STOCHASTIC ACCELERATION OF CHARGED PARTICLES

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Comparison of two schemes of stochastic acceleration of charged particles is carried out: at interacting with the field of wave with randomly changed phase and with the field of regular wave in the conditions of overlapping of non-linear resonances. In the absence of conditions of the resonances overlapping the spectral regions of random field which most effectively transfer their energy to particles are found. In all investigated cases, the presence of overlapping non-linear resonances led to more efficient schemes of stochastic acceleration and heating of particles in comparison with schemes involving random fields.

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1. INTRODUCTION

Now there are two basic mechanisms of stochastic acceleration and heating of charged particles by the fields of electromagnetic waves. First of all it is an energy transfer from random field to particles [1, 4, 5]. Such energy transfer has been specified for the first time in paper [1]. Then such schemes of the energy interchanging for heating and breakdown have been investigated in details in papers by Ya.B. Fainberg and V.I. Karas' (look, for example, [5] and references therein). Such schemes acceleration and heating are effective even in the absence of the resonant interaction of particles and fields. However, when it is possible to fulfill the conditions of the resonant interaction between waves and particles, more attractive scheme of resonant interaction of particles and regular waves under conditions, when the nonlinear resonances overlap, is seemed. The overlaps of nonlinear resonances provide the conditions under which the dynamics of the particles is similar to the dynamics of particles in the random field. Significant is the fact that the field acting on the particle is always greater than the field which acts on the particles in a random field (of course at the same energy acting on the particles of the field). As an example, of the effective heating of the particles by the field of regular waves can be caused by scheme of heating and acceleration of the particles in the plasma traps. Thus, at plasma heating in the magnetic traps, interaction of the charged particles with wave field occurs in narrow region of space. In this region, the particles are in resonance with the wave. After passing this region, they gain energy with high efficiency. Further dynamics of the particles is almost independent on the presence of an electromagnetic wave. Reflected from the magnetic mirrors, the charged particles fall back into the region of the resonant interaction. Repeated the passage of particles through resonance regions leads to their heating. In such scheme of heating it is supposed that at path from the resonance region to the magnetic mirror and back, particle in random way changes the phase relationship with the wave. The randomness origin is not discussed in most cases. Implicitly it is supposed that the regime of dynamic chaos or available fluctuations in the plasma are sufficient for such change. Perhaps these two reasons may exist simultaneously.

Also in the paper [2, 3] plasma heating by the regular field of a laser radiation in the conditions of overlapping of non-linear resonances was considered. In the same place comparing of efficiency of the energy transfer from field to the particles by this mechanism and random fields has been carried out.

It has been shown that heating by the regular fields is much more effective, than heating by random fields. However in this paper comparison has been provided with delta-correlated noise field. In this case energy of a wave is distributed on very broad spectrum ("spread"). In paper [6] attention has been applied on regular waves, phases of which on the average for the period in random way (jump) were changed. It was noted that in this case energy of the wave is concentrated in narrow spectrum. The analysis of particles dynamics in the field of these waves specified on efficiency of the energy transmission of such waves to energy of the charged particles. However the comparative analysis with efficiency of acceleration in regime with dynamic chaos it wasn't carried out. In the present paper we give such analysis.

2. DYNAMICS OF PARTICLES IN THE FIELD OF EXTERNAL ELECTROMAGNETIC WAVE

Let's consider charged particle moving in the external magnetic $\vec{H}_0$ field , guided along z axis and the field of plane electromagnetic wave with arbitrary polarization [2, 3]. Equations of motion may be written in

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the form:

$$\dot{\vec{p}} = (1 - \frac{\vec{k}\vec{p}}{\gamma})Re(\vec{\varepsilon}e^{i\Psi}) + \frac{\omega_H}{\gamma}[\vec{p}\vec{h}] + \frac{\vec{k}}{\gamma}Re(\vec{p}\vec{\varepsilon})e^{i\Psi}\,,$$
$$\dot{\vec{r}} = \vec{p}/\gamma; \qquad \dot{\Psi} = \vec{k}\vec{p}/\gamma - 1\,, \qquad (1)$$

where $\tau \equiv \omega t$, $\vec{h} \equiv \vec{H}/H_0$, $\omega_H \equiv eH_0/mc\omega$, $\Psi \equiv \tau - \vec{k}\vec{r} + \xi(\tau)$, $\xi(\tau)$ is stochastic function. Below we consider separately the dynamics of particle motion in the wave field with random phase and in the wave field when the random phase modulation is absent ($\xi(\tau) = 0$). At small strengths of the field we have non-relativistic motion of the particles. In this case, the solution of equation (1) for the velocity of the particles is expressed analytically. The average rate of energy change of electron under the action of the field pulse $g(t)$ with randomly varying phase can be characterized by the value of average power

$$P(t) = \frac{1}{t - t_0} Re \int_{t0}^{t} dt' g(t')v(t')\,. \qquad (2)$$

Here $v(t) = v_x(t) + iv_y(t) = \int_{t0}^{t} dt' \exp[i\omega_H(t'-t)g(t')$ is solution of the equations of motion (1) in the non-relativistic approximation, and the pulse of a field is $g(t) = \alpha_x E_0 \cos(\Psi(t)) + i\alpha_y E_0 \sin(\Psi(t))$. Thus the average rate of the electron energy change during the pulse for a linearly polarized wave can be represented as the Fourier components of the average spectral power at a frequency ω_H :

$$P_T = \frac{1}{2T}\left|\int_{T/2}^{T/2} Re(g(t'))e^{i\omega_H t'}dt'\right|^2 = \frac{2\pi^2}{T}|\tilde{g}_T\omega_H|^2. \qquad (3)$$

As can be seen from this relation, a maximum energy transfer will occur in the case where the frequency difference $\Delta\omega = \omega - \omega_H$ between the frequency of the external field and the cyclotron frequency is sufficiently small. Thus, at particles interaction with the wave field in the absence of the external magnetic field the maximum efficiency of energy transfer between field wave with randomly varying phase and the particle can be expected in the low frequency region. In the field of the wave with the regular phase of the interaction takes place under resonance condition

$$k_z v_{z0} + s\frac{\omega_H}{\gamma_0} - 1 = 0, \quad s = 0, \pm1, \pm2, ... \qquad (4)$$

When these conditions are fulfilled, shortened equations, describing the motion of the particle in an isolated nonlinear resonance, can be obtained

$$\dot{p}_\perp = \frac{(1 - k_z v_z)}{p_\perp} W_s \varepsilon_0 \cos\vartheta_s, \quad \dot{p}_z = \frac{k_z}{\gamma} W_s \varepsilon_0 \cos\vartheta_s,$$
$$\dot{\theta}_s = \Delta_s \equiv k_z v_z + s\frac{\omega_H}{\gamma} - 1, \qquad \dot{\gamma} = \frac{\varepsilon_0}{\gamma} W_s \cos\vartheta_s, \qquad (5)$$

where:

$$W_s \equiv \alpha_x p_\perp \frac{s}{\mu} J_s(\mu) - \alpha_y p_\perp J_s'(\mu) + \alpha_z p_z J_s(\mu)\,,$$
$$\mu \equiv k_z p_z/\omega_H\,.$$

We suppose that at the interaction of the particles with wave the change of the particle energy is small $\gamma = \gamma_0 + \tilde{\gamma}_s$, $|\tilde{\gamma}_s| << \gamma_0$.

Then, to determine $\tilde{\gamma}_s$ and θ_s one can obtain closed system of equations

$$\frac{d\tilde{\gamma}_s}{dt} = \frac{\varepsilon_0}{\gamma} W_s \cos\vartheta_s \qquad (6)$$

and define the width of the nonlinear isolated resonance

$$\Delta\theta_s = 4\left[\frac{(k_z^2 - 1)\varepsilon_0 W_s}{\gamma_0^2}\right] \; or \; \Delta\tilde{\gamma}_s = 4\left[\frac{\varepsilon_0 W_s}{(k_z^2 - 1)}\right]. \qquad (7)$$

Distance between neighboring resonances

$$\delta\gamma_s = \gamma_{0,s+1} - \gamma_{0,s} = \frac{\omega_H}{(1 - k_z^2)}\,. \qquad (8)$$

Change in the character the particle motion from regular to chaotic, is known to happen under the condition of overlapping of nonlinear resonances. This condition is:

$$\varepsilon_0 \geq \frac{\omega_H}{16W_s(1 - k_z^2)}\,. \qquad (9)$$

3. MODEL OF WAVE WITH RANDOMLY JUMPING OF PHASE

Above, we have an analytical expression for the transfer of energy to the charged particles in a random field for the non-relativistic case. Unfortunately, in the fields of high intensity (relativistic case) analytical expressions are difficult to obtain. Therefore, the following numerical methods will be used for investigation of the dynamics of charged particles in external fields of high tension. In this case, the main difficulty arises in the mathematical modeling of the external fields with random phases. In this section, we consider one of the modeling capabilities of such fields.

For a basis for formation of the wave with chaotically jumping phase, the travelling harmonic wave of kind $f(t, \vec{r}) = a\cos(\omega t - \vec{k}\vec{r} + \varphi_0)$ is taken (regular wave), to phase of which we will add stochastic function of time $\xi(t)$ with probability density having uniform distribution $f(t, \vec{r}) = a\cos(\omega t - \vec{k}\vec{r} + \varphi_0 + \xi(t))$. For a numerical analysis the scheme of numerical calculation which allows to vary the quantity of an interval of phases ($-\pi < \delta\varphi_0 < \pi$) in which there is the jump of phase change, is realized. Also the possibility of selecting the interval of time in which, at random moment of time, the phase jump occurs is realized. Duration of jump is supposed considerably smaller then the wave period. On plots Fig.1, as example, one can see the initial part of realization (length of 1000 period) time dependence of the wave field strength at random jump of the phase at each period of wave for interval of phases jump ($-\pi < \delta\varphi_0 < \pi$) and spectral density of power of this realization. From these plots it is visible that phase jump occurs at random moment of time at each period of the regular wave (Fig.1,a), and quantity of this jump also is random and lies in the range of phases $(-\pi, \pi)$. The spectrum (Fig.1,b) is widened enough with maximum near to unity.

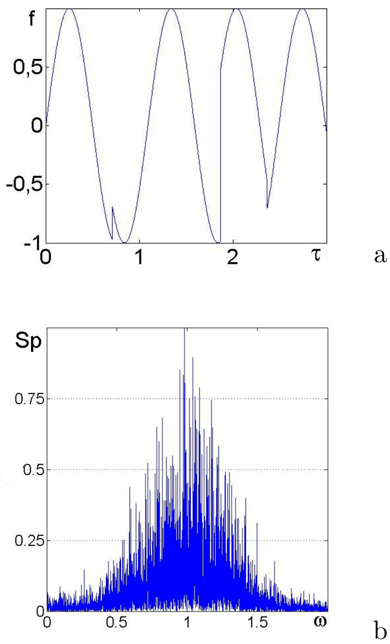


Fig.1. *Field of wave and spectrum*

On plots Fig.2 spectrums of wave with chaotically changing phase are given at various values of interval of time on which there is jumping of phase and quantity of interval of the phase jumps. One can see fromthese plots that, with increasing of the interval

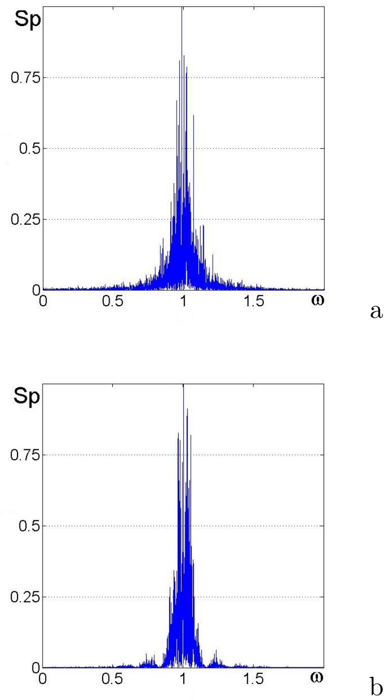


Fig.2. *Spectrum of wave.*
a – single period with jump $(-\pi/2 < \Delta\varphi_0 < \pi/2)$*;*
b – 5-th period with jump $(-\pi < \Delta\varphi_0 < \pi)$

of time on which there is jump of the phase and reduction of an interval of the phase jumps, the spectrum of the wave is considerably narrowed. Spectral band-
width reduction is proportional both to reduction of quantity of jump, and increasing interval of time in which this jump takes place.

4. DYNAMICS OF PARTICLES IN THE FIELD OF WAVE WITH RANDOM CHANGING PHASE

Let's consider charged particle moving in the external magnetic field $\vec{H}_0$, guided along $z-$ axis and the field of plane electromagnetic wave with arbitrary polarization [2, 3]. The dynamics of the particle obeys to the vector equation (1), in which $\xi(t)$ is the random function change under law described in the previous section. Numerical modeling of the particle motion in the field of wave with chaotically changing phase is carried out in the absence of a magnetic field $\vec{H}_0 = 0$ at various intervals of change of the phase jump $(-\pi < \Delta\varphi_0 < \pi)$ and various intervals of time in which, at random moment, there is the phase jump. Time dependence of the energy change for single particle with initial phase Ψ_0 and averaged on ensemble from 30 particles with initial phases from interval $(-\pi < \varphi_0 < \pi\ )$ for case

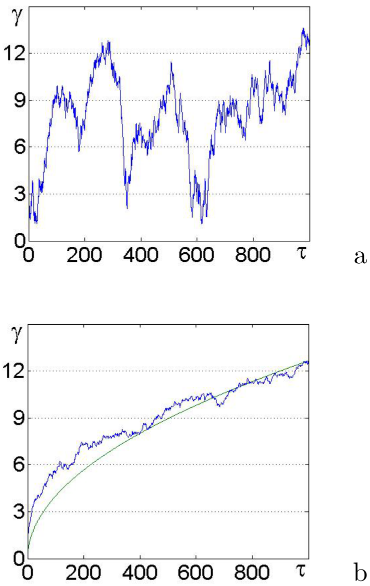
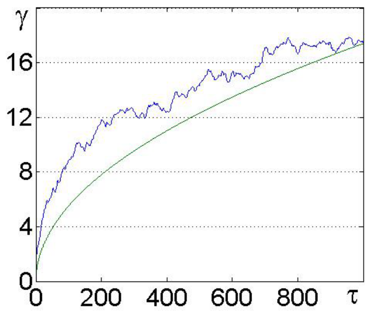


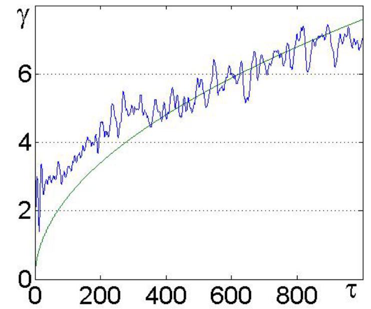
Fig.3. *Energy particle gain at the field.*
a – one particle; b – ensemble averaging

of single jump at period and interval of the phase jumps $((-\pi < \Delta\varphi_0 < \pi\ )$), is presented in Fig.3. On the sameplot, for comparison with the diffuse law of the energies growth with time, the curve of the time dependence of the energy change is given: $\gamma_d = \alpha\sqrt{t}$ at value of coefficient $\alpha = 0,4$. The parameter of the wave force has been chosen $\vec{\varepsilon} = e\vec{E}_0/mc\omega$. From the plot 3a one can see that single particle at interaction with field of wave in random way obtain and lose energy. However at ensemble averaging of particles (particle with various initial phases) certain regularity is observed.

On Fig.4 plots of time dependence of the particles energy, averaged on ensemble of 30 particles for various values of the phases jump $\Delta\varphi_0$ are given.



a



b

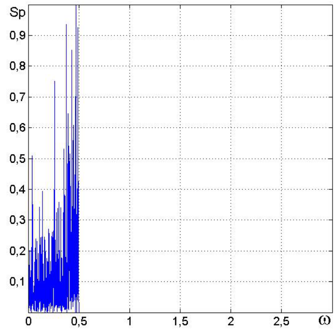
Fig.4. *Energy particle gain at the field with random jumping phase.*
a – single period with jump ($-\pi/2 < \Delta\varphi_0 < \pi/2$);
b – 5-th period with jump ($-\pi < \Delta\varphi_0 < \pi$)

From plots Fig.4 one can see that at ensemble averaging of particles the time dependence of middle energy has character close to diffused — smooth curves $\gamma_d = \alpha\sqrt{t}$ with $\alpha = 0.55$ - plot 4,a and $\alpha = 0.25$ — plot 4,b. There is a certain optimum quantity of an interval of jumps of phase at which heating of particles is most effective.

For a more detailed analysis of the influence of different parts of the spectrum in the dynamics of energy exchange of charged particles with the wave field with randomly changing phase of this wave identified three main region of the spectrum: a low, basic and high frequency. In this case, the missing parts of the frequency spectrum supplemented by zero values. With the help of the inverse Fourier transform has been restored realizations which correspond to each parts of the spectrum. Figs.5, 6, 7 shows plots the spectral power parts of spectrum and the corresponding initial part of restored realization ($-\pi < \varphi_0 < \pi$) for regions low $10^{-3}\omega_0 < \omega < 0.5\omega_0$, basic $0.5\omega_0 < \omega < 1.5\omega_0$ and the high frequency $\omega > 1.5\omega_0$.

For each of the reconstructed field realizations has been investigated the dynamics of particles in these fields for different values of the field amplitude.

For small amplitudes of the field strength of the wave parameter's $\varepsilon \leq 0.01$ or $\varepsilon \leq 0.1$, the main contribution to the energy exchange between the field and the particle is in the low frequency range. Indeed, at small strengths of field nonrelativistic motion of particles takes place. In this case, the average rate of change of the electron energy by pulse field $g(t)$



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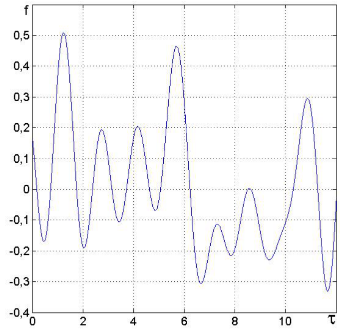
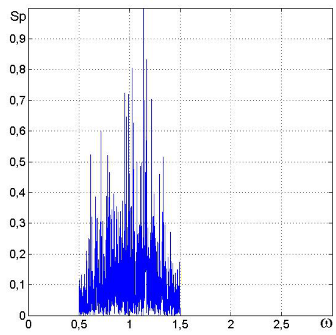


b

Fig.5. *a – low frequency part of the spectrum;*
b – initial part of realization corresponding to the low frequency part of the spectrum



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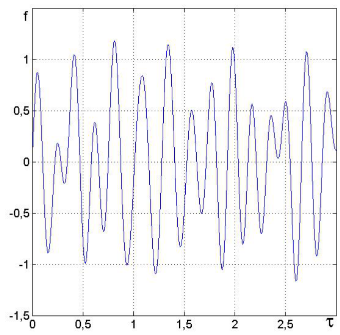
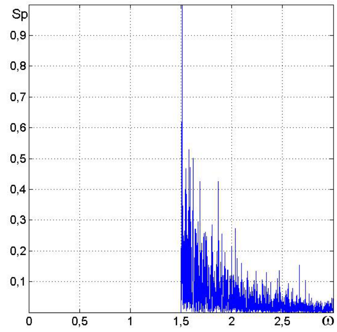


b

Fig.6. *a – main part of the spectrum;*
b – the initial part of realization corresponding to the main part of the spectrum

can be characterized by the value of average power. The expression for it is represented by formula (6).



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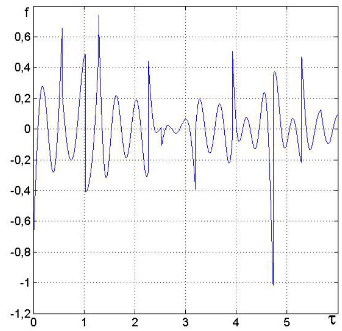

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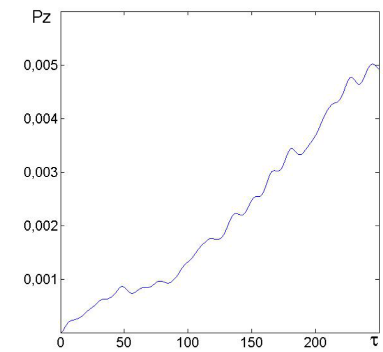
Fig.7. *a – high frequency part of the spectrum;*
b – initial part of realization corresponding to the high frequency main part of the spectrum



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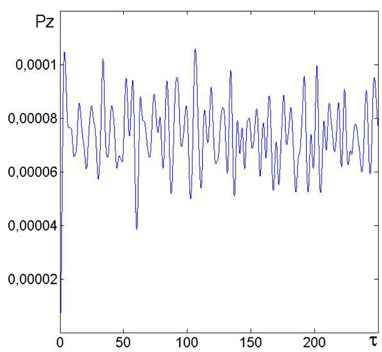
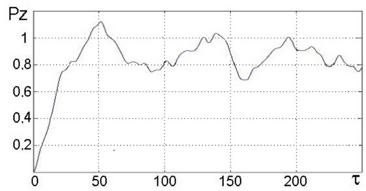


b

Fig.8. *The average pulse* $\varepsilon = 0.01$.
a – the low-frequency part of the spectrum;
b – the main part of the spectrum

As seen from this formula, the maximum energy transfer will occur in the case when the frequency difference $\Delta\omega = \omega - \omega_H$ between the frequency of the external field and the cyclotron frequency is small enough.

Thus, at interaction of particles with field of wave without external magnetic field, maximum efficiency of energy transfer between the field and the particle can be expected in the low frequency region. This is confirmed by numerical analysis. Figs.8-10 shows graphs of the longitudinal momentum



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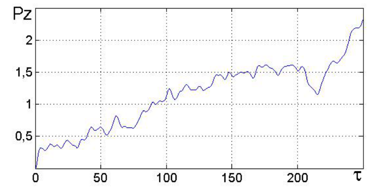

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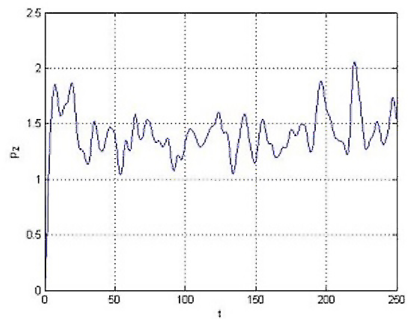
Fig.9. *The average pulse* $\varepsilon = 0.1$.
a – the low-frequency part of the spectrum;
b – the main part of the spectrum



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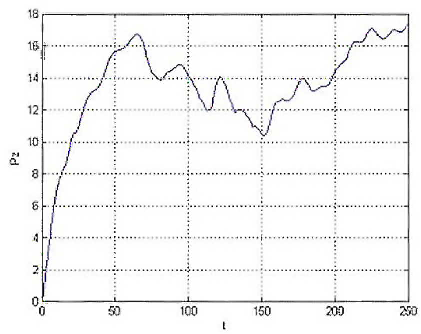


b

Fig.10. *The average pulse* $\varepsilon \geq 1$.
a – the low-frequency part of the spectrum;
b – the main part of the spectrum

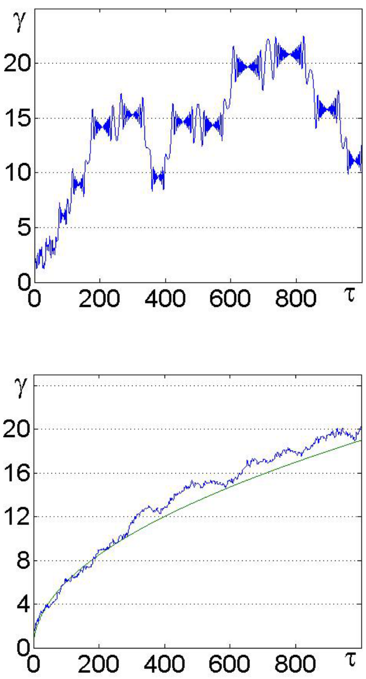
(energy)of the particles from time to time, averaged over an ensemble of 30 particles using the restored realization from various parts of the spectral expansion. Graphics averaged momentum for the high-frequency part of the spectrum are similar graphs for the middle part of the spectrum. As seen from these graphs for small field amplitude ($\varepsilon \geq 1$) a major role in the energy exchange of particles with particle field has low frequency.

At large values of the field amplitude ($\varepsilon \geq 1$) dynamics of the energy changes qualitatively. The main contribution to the energy transfer makes the main part of the spectrum (see Fig.10).

Thus, it is possible to speak that there is certain transitional process that consisting in the changing of the spectrum region, which provides the main contribution to the exchange energy of the particle and the field. This transitional process takes place in the interval of the field strength variation from $\varepsilon = 0.1$ to $\varepsilon = 1$.

5. DYNAMICS OF PARTICLES IN THE FIELD OF THE REGULAR WAVE IN THE CONDITIONS OF NON-LINEAR RESONANCES OVERLAPPING

Let's consider the dynamics of particles in the field of plane electromagnetic wave and in external constant magnetic field without random jumps of phase. This dynamics is described by the equations (1) at $\vec{H}_0 \neq 0$ and $\xi(t) = 0$. For maintenance of conditions of non-linear resonances overlapping, and also for the subsequent comparison of two methods of heating, parameter of the wave force is $\vec{\varepsilon} = 1$. In these conditions dynamics of charged particle, has stochastic character, with the characteristic random gain and loss of energy, Fig.11. However, as well as in the case with phase jumps, energy of the ensemble averaging particles



a

b

Fig.11. *Energy particle gain at overlap resonances. a – one particle; b – ensemble averaging*

energy changes under the law close to diffusion low with coefficient $\alpha = 0.6$ (Fig.11,b).

6. CONCLUSIONS

Finally, we will compare the efficiency of gain of the energy particles in random field and in the regular wave (in conditions overlapping of the resonances). At such comparison we will proceed from equality of energy in the wave with jumping phase and in the regular wave $W_N = W_r$. If the amplitude of the regular and random waves are the same, as we have seen above, gain of the particles energy in these fields is approximately the same $\Delta\gamma_N \approx \Delta\gamma_R$.

Energy of the noise wave: $W_N = \varepsilon_N^2 \Delta\omega_N$, $\Delta\omega_N \sim 0.4$. Energy of the regular wave: $W_R = \varepsilon_R^2 \Delta\omega_R$, $\Delta\omega_R \sim 1/Q$, Q - is quality factor. As $1/Q = 10^{-2}...10^{-7}$, higher level of energy of wave is necessary for achievement the same level of the particles energy in the field of wave with jumping phase. Greater efficiency of stochastic gain of energy of charged particles by field of the regular wave in the conditions of overlapping of non-linear resonances is caused by narrow spectral line of such radiation, and also presence of cyclotron resonances. It is possible to expect that at particle motion in the field of regular waves in the conditions of cyclotron resonances presence of rare random jumps, even without overlapping of non-linear resonances, will be also effective. In this case the spectrum is narrowed. The particle moves in an intensive field. Preliminary investigations of such dynamics of particles really show the efficiency of this scheme of stochastic gain of energy. Let's notice that such scheme of stochastic gain of energy is similar to the scheme of heating of particles in magnetic traps. Efficient difference is the long-term motion of particles in synchronism with wave (resonances) at which the gain of velocity of particles is proportional to time of resonant interaction with field. Short jumps in this case play a role of the phase of particle loss, relatively to the wave at motion from mirror to mirror in traps.

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СТОХАСТИЧЕСКОЕ УСКОРЕНИЕ ЗАРЯЖЕННЫХ ЧАСТИЦ

В. А. Буц, В. В. Кузьмин, А. П. Толстолужский

Проведено сравнение двух схем ускорения заряженных частиц – при взаимодействии с полем волны со случайно изменяющейся фазой и с полем регулярной волны в условиях перекрытия нелинейных циклотронных резонансов. Во всех рассмотренных случаях эффективность ускорения при выполнении условий перекрытия нелинейных циклотронных резонансов оказывается более высокой.

СТОХАСТИЧНЕ ПРИСКОРЕННЯ ЗАРЯДЖЕНИХ ЧАСТИНОК

В. О. Буц, В. В. Кузьмін, О. П. Толстолужський

Проведено порівняння двох схем прискорення заряджених частинок – при взаємодії з полем хвилі з фазою, що випадково змінюється, й з полем регулярної хвилі в умовах перекриття нелінійних циклотронних резонансів. У всіх розглянутих випадках ефективність прискорення при виконанні умов перекриття нелінійних циклотронних резонансів виявляється більш високою.