЯ ТОТОЖНИХ ЧАСТИНОК ЧЕРЕЗ ПРЯМОКУТНИЙ БАР'ЄР ПРИ ЇХ ОБМІННІЙ ВЗАЄМОДІЇ

Л. С. Марценюк

Робота присвячена вивченню впливу обмінних ефектів на час синхронного туннелювання тотожних частинок через прямокутний квантовий бар'єр. Показано, що такі ефекти істотно впливають на параметри туннелювання. Вперше розрахована зміна часу туннелювання тотожних частинок при урахуванні їх обмінної взаємодії в полі прямокутного квантового бар'єру.